Math 150A Midterm 2

(Dated: October 27, 2016)

Name:

SOLUTION

SID:

Write clearly and box all your answers. Simplify all formulas to the very end. No calculators allowed. Do not work out of memory, rather think before starting your calculations. Use the back for more space. Show all steps you are performing.

no

1) Graph $f(x) = \dfrac{x^2+12}{2x+1}$. Check for all asymptotes, vertical, horizontal and slant. Any cusps?

$x \neq -\frac{1}{2}$; neither odd/even; $\lim_{x\to\infty} f(x) = \infty$

$\lim_{x\to-\infty} f(x) = -\infty$

slant asymptote: $\lim_{x\to\infty} f(x)/x = m = \lim_{x\to\infty} \dfrac{x^2+12}{(2x+1)x} = \dfrac{1}{2} = m$

$\lim_{x\to\infty} f(x) - \frac{1}{2}x = b = \lim_{x\to\infty} \dfrac{x^2+12}{2x+1} - \dfrac{x}{2}$

$= \dfrac{\cancel{2x^2}+24-\cancel{2x^2}-x}{2(2x+1)} = -\dfrac{1}{4}$

$\boxed{y = \frac{1}{2}x - \frac{1}{4}}$ slant

$\lim_{x\to-\frac{1}{2}^-} f(x) = \dfrac{\text{pos}}{0^-} = -\infty$ $\qquad \lim_{x\to-\frac{1}{2}^+} f(x) = \dfrac{\text{pos}}{0^+} = +\infty$

$f'(x) = \dfrac{2x(2x+1) - 2(x^2+12)}{(2x+1)^2} = \dfrac{4x^2+2x-2x^2-24}{(2x+1)^2} = \dfrac{2(x^2+x-12)}{(2x+1)^2}$

$= \dfrac{2(x+4)(x-3)}{(2x+1)^2}$

$-\frac{1}{2}$

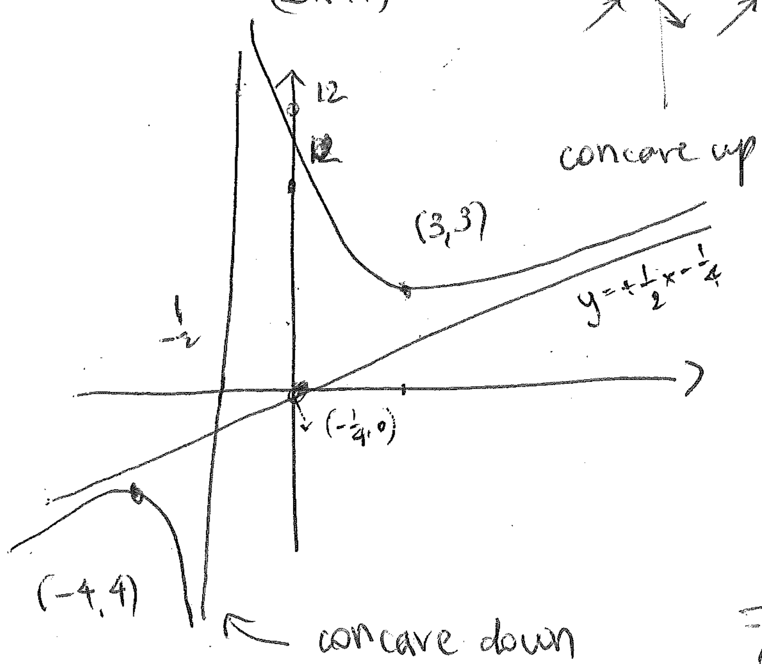
$-4 \quad 3$

$+ \quad - \quad - \quad +$

↗ ↘ ↘ ↗

max $\left(-4;\ \frac{28}{-7} = -4\right)$

min $\left(3;\ \frac{21}{7} = 3\right)$



$f''(x) = 2\,\dfrac{(2x+1)(2x+1)^2 - 2(2x+1)\,2(x+4)(x-3)}{(2x+1)^4}$

$= 2\,\dfrac{(2x+1)\left[\cancel{4x^2}+\cancel{4x}+1-\cancel{4x^2}-\cancel{4x}+48\right]}{(2x+1)^4}$

$= \dfrac{2(2x+1)\cdot 49}{(2x+1)^4}$

$-\frac{1}{2}$

$\cap \quad \cup$

no

2) Graph $f(x) = \frac{1+x}{\sqrt{x}}$. Check for all asymptotes, vertical, horizontal and slant. Any cusps?

$x \neq 0$

$\boxed{x > 0}$

$\lim_{x \to \infty} f(x) = \infty$

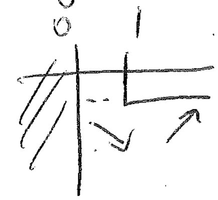
$\lim_{x \to -\infty} f(x)$ DNE!

