Noise Induced State Transitions of 2D Interacting, Self-propelled Particle Systems

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We consider a system of self-propelled agents interacting via pairwise attractive and repulsive Morse potentials and subject to gaussian noise in two dimensions. Earlier work showed that depending on interaction parameters, a catastrophic and an H–stable regime could arise, with diverse aggregation morphologies including mills, rings, clumps, flocks in the catastrophic regime and rigid body rotators and flocks in the H–stable one. Here, we consider both regimes and investigate the role of noise in promoting transitions between patterns, and in causing swarm disassembly. We find that within the catastrophic regime increasing noise intensity leads to a first order transition between translational flocks and compact stationary swarms and yet higher noise levels lead to swarm breakup. Within the H-stable regime instead we find .... Hysterisis.

The aggregation of self–propelled particles into coherent patterns is a ubiquitous process found in many chemical, physical, biological, and engineered systems [1]. Patterns may be static or dynamic and may arise over several spatio–temporal scales. For example, interacting droplets or ferromagnetic particles floating on fluid layers may form ordered structures [2, 3], organic molecules adsorbed on surfaces may self assemble into monolayers [4] while actin filaments, cells, myxobacteria and flagellated bacteria may form colonies, swarms or biofilms [5, 6, 7]. Interactions among members of these ensembles may be electrostatic, chemotactic, hydrophobic, electromagnetic, of Van der Waals type, and may also depend on the surrounding environment or cell culture.

More complex organisms such as insects, animals and humans also self-assemble, forming schools of fish, flocks of birds, locust swarms, or moving crowds that have inspired a new generation of mathematical modelers. In all these systems, agents organize following direct visual, tactile, auditory or other sensory couplings, giving rise to coherent bodies that may impart protection, enhanced mobility or other advantages to their members.

As our understanding of biological and biologically inspired self-assembly increases there is also great interest in applying this knowledge to design and create non–biological inanimate systems with novel properties and control possibilities [8]. Swarms of multiple, task-specific entities, such as unmanned land vehicles, search robots, underwater gliders, aerial drones have been tested. Expanded fabrication capabilities at the nano and mesoscale levels may also lead to the possibility of creating swarming nanostructures to monitor the presence of pathogens in seawater [9], or for biomedical purposes within the human body [10].

Beginning with the seminal work of Viczek and collaborators in the mid-ninties, and using the basic ingredients of direct interaction among agents, self–propulsion and the absence of central coordination, many discrete,

rule-based, kinetic and hydrodynamic models have been presented. Some of these descriptions have considered idealized, portable swarming systems, while others have focused on specific organisms or vehicles, introducing ad–hoc behaviors, experimentally derived parameters and validated field testing.

One of the most studied discrete models within the swarming literature was initially introduced in Ref. **??** and is characterized by a set of individual self-propelled particles interacting via a repulsive–attractive potential. The deterministic model has been studied in detail and several characterizations of the resulting morphologies and their behavior as a function of parameter choices have been presented in the literature, both in two and three dimensions. In this paper we consider the same system subject to random noise and illustrate behaviors and trends

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I. MODEL

We model our multi-particle swarm by solving the following differential equations that describe each individual particle's position and velocity respectively

$$\frac{d\overrightarrow{x_i}}{dt} = \overrightarrow{v_i} \tag{1}$$

$$m_i \frac{d\overrightarrow{v_i}}{dt} = \alpha \overrightarrow{v_i} - \beta |\overrightarrow{v_i}|^2 \overrightarrow{v_i} - \overrightarrow{\bigtriangledown} U_i \tag{2}$$

where m_i is the mass of the particle under consideration, making (2) represent the force on the particle. The first two terms represent the self-propulsion term subject to a frictional force. Assuming a particle is alone, its absolute velocity will accelerate or decelerate to $\sqrt{\frac{\alpha}{\beta}}$ and remain

there, therefore α and β can be used to tune the intrinsic terminal velocity of each particle. The potential term, U_i, is given by the Morse potential

$$U_i = \sum_{j \neq i} -C_a e^{\frac{|\overrightarrow{x}_i - \overrightarrow{x}_j|}{l_a}} + C_r e^{\frac{|\overrightarrow{x}_i - \overrightarrow{x}_j|}{l_r}} \qquad (3)$$

and describes the pairwise interaction between the current particle and all the surrounding particles. C_a and C_r represent the amplitude of the each pairwise forces of attraction and repulsion respectively as l_a and l_r represent the range of the these pairwise forces. The relationship between the parameter space of $C = \frac{C_r}{C_a}$ and $l = \frac{l_r}{l_a}$ and the swarm's stationary lattice structures has been thoroughly investigated [?]. The parameter space can be partitioned into regions where swarms have fundamentally similar morphological structures as in Figure ?. For this paper, we focus on the biologically relevant, catastrophic region labeled VII. The term catastrophic refers to the contracting behavior of the radius of the swarm as the number of particles increases, differing from H-stable swarms that do not contract or collapse [?]. In this parameter region, we find the swarm settles on one of two steady state structures: a coherent flock or a circular mill, as illustrated in Figure ?. In the flocking configuration, every particle and subsequently the center of mass of the swarm has absolute velocity given by $\sqrt{\frac{\alpha}{\beta}}$. The same is true for each particle in the milling configuration, but depending on the radius of the particle's orbit around the center of mass, the particle's angular velocity varies. When given random initial positions and velocities, the choice of steady state structures the swarm makes is highly dependent on the initial amplitude of the velocity of the center of mass, where significantly large velocity will increase the likely-hood of the swarm settling into a flock.

In order to implement noise into the system, we alter equation (2) to

$$m_i \frac{d\overrightarrow{v_i}}{dt} = \alpha \overrightarrow{v_i} - \beta |\overrightarrow{v_i}|^2 \overrightarrow{v_i} - \overrightarrow{\nabla} U_i + \overrightarrow{\xi_i}(t) \qquad (4)$$

where $\overrightarrow{\xi_i}(t)$ is a vector valued noise function associated with the given particle. We choose a Gaussian distribution for $\overrightarrow{\xi_i}(t)$ where each component has a normal distribution with a mean of 0 and variance of σ^2 that we vary in our experiment. For a single particle, or when $U_i = 0, \forall i$, we already know that $|\overrightarrow{v_i}(t)| = \sqrt{\frac{\alpha}{\beta}}$ when $\overrightarrow{\xi_i}(t) = 0$. But with a non-zero noise term, the particle velocity jitters around the value $\sqrt{\frac{\alpha}{\beta}}$, while the direction drifts as a continuous random walk. Because of the

drift in direction, when a whole swarm is in a flocking configuration, a change in the direction of any individual from the already established direction of the flock will decrease the component of the center of mass's velocity in that direction. Since the component parallel to the flock's direction of the center of mass is the dominant term of the absolute velocity, any jitter in the individuals result in a net decrease in the absolute velocity of the center of mass. In other words, if $\overrightarrow{v^*}$ is the velocity of the center of mass of a swarm initialized with a flocking configuration and $\sigma^2 = 0$, then $|\overrightarrow{v^*}| = \sqrt{\frac{\alpha}{\beta}}$. But as we increase σ^2, we'll see $|\overrightarrow{v^*}|$ approach 0.

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