

Math 150A Midterm 2

(Dated: October 29 2014)

Name:

MARLA

SID:

Write clearly and box all your answers. Simplify all formulas to the very end. No calculators allowed. Do not work out of memory, rather think before starting your calculations. Use the back for more space. Show all steps you are performing.

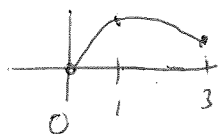
November 4th is Election Day. For all those who are eligible to vote, please don't forget to educate yourselves on the issues, and to cast your ballots. This is only way we can have a functional democracy.

- 1) Find the local and absolute extreme values of $f(x) = \frac{x}{x^2 - x + 1}$ on the interval $[0, 3]$

$$\boxed{\begin{array}{l} \text{min} \\ \text{max} \end{array} \left. \begin{array}{l} f(0) = 0 \\ f(3) = \frac{3}{7} \end{array} \right\}}$$

crit. point $x=1$
local max

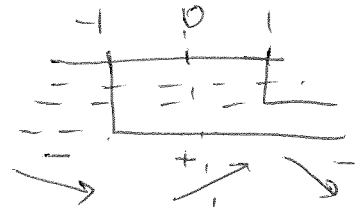
$$\boxed{\text{max } f(1) = 1}$$



$$f'(x) = \frac{x^2 - x + 1 - (2x - 1)x}{(x^2 - x + 1)^2}$$

$$= \frac{x^2 - x + 1 - 2x^2 + x}{(x^2 - x + 1)^2} = \frac{-x^2 + 1}{(x^2 - x + 1)^2}$$

$$= \frac{-(1+x)(x-1)}{(\quad)^2}$$



- 2) A cylindrical tank with radius 5 meters is being filled with water at a rate of $3 \text{ m}^3/\text{min}$. How fast is the height of the water increasing?



$$\begin{aligned} V &= \pi r^2 h \\ &= \pi \cdot 25 h \end{aligned}$$

$$\frac{dV}{dt} = 3$$

$$\frac{dV}{dt} = 25\pi \frac{dh}{dt}$$

$$\boxed{\frac{dh}{dt} = \frac{3}{25\pi}}$$

- 3) Find the derivative of $\cos \sqrt{\sin(\tan \pi x)}$

$$\leftarrow -\sin \sqrt{\sin(\tan \pi x)} \cdot \frac{\cos(\tan \pi x) \cdot \sec^2 \pi x \cdot \pi}{2 \sqrt{\sin(\tan \pi x)}}$$

4) Find dy/dx by implicit differentiation for $\sqrt{x+y} = 1 + x^2y^2$

$$\frac{1}{2\sqrt{x+y}} (1 + y') = 2xy^2 + 2x^2yy'$$

$$y' = \frac{2xy^2 - \frac{1}{2\sqrt{x+y}}}{\frac{1}{2\sqrt{x+y}} - 2x^2y} = \frac{4xy^2 - \frac{1}{1+x^2y^2}}{\frac{1}{1+x^2y^2} - 4x^2y} = \frac{4xy^2(1+x^2y^2) - 1}{1 - 4x^2y(1+x^2y^2)}$$

5) Find the limit or show it does not exist $\lim_{x \rightarrow \infty} \sqrt{x^2 + ax} - \sqrt{x^2 + bx}$

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{\sqrt{x^2 + ax} - \sqrt{x^2 + bx}}{\sqrt{x^2 + ax} + \sqrt{x^2 + bx}} &= \frac{x^2 + ax - x^2 - bx}{\sqrt{x^2 + ax} + \sqrt{x^2 + bx}} = \frac{(a-b)x}{2x} = \frac{a-b}{2} \end{aligned}$$

6) Show that the equation $2x - 1 - \sin x = 0$ has exactly one real root. Use Rolle's theorem.

$$f(0) = -1 < 0$$

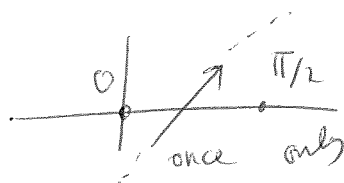
$$f\left(\frac{\pi}{2}\right) = \pi - 1 - 1 > 0$$

$$f(x) = 2x - 1 - \sin x$$

at least one root between $0, \frac{\pi}{2}$

$$f'(x) = 2 - \cos x > 0 \text{ always increasing}$$

\Rightarrow MUST HAVE ONLY ONE ZERO



7) Does there exist a function f such that $f(0) = -1$, $f(2) = 4$ and $f'(x) \leq 2$ for all x ? Use the Mean Value Theorem to prove or disprove your answer.