$f(x) = x^{-\frac{1}{2}} + x^{\frac{1}{2}}$

$f'(x) = -\frac{1}{2} x^{-3/2} + \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2} x^{-3/2}(-1 + x)$

$\frac{x\, x^{-1/2}}{x}$

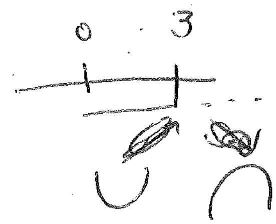
$f'(x) \geq 0$ for $x \geq 1$

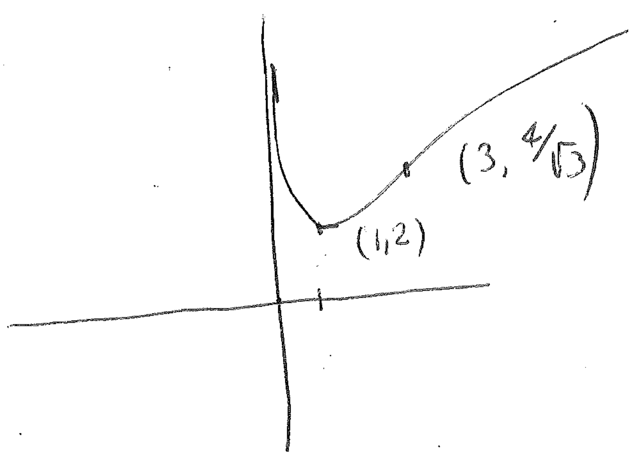


min @ $x = 1$, $y = 2$ $(1, 2)$

$f''(x) = +\frac{3}{4} x^{-5/2} - \frac{1}{4} x^{-3/2} = \frac{1}{4} x^{-5/2}(3 - x)$

$x^{-1} \cdot x$





$\lim_{x \to 0^+} \frac{1+x}{\sqrt{x}} = \frac{\text{pos}}{0^+} = +\infty$

inflection point

$\left(3, \frac{4}{\sqrt{3}}\right)$

3) Locate the critical points of $f(x) = -x^3 + 9x$; identify its absolute maximum and minimum on $[-4, 3]$.

$f'(x) = -3x^2 + 9 = -3(x^2 - 3) = -3(x - \sqrt{3})(x + 3)$

$-4 \quad -\sqrt{3} \quad \sqrt{3} \quad 3$

$(x - \sqrt{3})$
$(x + \sqrt{3})$
-1

$- \quad + \quad -$

$\searrow \quad \nearrow \quad \searrow$

local min $(-\sqrt{3}, -\sqrt{3} \cdot 6)$

local max $(\sqrt{3}, +\sqrt{3} \cdot 6)$

check

$(-4, -36 + 64) = (-4, 28)$

$(3, 0) \qquad (3, 0)$

$(-\sqrt{3}, -6\sqrt{3})$ ABSOLUTE MIN

$(\sqrt{3}, 6\sqrt{3})$ LOCAL MAX

$(-4, 28)$ ABS MAX

$(3, 0)$

4) Locate the critical points of $f(x) = \sin 3x$; identify its absolute maximum and minimum on $[-\frac{\pi}{4}, \frac{\pi}{3}]$.

$f'(x) = 3\cos 3x = 0$ at $3x = \frac{\pi}{2} + n\pi$

$x = \frac{\pi}{6} + \frac{n\pi}{3}$

$x = \underline{\frac{\pi}{6}}$; ; $\cancel{\frac{\pi}{3} + \frac{\pi}{6}}$, $\frac{\pi}{6} - \frac{\pi}{3} = \underline{-\frac{\pi}{6}}$

not in interval

$-\frac{\pi}{4} \quad -\frac{\pi}{6} \quad \pi/6 \quad \frac{\pi}{3}$

$f'(x) \geq 0$ if $\cos 3x$ in 1^{st} or 4^{th}

$-\frac{\pi}{2} \leq 3x \leq \frac{\pi}{2} \qquad -\frac{\pi}{6} \leq x \leq \frac{\pi}{6}$

$\boxed{\text{MAX } f(\frac{\pi}{6}) = 1}$ $\boxed{f(-\frac{\pi}{6}) = -1}$ MIN

$f(\frac{\pi}{3}) = 0 \qquad f(-\frac{\pi}{4}) = -\frac{1}{\sqrt{2}}$

5) A rectangle with area $A = 64$ has perimeter $P(x) = 2x + 128/x$ where x is the length of one side of the rectangle. Find the absolute minimum of the perimeter. What kind of rectangle is this?

