Math 150A Midterm 1

(Dated: September 28th 2016)

Name: SOLUTION

SID:

Write clearly and box all your answers. Simplify all formulas to the very end. No calculators allowed. Do not work out of memory, rather think before starting your calculations. Use the back for more space. Show all steps you are performing.

1) Use the $x+h$ definition of the limit to find the equation of the tangent line at the point $x=1$ of the function $f(x)=\sqrt{x+3}$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\sqrt{x+3+h}-\sqrt{x+3}}{h}$$

$$=\lim_{h\to 0}\frac{(\cancel{x}+\cancel{3}+\cancel{h})-(\cancel{x}+\cancel{3})}{\cancel{h}[(\sqrt{x+3+h})+\sqrt{x+3})}=\frac{1}{2\sqrt{x+3}}\quad @\ x=1=\frac{1}{4}$$

$(1,2)$

$$y=\frac{1}{4}x+b\qquad 2=\frac{1}{4}+b\qquad b=\frac{7}{4}\qquad \boxed{y=\frac{1}{4}x+7/4}$$

2) Evaluate $\lim\limits_{x\to 0}\frac{3}{\sqrt{16+3x}+4}$

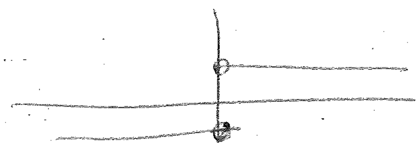
$$=\boxed{\frac{3}{8}}$$

3) Find the $\lim_{x\to 1^+}$, $\lim_{x\to 1^-}$ for $f(x)=\frac{x-2}{(x-1)^3}$ and then state whether $\lim_{x\to 1} f(x)$ exists or not

$$\lim_{x\to 1^+}\frac{x-2}{(x-1)^3}=-\infty\qquad \lim_{x\to 1^-}\frac{x-2}{(x-1)^3}=+\infty$$

$$\lim_{x\to 1}\frac{x-2}{(x-1)^3}=\text{DNE}$$

4) Consider the function $f(x) = \frac{|x|}{x}$ for $x \neq 0$. Sketch its graph on the interval $[-3, 3]$ and determine whether $\lim_{x\to 0} f(x)$ exists or not



$\lim_{x\to 0} f(x)$ DNE as

$\lim_{x\to 0^+} f(x) = 1$

and they are different

$\lim_{x\to 0^-} f(x) = -1$

5) Find the slant asymptote of the curve $f(x) = \frac{3x^2 - 2x + 5}{3x + 4}$

$$m = \lim_{x\to\infty} \frac{3x^2 - 2x + 5}{(3x+4)\,x} = 1$$

$$b = \lim_{x\to\infty} f(x) - mx = \lim_{x\to\infty} \frac{3x^2 - 2x + 5}{(3x+4)} - x =$$

$$\lim_{x\to\infty} \frac{\cancel{3x^2} - 2x + 5 - \cancel{3x^2} - 4x}{3x+4} = -\frac{6}{3} = -2 \quad \boxed{y = x - 2}$$

6) Let $f(x) = x^2 - 4$, $g(x) = x^3$, $F(x) = \frac{1}{x-3}$. Evaluate $g \circ F \circ f(x)$, and $F \circ F(x)$. Simplify to the end.

$$g(F(f(x))) = g\left(\frac{1}{x^2 - 7}\right) = \boxed{\left(\frac{1}{x^2-7}\right)^3} = \frac{1}{x^6 - 3 \cdot 7x^4 + 3 \cdot 49x^2 - 49 \cdot 7}$$

$$= \frac{1}{x^6 - 21x^4 + 147x^2 - 343}$$

$$F \circ F = \frac{1}{\frac{1}{x-3} - 3} = \frac{x-3}{1 - 3x + 9} = \boxed{\frac{x-3}{10 - 3x}}$$

10) Show that $-|x| \le x \sin \frac{1}{x} \le |x|$ for $x \neq 0$. Use the sqeeze theorem to prove that $\lim_{x\to 0} x \sin \frac{1}{x} = 0$.

