Math 150A Midterm 1

(Dated: September 24 2014)

Name: MARIA SOLUTIONS

SID:

Write clearly and box all your answers. Simplify all formulas to the very end. No calculators allowed. Do not work out of memory, rather think before starting your calculations. Use the back for more space. Show all steps you are performing.

1) Differentiate using the definition of derivative $y = \dfrac{x^2-1}{2x-3}$

$$\lim_{h\to 0} \frac{\dfrac{(x+h)^2-1}{2(x+h)-3} - \dfrac{x^2-1}{2x-3}}{h} = \lim_{h\to 0} \frac{1}{h} \frac{[(x+h)^2-1](2x-3) - (x^2-1)(2(x+h)-3)}{(2(x+h)-3)(2x-3)}$$

$$= \lim_{h\to 0} \frac{1}{\cancel{h}} \frac{\cancel{(x^2-1)(2x-3)} + 2x\cancel{h}(2x-3) + h^2(2x-3) - \cancel{(x^2-1)(2x-3)} - (x^2-1)2\cancel{h}}{(2(x+h)-3)(2x-3)} =$$

$$= \frac{2x(2x-3) - 2(x^2-1)}{(2x-3)^2} = \frac{2x^2-6x+2}{(2x-3)^2} = \boxed{\frac{2(x^2-3x+1)}{(2x-3)^2}}$$

2) Differentiate $y = \dfrac{x^2+4x+3}{\sqrt{x}}$

$$= x^{3/2} + 4x^{1/2} + 3x^{-1/2}$$

$$= \frac{3}{2}x^{1/2} + \frac{4}{2\sqrt{x}} - \frac{3}{2}x^{-3/2} = \boxed{\frac{3}{2}\sqrt{x} + \frac{2}{\sqrt{x}} - \frac{3}{2x\sqrt{x}}}$$

$y = 1 + 4x^2 + 4x$

3) Find the equation of the tangent and normal lines to the curve $y = (1+2x)^2$ at $(1, 9)$.

$m = y'(x=1, y=9)$

$y' = 2(1+2x)\cdot 2$

$m = 2(1+2\cdot 1)\cdot 2 = 4\cdot 3 = 12$

or

$y' = 8x + 4 = 8\cdot 1 \cdot 4 = 12$

$y = 12x + b$ $\quad 9 = 12 + b$ $\quad b = -3$ $\quad \boxed{y = 12x - 3}$

normal $y = -\frac{1}{12}x + b$ $\quad 9 = -\frac{1}{12} + b$ $\quad b = \frac{109}{12}$ $\quad \boxed{y = -\frac{1}{12}x + \frac{109}{12}}$

4) Find the limit, if it exists $\lim_{x \to 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{x}$ $\cdot \frac{\sqrt{1+x} + \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}$

$= \frac{\cancel{1}+x-\cancel{1}+x}{x \cdot 2} = \frac{2\cancel{x}}{2\cancel{x}} = 1$

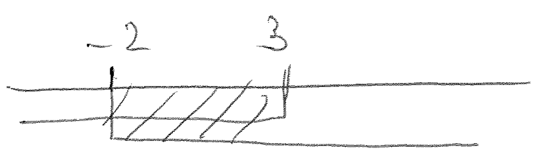
5) Evaluate the limit, if it exists $\lim_{x \to -4} \frac{\sqrt{x^2+9} - 5}{x+4}$ $\cdot \frac{\sqrt{x^2+9}+5}{\sqrt{x^2+9}+5}$

$= \frac{x^2+9-25}{(x+4)(\sqrt{x^2+9}+5)} = \frac{(x-4)\cancel{(x+4)}}{\cancel{(x+4)}(\sqrt{x^2+9}+5)} =$

$\frac{-8}{10} = -\frac{4}{5}$

6) Find the domain of the function $y = \sqrt{3-x} - \sqrt{2+x}$

$3 - x \geq 0$ and $2 + x \geq 0$



$\boxed{-2 \leq x \leq 3}$

7) Find the functions $f \circ g$, $g \circ f$, $f \circ f, g \circ g$ for $f(x) = x + \frac{1}{x}$ and $g(x) = \frac{x+1}{x+2}$. Simplify to the end.

