

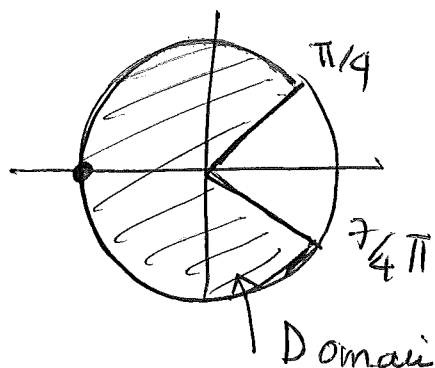
$$f(x) = x + \cot\left(\frac{x}{2}\right)$$

$$\left[\frac{\pi}{4}, \frac{7\pi}{4}\right] \quad \# 56 \text{ pag } 205$$

$$f'(x) = 1 + \left(-\csc^2\left(\frac{x}{2}\right)\right) \frac{1}{2}$$

$$= 1 - \frac{1}{\left(\sin^2 \frac{x}{2}\right) \cdot 2}$$

$$= \frac{2\sin^2 \frac{x}{2} - 1}{2\sin^2 \frac{x}{2}}$$



exclude

$$\sin \frac{x}{2} = 0$$

$$\frac{x}{2} \neq 0, \pi, 2\pi$$

$$x \neq 2\pi, 4\pi, 0$$

OK

critical points:

$$\sin^2 \frac{x}{2} = \frac{1}{2}$$

$$f'(x) = 0$$

$$\sin^2 \frac{x}{2} = 0$$

$$f'(x) \text{ DNE}$$

$$\sin \frac{x}{2} = \pm \frac{1}{\sqrt{2}}$$

$$\frac{x}{2} = \frac{\pi}{4}, \frac{3\pi}{4}$$

$$\sin \frac{x}{2} = +\frac{1}{\sqrt{2}}$$

$$\frac{x}{2} = \frac{5\pi}{4}, \frac{7\pi}{4}$$

$$\sin \frac{x}{2} = -\frac{1}{\sqrt{2}}$$

$$x = \frac{\pi}{2}, \frac{3}{2}\pi \quad \text{on!}$$

$$x = \frac{5}{2}\pi, \frac{7}{2}\pi \quad \text{outside of domain}$$

$$\sin \frac{x}{2} = 0 \quad \frac{x}{2} = 0, \pi, 2\pi \quad \text{outside of domain}$$

check

$$f\left(\frac{\pi}{2}\right) = \frac{\pi}{2} + \frac{\cos \pi/2}{\sin \pi/2} = \left(\frac{\pi}{2}\right) \approx 1.5 \quad (\text{min})$$

$$f\left(\frac{3}{2}\pi\right) = \frac{3}{2}\pi + \frac{\cos 3/2\pi}{\sin 3/2\pi} = \left(\frac{3}{2}\pi\right) \approx 4.6 \quad (\text{max})$$

$$f\left(\frac{\pi}{4}\right) = \frac{\pi}{4} + (1) = \frac{\pi}{4} + 1 \approx 1.8$$

$$f\left(\frac{7}{4}\pi\right) = \frac{7}{4}\pi + (-1) = \frac{7\pi}{4} - 1 \approx 4.6$$

compare:

$$\frac{7}{4}\pi - 1 \stackrel{?}{>} \frac{3}{2}\pi$$

$$\frac{7\pi}{4} - 1 \stackrel{?}{>} \frac{6\pi}{4} \quad \pi > 4 \quad \text{no!} \quad = 0$$

$$\frac{7}{4}\pi - 1 < \frac{3}{2}\pi \quad \uparrow \quad \text{max}$$