$-1 \le \sin\frac{1}{x} \le 1$ multipl(i)es by $|x|$

$-|x| \le |x| \sin\frac{1}{x} \le |x|$

$-|x| \le (x) \sin\frac{1}{x} \le |x| \sin\frac{1}{x} \le |x|$

$\Rightarrow -|x| \le x \sin\frac{1}{x} \le |x|$ $\lim_{x\to 0}$ on all sides

$\Rightarrow 0 \le \lim_{x\to 0}(\sin\frac{1}{x})x \le 0$

BY SQUEEZE THM.

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11) Consider $f(x) = \dfrac{\sqrt{x^2+2x+6}-3}{x-1}$. Find $\lim_{x\to\infty} f(x)$, $\lim_{x\to-\infty} f(x)$ and all the vertical asymptotes $x = a$ of $f(x)$. For each asymptote evaluate $\lim_{x\to a} f(x)$.

$$\lim_{x\to\infty} \frac{\sqrt{x^2+2x+6}-3}{x-1} = \frac{|x|\left(\sqrt{1+\cancel{\frac{2}{x^2}}+\cancel{\frac{6}{x^2}}}-\cancel{\frac{3}{|x|}}\right)}{x-1} = +1$$

$$\lim_{x\to-\infty} \frac{\sqrt{x^2+2x+6}-3}{x-1} = \frac{|x|\left(\sqrt{1+\cancel{\frac{2}{x^2}}+\cancel{\frac{6}{x^2}}}-\cancel{\frac{3}{|x|}}\right)}{x-1} = -1$$

$x=1$ V. Asymptote? $\lim_{x\to 1} \dfrac{x^2+2x+6-9}{(x-1)(\sqrt{x^2+2x+6}+3)} = \dfrac{(x+3)\cancel{(x-1)}}{\cancel{(x-1)}\sqrt{}}$

$\downarrow$

$\frac{4}{6} = \frac{2}{3}$

12) Identify the symmetry of the functions $f(x) = x^5 - x^3 - 2$, $g(x) = x^4 + 5x^2 - 12$, $h(x) = x|x|$.

$f(x)$ neither odd nor even

$g(x)$ even ; $g(-x) = g(x)$

$h(x)$ odd $h(-x) = -h(-x)$

7) Using the a definition of the derivative, find the equation of the tangent line to the point $a = 1$ of the function $f(x) = 1/x^2$.

$$\lim_{x\to a} \frac{f(x)-f(a)}{x-a} = \lim_{x\to a} \frac{\frac{1}{x^2}-\frac{1}{a^2}}{x-a} = \lim_{x\to a} \frac{\overset{-1}{\cancel{(a-x)}}(a+x)}{a^2x^2\cancel{(x-a)}}$$

$$= -\frac{2a}{a^4} = -\frac{2}{a^3} \quad @ \quad a=1 \quad \underline{-2} \quad m=-2$$

$(1,1)$

$y = -2x+b$

$1 = -2+b$

$b=3$

$$\boxed{y=-2x+3}$$

8) Compute $\lim_{x\to -5^+} f(x)$, $\lim_{x\to -5^-} f(x)$, $\lim_{x\to -5} f(x)$, $\lim_{x\to 5^+} f(x)$, $\lim_{x\to 5^-} f(x)$, $\lim_{x\to 5} f(x)$, for

A B C D E F

$$f(x) = \begin{cases} 0 & \text{if } x \le -5 \\ \sqrt{25-x^2} & \text{if } -5 < x < 5 \\ \sqrt{3x} & \text{if } x \ge 5 \end{cases} \tag{1}$$

A $= 0$

B $= 0$

C $= 0$ since A $=$ B

E $= \sqrt{15}$

F $= 0$

G $=$ DNE since

E $\neq$ F

9) Evaluate $\lim_{x\to a} = \dfrac{x^3-a^3}{x-a}$.

$$= \lim_{x\to a} \frac{\cancel{(x-a)}(x^2+ax+a^2)}{\cancel{(x-a)}} = \boxed{3a^2}$$