$f \circ g$ $\quad \frac{x+1}{x+2} + \frac{x+2}{x+1} = \frac{x^2+2x+1+x^2+4x+4}{(x+2)(x+1)} = \boxed{\frac{2x^2+6x+5}{(x+1)(x+2)}}$

$g \circ f$ $\quad \frac{x+\frac{1}{x}+1}{x+\frac{1}{x}+2} = \boxed{\frac{x^2+x+1}{x^2+2x+1}}$

$f \circ f$ $\quad x+\frac{1}{x}+\frac{1}{x+\frac{1}{x}} = \frac{\left(x+\frac{1}{x}\right)^2+1}{x+\frac{1}{x}} = \frac{x^2+\frac{1}{x^2}+2+1}{x+\frac{1}{x}} = \boxed{\frac{x^4+3x^2+1}{x^3+x}}$

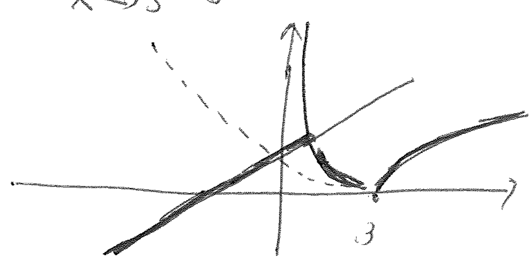
$g \circ g$ $\quad \frac{x+1}{x+2}+1 \Big/ \frac{x+1}{x+2}+2 = \frac{x+1+x+2}{x+1+2x+4} = \boxed{2x+3/3x+5}$

8) Where is $f(x)$ is discontinuous and why? Sketch the function.

$$f(x) = \begin{cases} x+1 & \text{if } x \leq 1 \\ 1/x & \text{if } 1 \leq x < 3 \\ \sqrt{x-3} & \text{if } x \geq 3 \end{cases} \tag{1}$$

$\lim_{x \to 1^-} f(x) = 2$ $\qquad \lim_{x \to 1^+} f(x) = 1$ $\qquad$ 1 is a discontinuity

$\lim_{x \to 3^-} f(x) = \frac{1}{3}$ $\qquad \lim_{x \to 3^+} f(x) = 0$ $\qquad$ 3 ″



9) Find an equation of the tangent line to the curve $y = \sqrt{x}$ for $x = 1$. $\qquad x=1 \quad y=1$

$y' = \frac{1}{2\sqrt{x}}$ $\qquad m = \frac{1}{2}$ $\qquad y = \frac{1}{2}x + b$

$1 = \frac{1}{2} + b$ $\qquad b = \frac{1}{2}$

$\boxed{y = \frac{1}{2}x + \frac{1}{2}}$

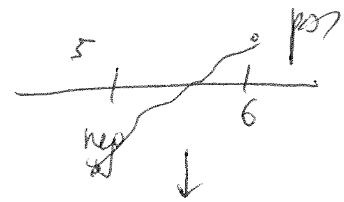
10) Prove that the following equation has at least one real root $\sqrt{x-5} = \dfrac{1}{x+3}$

this is the same as proving that $f(x) = \sqrt{x-5} - \dfrac{1}{x+3} = 0$

so, since $f(x)$ is continuous everywhere except at $x = -3$ we can try

$x = 5 \quad f(x) = -\dfrac{1}{8} < 0$

$x = 6 \quad f(x) = 1 - \dfrac{1}{9} > 0$



there must be at least one zero, hence $f(x) = 0$ between $x = 5$, $x = 6$

11) Find the derivative of $f(x) = (1 + 2x^2)(x - x^2)$ in two ways: by using the product rule and by performing the multiplication first. Do your results agree?

P.R. $4x(x - x^2) + (1 + 2x^2)(1 - 2x)$

$= 4x^2 - 4x^3 + 1 - 2x + 2x^2 - 4x^3 = \boxed{-8x^3 + 6x^2 - 2x + 1}$

Mult. $x + 2x^3 - x^2 - 2x^4 = -2x^4 + 2x^3 - x^2 + x$

$y' = \boxed{-8x^3 + 6x^2 - 2x + 1}$

12) The equation for the height of a rock as a function of time is $h(t) = 10t - 2t^2$. Find the velocity of the rock after 1 second and at time $t = a$. When will the rock hit the surface? With what velocity will the rock hit the surface?

$v = 10 - 4t$

after 1 sec $= 10 - 4 = 6$

at $a = 10 - 4a$

hit surface at $t = 0$ or $t = 5$

velocity $= 10 - 4 \cdot 5 = -10$