1) Given that x, x^2 and $1/x$ are solutions of the homogeneous solution corresponding to $x^2y''' + x^2y'' - 2xy' + 2y = 2x^4$, find the solution to the full non homogeneous system.

2) Find a general formula for a particular solution of the differential equation $y''' - 3y'' + 3y' - y = g(t)$. Find the solution if $g(t) = e^t t^{-2}$.

3) Find the general solution of $y''' + 5y'' + 6y' + 2y = 0$

4) Consider the harmonic oscillator system in the presence of an external force:

$$mu'' + ku = F_0 \cos(\omega t)$$

i) Find the general solution for $\omega \neq \omega_0$ where $\omega_0 = \sqrt{k/m}$ such that $u(t = 0) = 0$ and $u'(t = 0) = 0$.
ii) Use the trigonometric identities for $\cos(2A - 2B) = -2\sin(A + B)\sin(A - B)$ to re-express your solution. Consider the case where ω is very close to ω_0. Of the two sinusiodal curves, which one is the fastest oscillating? You can consider the other sinusoidal term an amplitude modulation. Sketch the solution.
iii) Consider now the limit of your solution as $\omega \to \omega_0$. Find the solution from the beginning with $\omega_= \omega_0$ and check that this solution agrees with the limit of the previous one.

5) Solve the following $y'' + 4y = 3\sin 2t$ with $y(0) = 2$ and $y'(0) = -1$.

6) Using the series solution method find the solution to $(4-x^2)y''+2y = 0$ about the point $x_0 = 0$.

7) Solve the following: $(3y^2 + 2xy)dx = (2xy + x^2)dy$

8) Consider the Riccati equation $y' = q_1(t) + q_2(t)y + q_3(t)y^2$ where q_1, q_2, q_3 are functions of t. Suppose that a particular solution $y_1(t)$ is known. A more general solution containing one arbitrary constant can be obtained through the substitution $y(t) = y_1(t) + \frac{1}{v(t)}$. Show that $v(t)$ satisifes the differential equation $v' = -(q_2 + 2q_3y_1)v - q_3$. Note that $v(t)$ will contain an arbitrary constant.

9) Using the method outlined in 8) given the particular solution y_1 solve the following Riccati equation $y' = 1 + t^2 - 2ty + y^2 \qquad y_1(t) = t$.

10) Determine whether the following functions form a linearly dependent set of solutions or not. If they are not independent express one of them in terms of the others:

$$f_1(t) = 2t - 3, \qquad f_2(t) = t^2 + 1, \qquad f_3(t) = 2t^2 - t, \qquad f_4(t) = t^2 + t + 1$$