Evaluate the following integrals:

1. $\int_0^{\pi/4} \frac{x \sin x}{\cos^3 x}\, dx$

2. $\int \frac{12^{\sqrt{x}}}{\sqrt{x}}\, dx$

3. $\int \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}\, d\theta$

4. $\int_0^{\ln 10} \frac{e^x \sqrt{e^x - 1}}{e^x + 8}$

5. $\int x\sqrt{x+1}$

6. $\int (x^3 + 4x)\, e^{(x^4 + 8x^2)}\, dx$

7. $\int \tan^5 x \sec^4 x\, dx$

8. $\int_0^{1/\sqrt{2}} y \arctan y^2\, dy$

9. $\int \frac{\sin x}{1 + \cos x}\, dx$

10. $\int \frac{dx}{\sqrt{1-4x^2}}$

11. $\int_0^1 \frac{1}{2-3x}\,dx$

12. $\int_0^1 \frac{x-1}{2\sqrt{x}}\,dx$

13. $\int_0^4 \frac{\ln x}{\sqrt{x}}\,dx$

14. $\int_1^2 x^5 \ln x\,dx$

15. $\int \frac{x+1}{9x^2+6x+5}\,dx$

16. Find the length of the curve defined by $y = \frac{x^4}{16} + \frac{1}{2x^2}$ for $1 \le x \le 2$

17. same as above for $y = 2\ln\left(\sin\frac{x}{2}\right)$ for $\frac{\pi}{3} \le x \le \pi$

18. Sketch the polar curve

$r = \sin 4\theta$

19 Find the polar equation for

$x + y = 2, \quad x^2 + y^2 = 2$

20 Where does the following curve have vertical or horizontal tangents?

$$x = 2b\cos t - b\cos 2t$$
$$y = 2b\sin t - b\sin 2t$$

21. Consider

$$x = \frac{3t}{1+t^3} \qquad y = \frac{3t^2}{1+t^3}$$

Show that if (a, b) belongs to the curve, so does (b, a). Find the points on the curve where the tangent lines are horizontal or vertical. Show that a Cartesian representation is $x^3 + y^3 = 3xy$. Show that the polar representation is $r = \dfrac{3\sec\theta\tan\theta}{1+\tan^3\theta}$

22. Determine whether the following series converge, or diverge. If the series is convergent, find its limit.

22 $a_n = \dfrac{n^3 + 2}{2n^3 + 1}$

23 $a_n = \dfrac{9^{n+1}}{11^n}$

24 $a_n = \dfrac{2n^3}{1 + n^2}$

25 $a_n = \dfrac{\ln n}{\sqrt{n}}$

26 $a_n = \left(1 + \dfrac{4}{n}\right)^{4n}$

27 $a_n = \dfrac{n \sin n}{n^2 + 2}$

Find the sum

28 $\displaystyle\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{2^{3n+1}}$

29 $\sum_{n=1}^{\infty} \frac{1}{n(n+3)}$

30 $\sum_{n=1}^{\infty} \left[\tan^{-1}(n+1) - \tan^{-1} n \right]$

31 $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^n}{4^{2n}}$

32 $\sum_{n=0}^{\infty} n\left(\frac{1}{2}\right)^n$

33 $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$

34. Determine whether the following are convergent or divergent

$$\sum_{n=2}^{\infty} \frac{1}{n^2-1}$$

Use the decomposition

$$\frac{1}{n^2-1} = \frac{3}{4} + \frac{A}{2n} + \frac{B}{2n+2}$$

to find A and B and eventually the value of the sum. Show

$$\sum_{n=2}^{\infty} \frac{1}{n^2-1} = \frac{3}{4}$$

35. Using the above evaluate

$$\sum_{n=4}^{\infty} \frac{1}{n^2-1}$$

36. Determine whether the following is convergent or ~~divergent~~ divergent:

$$\sum_{n=0}^{\infty} \frac{4n^2 - n^3}{10 + 2n^2}$$

37. Find the following sum using partial fractions:

$$\sum_{n=1}^{\infty} \frac{1}{n^2 + 4n + 3}$$

38. Find the following

$$\sum_{n=1}^{\infty} \frac{4}{3(n^2+4n+3)} - 9^{-n+2}\,4^{n+1}$$

39. Determine if the following is convergent or divergent

$$\sum_{n=0}^{\infty} n\bar{e}^{n^2}$$

40. Same as 40) for

$$\sum_{n=0}^{\infty} \frac{1}{n+3^n}$$

41. Consider

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$$

differentiate both sides with respect to x to prove that

$$\sum_{n=1}^{\infty} n x^{n-1} = + \frac{1}{(1-x)^2} \quad , \text{ for } |x|<1$$

42. Calculate

$$\sum_{n=1}^{\infty} n\left(\frac{1}{3}\right)^n$$

43. Determine convergence or divergence

$$\sum_{n=1}^{\infty} \frac{n}{n^2 - \cos^2 n}$$

44. Same as above for

$$\sum_{n=2}^{\infty} \frac{4n^2 + n}{\sqrt[3]{n^7 + n^3}}$$

45. Find the limit of the sequence

$$\lim_{n\to\infty} \left(1 + \frac{2}{n}\right)^n$$

46. Determine if the sequence is increasing, decreasing or non-monotonic. Is the sequence bounded?

46. $a_n = (-2)^{n+1}$

47. $a_n = \dfrac{2n-3}{3n+4}$

48. $a_n = n + \dfrac{1}{n}$

49. $a_n = \dfrac{2n}{n^2+1}$

50. $a_n = 2 + \dfrac{(-1)^n}{n}$