$xy = 64$

$P(x) = 2x + \frac{128}{x}$

$P'(x) = 2 - \frac{128}{x^2}$

$\frac{2(x^2 - 64)}{x^2}$

$(x - 8)$
$(x + 8)$

$-8 \quad 0 \quad 8$

$\searrow \quad \nearrow$

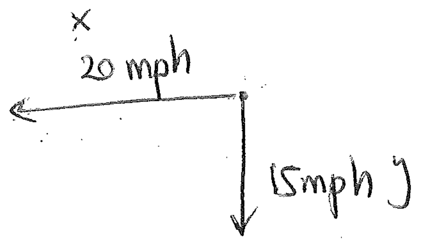
min @ $x = 8$

if $A = 64 \Rightarrow y = 8$

rectangle is a square!

$\frac{dD}{dt} = 25$ mph $\qquad D = \text{dist}$

6) Two boats leave port at the same time. One travels west at 20 mph. The other travels south at 15 mph. At what rate is the distance between them changing 30 min after they leave port?



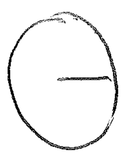
@ $t = \frac{1}{2}$ h $\qquad x = 10$ miles, $y = \frac{15}{2}$ miles

$D = \text{dist} = \sqrt{x^2+y^2}$

$\frac{d\,\text{dist}}{dt} = \frac{\cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt}}{\cancel{2}\sqrt{\ }} = \frac{10 \cdot 20 + \frac{15}{2} \cdot 15}{\sqrt{10^2 + 15^2/4}} =$

$= 200 + 225/2 \Big/ \sqrt{100 + 225/4} = \frac{625}{2} \cdot \frac{2}{\sqrt{625}} = 25$

7) A circle has a 50 foot radius. It begins shrinking at a rate of 2 feet per minute. What is the rate of change of the area when the radius is 10 feet? [Hint: What is the sign of the rate of change if shrinking?]



$A = \pi r^2 \qquad \frac{dr}{dt} = -2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi r(-2) = -4\pi r \Big|_{r=10} = -40\pi$

8) Use implicit differentiation to find $y'(x)$ for $\sin x \cos y = \sin x + \cos y$.

$\cos x \cos y - \sin x \sin y \cdot y' = \cos x - \sin y \cdot y'$

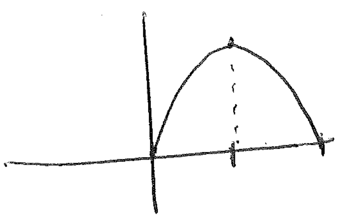
$y' = \frac{\cos x - \cos x \cos y}{\sin y - \sin x \sin y} = \frac{\cos x(1-\cos y)}{\sin y(1-\sin x)}$

9) Find the derivative of $\sin(4\cos x)$.

$-4\cos(4\cos x)\sin x$

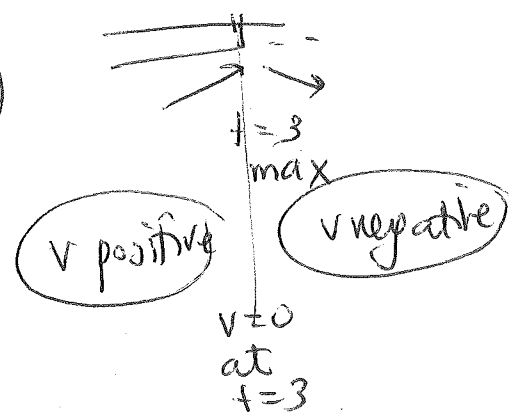
10) The motion of an object as a function of time is given by $f(t) = 18t - 3t^2$ where $t \geq 0$. Graph $f(t)$ and determine when is its velocity increasing, decreasing or stationary. Determine its acceleration.

$f(t) = 18t - 3t^2$ $\quad v = \frac{-18}{-6} = 3 \quad (3, 54-27) = (3, 27)$



$v = f'(t) = 18 - 6t$

$6(3-t)$



$f''(t) = a = -6$

11) What is $\lim_{x\to 0} \frac{\sin x}{x}$? Using this result, find $\lim_{x\to 0} \frac{\tan 7x}{x}$ and $\lim_{x\to 0} \frac{\sin 3x}{\tan 4x}$.

$= 1$

$\lim_{x\to 0} \frac{\sin 7x}{\cos 7x \cdot 7x} \cdot 7 = 7$ ←

$\lim_{x\to 0} \frac{\sin 3x}{\sin 4x} \cdot \frac{\cos 4x}{3x} \cdot \frac{3x}{4x} \cdot (4x)$

$\frac{3}{4}$ ←