Math 150A Midterm 3

(Dated: November 21, 2016)

Name:

SOLUTIONS

SID:

MATH 150 A

Midterm 3

Write clearly and box all your answers. Simplify all formulas to the very end. No calculators allowed. Do not work out of memory, rather think before starting your calculations. Use the back for more space. Show all steps you are performing.

1) A 3D box will be built where the sum of the length, height and width must be 108 inches. What are the dimensions and the volume of a square based box with the greatest volume under this constraint?

$l + w + h = 108$ SQUARE BASED $\downarrow$ $l = w$

$2w + h = 108$

$V = w^2 h$ with $h = 108 - 2w$

$= w^2(108 - 2w) = 108w^2 - 2w^3$ MAXIMIZE!

0 36

$V' = 2 \cdot 108 w - 2 \cdot 3 w^2 = 2w(108 - 3w)$

max at $w = 36$!

$h = 108 - 72 = 36$

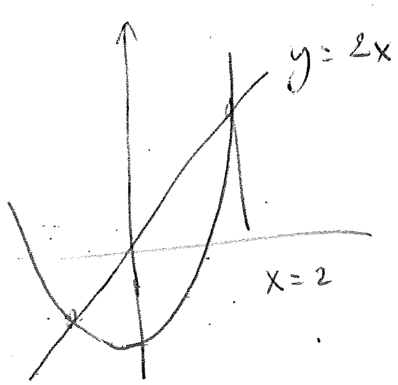
CUBE! $V = 36^3$

2) Evaluate $\lim\limits_{x \to 0} \dfrac{1 - \cos 3x}{x^2}$

$\frac{0}{0}$

$\lim\limits_{x \to 0} \dfrac{+\sin 3x \cdot 3}{2x} = \lim\limits_{x \to 0} \dfrac{3}{2} \cdot \dfrac{3 \cos 3x}{1} = \dfrac{9}{2}$

3) Find the area of the region bounded by $y = 2x$ and $y = 2x^2 - 4$



$2x = 2x^2 - 4$

$x^2 - x - 2 = 0$

$(x - 2)(x + 1) = 0$

$x = 2 \quad x = -1$

$$\int_{-1}^{2} (2x - 2x^2 + 4)\, dx = x^2 - \frac{2}{3}x^3 + 4x \Big|_{-1}^{2} = 4 - \frac{16}{3} + 8 - 1 - \frac{2}{3} + 4$$

$$= \frac{45 - 18}{3} = \frac{27}{3} = 9$$

4) Evaluate $\int_0^4 x(x-2)(x-4)dx$ =

$$\int_0^4 (x^3 + 8x - 6x^2)dx = \frac{1}{4}x^4 + 4x^2 - 2x^3 \Big|_0^4$$

$$= 4^3 + 4^3 - 2\cdot 4^3 = 0$$

5) Evaluate $\int_0^{\pi/4} \frac{\sin\theta}{\cos^3\theta}d\theta$

$\cos\theta = u$

$-\sin\theta \, d\theta = du$

$$\int_1^{\frac{1}{\sqrt{2}}} \frac{-du}{u^3} = \int_{\frac{1}{\sqrt{2}}}^1 \frac{du}{u^3} = -\frac{1}{2}u^{-2}\Big|_{1/\sqrt{2}}^1$$

$$= -\frac{1}{2} + \frac{1}{2}\frac{1}{(1/\sqrt{2})^2} =$$

$$-\frac{1}{2} + \frac{1}{2}2 = \frac{1}{2}$$

if $\theta = \pi/4$ $u = \frac{1}{\sqrt{2}}$

$\theta = 0$ $u = 1$

6) Evaluate $\int_0^3 \frac{x^2+1}{\sqrt{x^3+3x+4}}dx$

$x^3 + 3x + 4 = u$

$(3x^2+3)dx = du$

$(x^2+1)dx = du/3$

$$= \int_4^{40} \frac{du}{3\sqrt{u}} = \frac{1}{3}\left(2u^{1/2}\right)\Big|_4^{40}$$

$u^{-1/2}$

$$= \frac{2}{3}\left(\sqrt{40} - \sqrt{4}\right)$$

$$= \frac{4}{3}\left(\sqrt{10} - 1\right)$$

$x = 3$; $u = 27 + 9 + 4$
$= 40$

$x = 0$ $u = 4$

7) Find the antiderivative F of $f(x) = x^5 - 2x^{-2} + 1$ such that $F(1) = 0$

$$F = \frac{1}{6}x^6 + 2\,x^{-1} + x + C \qquad F(1) = 0$$

$$0 = \frac{1}{6} + 2 + 1 + C \qquad C = -3 - \frac{1}{6} = \frac{-19}{6}$$

$$\boxed{F = \frac{1}{6}x^6 + \frac{2}{x} + x - \frac{19}{6}}$$

8) What two nonnegative real numbers with a sum of 23 have the largest possible product?

$$S = x + y = 23 \qquad y = 23 - x) \qquad x = y = 23/2$$

$$P = x \cdot y = \text{maximize}$$

$$P = x(23 - x) \qquad P' = 23 - 2x \qquad \geq 0$$

$23/2$

max at $23/2$

9) Evaluate $\int \frac{x}{\sqrt{4 - 9x^2}}\,dx$

$$4 - 9x^2 = u$$
$$-18x\,dx = du$$
$$x\,dx = -\frac{du}{18}$$

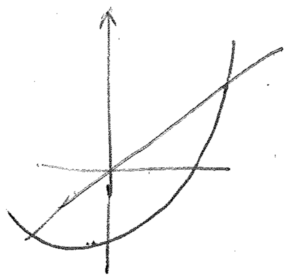
$$\int \frac{-du}{18\sqrt{u}} \qquad u^{-1/2}$$

$$= -\frac{1}{18}\left(2u^{1/2}\right) + C$$

$$= -\frac{1}{9}u^{1/2} + C$$

$$\boxed{-\frac{1}{9}\sqrt{4 - 9x^2} + C}$$

10) Find the area between $y = x$ and $y = x^2 - 2$



$x^2 - 2 - x = 0 \quad (x-2)(x+1) = 0$

$x = 2$
$x = -1$ intersect

$$\int_{-1}^{2} (x - x^2 + 2)\,dx = \left(\frac{1}{2}x^2 - \frac{1}{3}x^3 + 2x\right)\Big|_{-1}^{2}$$

$$= 2 - \frac{8}{3} + 4 - \frac{1}{2} - \frac{1}{3} + 2 = 8 - 3 - \frac{1}{2} = \frac{9}{2}$$

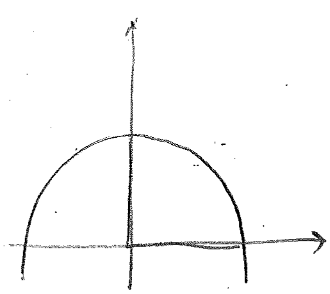
$5 - \frac{1}{2}$

note it is exactly half of part 3

11) Evaluate $\lim_{x \to 0} \frac{2 \tan x}{\sec^2 x}$

$$\lim_{x \to 0} \frac{2 \sin x \cos^{\cancel{2}} x}{\cancel{\cos x}} = 0$$

12) Find the area of the region above the x axis and bounded by $f(x) = 4 - x^2$.



$x = 2$

$$2\int_{0}^{2} (4 - x^2)\,dx$$

$$= 2\left(4x - \frac{1}{3}x^3\right)\Big|_{0}^{2}$$

$$= 2\left(8 - \frac{8}{3}\right) = 16 \cdot \frac{2}{3} = \frac{32}{3}$$