Math 150A Midterm 3

(Dated: November 24 2014)

Name:

MAKA

SID:

Write clearly and box all your answers. Simplify all formulas to the very end. No calculators allowed. Do not work out of memory, rather think before starting your calculations. Use the back for more space. Show all steps you are performing.

1) The sum of two positive numbers is 16. What is the smallest possible value of the sum of their squares?

$x\,,\; y = 16 - x$

$x^2 + (16 - x)^2 = $ sum of squares

$\cancel{2}x + \cancel{2}(16 - x) = 0$
(-1)

$x - 16 + x \geq 0 \qquad x - 8 \geq 0$

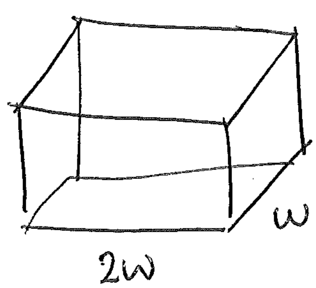
8

min

sum min $\quad x = 8,\; y = 8$

$\boxed{S = 128}$

2) A rectangular storage container with an open top is to have a volume of 10 m^3. The length of its base is twice the width. Material for the base costs \$10 per square meter. Material for the sides costs \$6 per square meter. Find the cost of materials for the chapest such container.



$V = 10\ m^3$

$10 = 2w^2 h \qquad h = 5/w^2$

base area $= 2w^2 \qquad$ cost $\quad 10 \cdot 2w^2 = \underline{20w^2}$

side $= (2w \cdot h + wh) \cdot 2 \qquad$ cost $\quad 12(2wh + wh) = 36wh$

$20w^2 + 36\cancel{w}\dfrac{5}{w^{\cancel{2}}} = 20w^2 + \dfrac{180}{w}$

$= 20\left(w^2 + \dfrac{9}{w}\right)$

min = $\quad 2w - \dfrac{9}{w^2} = \dfrac{2w^3 - 9}{w^2}$

$w \geq \sqrt[3]{9/2}$

$= 20\left[\left(\tfrac{9}{2}\right)^{2/3} + \dfrac{9}{\sqrt[3]{9}}\sqrt[3]{2}\right] = \boxed{20\left(\tfrac{9}{2}\right)^{2/3} + 180\left(\tfrac{2}{9}\right)^{1/3}}$

3) Find f if $f'' = 6x + \sin x$

$$f' = 3x^2 - \cos x + C$$

$$f = x^3 - \sin x + Cx + D$$

4) Evaluate the integral $\int_0^2 (y-1)(2y+1)dy$

$$\int_0^2 (2y^2 + y - 2y - 1)\,dy = \int_0^2 2y^2 - y - 1$$

$$= \frac{2}{3}y^3 - \frac{1}{2}y^2 - y \Big|_0^2$$

$$= \frac{16}{3} - 2 - 2 = \frac{16-12}{3} = 4/3$$

5) Evaluate the integral $\int_1^9 \frac{3x-2}{\sqrt{x}}dx$

$$\int_1^9 \left(3\sqrt{x} - 2x^{-1/2}\right)dx$$

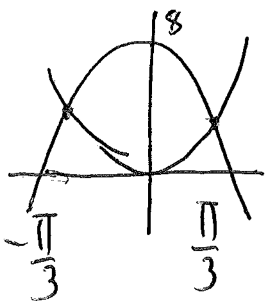
$$= \cancel{3}\cdot\frac{2}{\cancel{3}}x^{3/2} - 2\frac{2}{\cancel{1}}x^{1/2} = 2x^{3/2} - 4x^{1/2}\Big|_1^9 = 44$$

$$= 2\cdot 27 - 4\cdot 3 - 2 + 4 = 54 + 4 - 14$$

6) Evaluate the integral $\int x^2 \cos(x^3)dx$

$$= \frac{1}{3}\sin x^3 + C$$

7) Sketch the region enclosed by the given curves and find its area $y = \sec^2 x$, $y = 8\cos x$, $-\frac{\pi}{3} \leq x\frac{\pi}{3}$.



$$\sec^2 x = 8\cos x \Rightarrow \cos^3 x = \frac{1}{8}$$

$$\cos x = \pm\frac{1}{2}$$

$$A = 2\int_0^{\pi/3} (8\cos x - \sec^2 x)\,dx = 2\left[8\sin x - \tan x\right]\Big|_0^{\pi/3}$$

$$= 2\left[\frac{\cancel{8}^{4}\cdot\sqrt{3}}{\cancel{2}} - \sqrt{3}\right) = 6\sqrt{3}$$

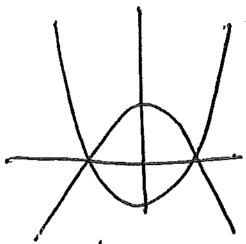
8) Find the average of the function on the given interval $y(t) = t^2(1+t^3)^4$ between $[0, 4]$

$$\frac{1}{4}\int_0^4 t^2(1+t^3)^4\,dt$$

$$t^3 + 1 = u$$
$$3t^2\,dt = du$$

$$\frac{1}{4}\int_1^{65} u^4\,du\,\frac{1}{3} = \frac{1}{5}\,\frac{1}{12}\left(65^5 - 1\right) = \frac{65^5 - 1}{60}$$

9) Sketch the region enclosed by the given curves and find its area $y = \cos \pi x$, $y = 4x^2 - 1$



$\cos \pi x = 4x^2 - 1$ $\quad$ $x = \pm \frac{1}{2}$

$$2\int_0^{1/2} \left(\cos \pi x - 4x^2 + 1\right) dx = 2\left[\frac{\sin \pi x}{\pi} - \frac{4}{3}x^3 + x\right]\Bigg|_0^{1/2}$$

$$= \frac{2}{\pi} - \frac{8}{3}\,\frac{1}{8} + \frac{2}{2} = \frac{2}{\pi} + \frac{2}{3}$$

10) Evaluate the integral $\int \sqrt{\cot x}\,\csc^2 x dx$

$\cot x = u$

$-\csc^2 x\,dx = du$

$$\int \sqrt{u}\,(-1)\,du$$

$$= -\frac{2}{3}u^{3/2} + C$$

$$-\frac{2}{3}(\cot x)^{3/2} + C$$