$$f'(c) = \frac{f(2) - f(0)}{2 - 0} = \frac{4 - (-1)}{2} = \frac{5}{2}$$

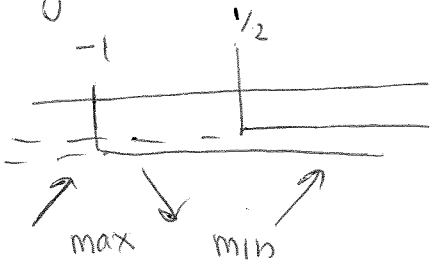
between
 $0, 2$

$$f'(c) = \frac{5}{2} \quad \text{so} \quad f'(x) \text{ cannot be } \leq 2 \text{ for all } x$$

$$0 \leq c \leq 2$$

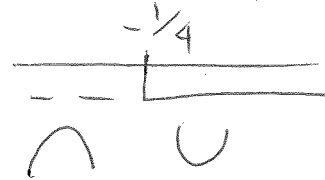
8) Find the intervals upon which $f(x) = 4x^3 + 3x^2 - 6x + 1$ is increasing or decreasing. Find its local maximum and minimum. Sketch the function. Find concavity and convexity and inflection points.

$$f'(x) = 12x^2 + 6x - 6 = 6(2x^2 + x - 1) = 6(2x - 1)(x + 1)$$



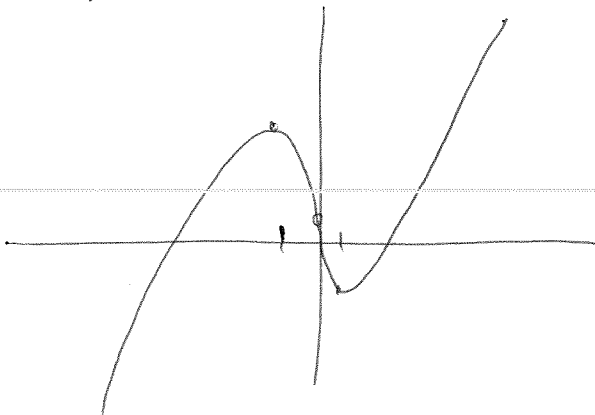
$$f''(x) = 24x + 6$$

$$x \geq -\frac{1}{4}$$



$$f(-1) = 6$$

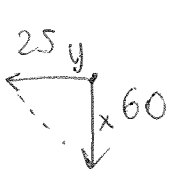
$$f\left(\frac{1}{2}\right) = -\frac{3}{4}$$



$$\lim_{x \rightarrow \infty} f(x) = +\infty$$

$$\lim_{x \rightarrow -\infty} f(x) = -\infty$$

- 9) Two cars start moving from the same point. One travels south at 60 mi/hr and the other travels west at 25 mi/h. At what rate is the distance between them increasing two hours later?



$$l = \text{distance} = \sqrt{x^2 + y^2}$$

$$l^2 = \text{distance}^2 = [(60)^2 + (25)^2] t^2$$

$$x = 60t$$

$$y = 25t$$

dista

$$\frac{d}{dt} l^2 = [60^2 + 25^2] \cdot 2t$$

$$\frac{dl}{dt} = \frac{3600 + 625}{\sqrt{3600 + 625}}$$

65 mph

$$(65) = \frac{845}{13} = \frac{4225}{8\sqrt{169}}$$

$$= \frac{4225}{\sqrt{4225}}$$

- 10) Sketch $y(x) = \frac{x}{\sqrt{x^2+1}}$. Find slant asymptotes if they exist and evaluate the concavity.

$$f(x) \geq 0 \quad \text{for } x > 0$$

$$f(x) \leq 0 \quad \text{for } x < 0$$

$$f(x) = 0 \quad \text{at } x = 0$$

$f(x)$ odd

$$\lim_{x \rightarrow \infty} \frac{x}{\sqrt{x^2+1}} = 1$$

$$\lim_{x \rightarrow -\infty} \frac{x}{\sqrt{x^2+1}} = -1$$

$$f'(x) = \frac{\sqrt{x^2+1} - \frac{2x^2}{2\sqrt{x^2+1}}}{(x^2+1)}$$

$$= \frac{x^2+1 - x^2}{(x^2+1)\sqrt{x^2+1}} \geq 0 \quad \text{always increasing}$$

$$= \frac{1}{(x^2+1)^{3/2}} = (x^2+1)^{-3/2}$$

$$f''(x) = -\frac{3}{2} (x^2+1)^{-5/2} \cdot 2x$$

