Consider a two-dimensional spatial domain Ω. Let $s(\mathbf{x},t)$ represent the density of solitary locusts, and $g(\mathbf{x},t)$ the density of gregarious ones. Our model accounts for movement and for flux from one phase to the other. The equations take the form

$$\dot{s} + \nabla \cdot (v_s s) = -f_2(\rho)s + f_1(\rho)g \tag{1a}$$
$$\dot{g} + \nabla \cdot (v_g g) = \quad f_2(\rho)s - f_1(\rho)g. \tag{1b}$$

Define the total density

$$\rho = s + g \tag{2}$$

and the total mass

$$M = \int_\Omega \rho \, d\Omega. \tag{3}$$

Solitarious locusts should display repulsion, and gregarious locusts should display attraction with short range repulsion. Assuming pairwise, superposed social interactions, we have the velocity terms

$$v_s = -\nabla(Q_s * \rho) \tag{4a}$$
$$v_g = -\nabla(Q_g * \rho) \tag{4b}$$

where the repulsive potential Q_s and the attractive potential Q_g are

$$Q_s = R_s \mathrm{e}^{-|x|/r_s} \tag{5a}$$
$$Q_g = R_g \mathrm{e}^{-|x|/r_g} - A_g \mathrm{e}^{-|x|/r_a}. \tag{5b}$$

Now consider the density dependent rates of solitarization f_1 and gregarization f_2. We take

$$f_1(\rho) = \frac{\delta_1}{1 + (\rho/k_1)^2} \tag{6a}$$
$$f_2(\rho) = \frac{\delta_2 (\rho/k_2)^2}{1 + (\rho/k_2)^2}. \tag{6b}$$

This model has ten parameters, $\delta_{1,2}$, $k_{1,2}$, $R_{s,g}$, $r_{s,g}$, A_g, and a_g. To reduce the number of parameters, we nondimensionalize. Let

$$\widetilde{\rho} = \rho/k_1, \qquad \widetilde{M} = M/k_1, \qquad \widetilde{g} = g/k_1, \qquad \widetilde{s} = s/k_1, \qquad \widetilde{t} = x/r_s, \qquad \widetilde{t} = t\delta_1. \tag{7}$$

Then define new, dimensionless parameters

$$\widetilde{\delta}_2 = \delta_2/\delta_1, \quad \widetilde{k}_2 = k_2/k_1, \quad \widetilde{r}_g = r_g/r_s, \quad \widetilde{a}_g = a_g/r_s, \quad \widetilde{R}_{s,g} = R_{s,g}/\beta, \quad \widetilde{A}_g = A_g/\beta \tag{8}$$

where for convenience we define

$$\beta = \frac{r_s^2 \delta_1}{k_1}. \tag{9}$$

We substitute the nondimensionalization into (1) through (6) and drop hats on variables and parameters to obtain

$$\dot{s} + \nabla \cdot (v_s s) = -f_2(\rho)s + f_1(\rho)g \tag{10a}$$
$$\dot{g} + \nabla \cdot (v_g g) = \quad f_2(\rho)s - f_1(\rho)g \tag{10b}$$

where

$$v_s = -\nabla(Q_s * \rho) \tag{11a}$$
$$v_g = -\nabla(Q_g * \rho) \tag{11b}$$

with

$$Q_s = R_s \mathrm{e}^{-|x|} \tag{12a}$$
$$Q_g = R_g \mathrm{e}^{-|x|/r_g} - A_g \mathrm{e}^{-|x|/r_a} \tag{12b}$$

and

$$f_1(\rho) = \frac{1}{1+\rho^2} \tag{13a}$$
$$f_2(\rho) = \frac{\delta_2 \left(\rho/k_2\right)^2}{1+\left(\rho/k_2\right)^2}. \tag{13b}$$

Ω now signifies the new nondimensionalized spatial domain, whose area we call A.

For this dimensionless model, define the total number of solitary locusts and gregarious locusts,

$$S = \int_\Omega s \, d\Omega \tag{14a}$$
$$G = \int_\Omega g \, d\Omega \tag{14b}$$

so that

$$S + G = M. \tag{15}$$

In simulations of the particle system analogous to (10), we observe mass-balanced states in which gregarious and solitarious locusts segregate. We attempt a rough calculation of such solutions. The solitarious locusts are spread throughout most of Ω, covering an area approximately equal to A. The gregarious locusts are concentrated in a clump whose area we call α, which may presumably be estimated from the gregarious potential (12b). Therefore, the local densities that solitarious and gregarious locusts will sense in their respective patches are

$$s = S/A, \qquad g = G/\alpha. \tag{16}$$

At mass balance, for the segregated state, the number flux (as opposed to density flux) of gregarious locusts becoming solitarized per unit time is $f_1(G/\alpha)\cdot G$. Similarly, the number flux of solitarious locusts becoming gregarized is $f_2(S/A)\cdot S$. Equating these expressions and substituting from (13), we have

$$\frac{G}{1+\left(G/\alpha\right)^2} = \frac{\delta_2 S^3/\left(Ak_2\right)^2}{1+S^2/\left(Ak_2\right)^2}. \tag{17}$$

To find the mass-balanced states, we must solve (17). To simplify this calculation, we define

$$\widehat{S} = S/M, \qquad \widehat{G} = G/M, \tag{18}$$

so that

$$\widehat{S} + \widehat{G} = 1. \tag{19}$$

Substituting (18) into (17) and dividing through by M yields

$$\frac{\widehat{G}}{1 + c_3\widehat{G}^2} = \frac{c_1\widehat{S}^3}{1 + c_2\widehat{S}^2} \tag{20}$$

where

$$c_1 = \frac{\delta_2 M^2}{A^2 k_2^2}, \qquad c_2 = \frac{M^2}{A^2 k_2^2}, \qquad c_3 = \frac{M^2}{\alpha^2}. \tag{21}$$