Math 150A Answer Sheet

1 a. $\lim_{x\to 1} \frac{(x-2)\cancel{(x-1)}}{\cancel{(x-1)}} = x-2 = -1$

$1-2$

b. $\lim_{x\to 0} \frac{x-1}{x} = \frac{0-1}{0} = \frac{-1}{0} =$ does not exist.

d. $\lim_{x\to 0^-} \frac{x-|x|}{x} = \lim_{x\to 0^-} \frac{x-(-x)}{x} =$

$\lim_{x\to 0^-} \frac{2\cancel{x}}{\cancel{x}} = 2$

e. $\lim_{x\to\infty} \frac{2x^2-5x+6}{8+x+3x^2}$ $\lim_{x\to\infty} \frac{\frac{2x^2}{x^2} - \frac{5x}{x^2} + \frac{6}{x^2}}{\frac{8}{x^2} + \frac{x}{x^2} + \frac{3\cancel{x^2}}{\cancel{x^2}}} =$

$\lim_{x\to\infty} \frac{2 - \frac{5}{x} + \frac{6}{x^2}}{\frac{8}{x^2} + \frac{1}{x} + 3} = \frac{2}{3}$

f. $\lim_{x\to\infty} \frac{\sin x}{x} = 0$.

(since the numerator is bounded by ±1 and the denominator grows without bound)

2. $f(x) = \begin{cases} ax & x<3 \\ bx^2+x+9 & x\geq 3 \end{cases}$

$f(x) = ax \qquad g(x) = bx^2+x+9$

$f(x) = g(x) \quad = \quad ax = bx^2+x+9 =$

$f(3) = g(3) \quad = \quad a(3) = b(3)^2+3+9 = \underline{3a = 9b+12}$

(substitute

$f'(x) = g'(x) \quad = \quad a = 2bx+1 =$

$f'(3) = g'(3) \quad \quad a = 2b(3)+1 = \underline{a = 6b+1}$

<u>substitute:</u>

$3(6b+1) = 9b+12$

$18b+3 = 9b+12 =$

$9b = 9$

$\boxed{b=1}$

<u>substitute 1 & solve for a:</u>

$3a = 9(1)+12$

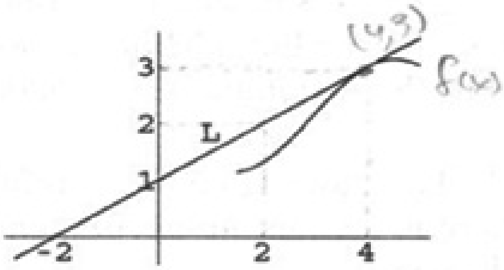
$3a = 9+12$

$3a = 21$

$\boxed{a=7}$

3. $f(4) = 3$

$f'(4) = \frac{3-2}{4-2} = \frac{1}{2}$



4. $f(x) = x|x|$

$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$

$f'(0) = \lim_{h\to 0} \frac{f(0+h) - f(0)}{h}$

$f(0+h) = (0+h)|0+h| = h|h|$

$f(0) = 0|0| = 0$

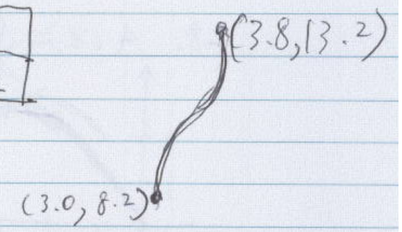
$f'(0) = \lim_{h\to 0} \frac{h|h| - 0}{h} = \lim_{h\to 0} \frac{h|h|}{h} = \lim_{h\to 0} |h|$

$f'(0) = \lim_{h\to 0} |h| = 0$

5.

x	3.0	3.2	3.4	3.6	3.8
$f(x)$	8.2	9.5	10.5	11.0	13.2

estimate $f'(3.2)$, $f'(3.5)$.



$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$f'(3.2) = \frac{9.5 - 8.2}{3.2 - 3.0} = 6.5$, $f'(3.2) = \frac{10.5 - 9.5}{0.2} = 5$

$\frac{(6.5 + 5)}{2} = 5.75$

$f'(3.5) = \frac{11.0 - 10.5}{3.6 - 3.4} = 2.5$

6a) $f(x) = \frac{1}{9}x^5 + x^{\frac{2}{3}} - \frac{1}{x^6}$

$= \frac{1}{9}x^5 + x^{\frac{2}{3}} - x^{-6}$

$= \left(5 \cdot \frac{1}{9}\right)x^{5-1} + \frac{2}{3}x^{\frac{2}{3}-1} - (-6)x^{-6-1}$

$= \frac{5}{9}x^4 + \frac{2}{3}x^{-\frac{1}{3}} + 6x^{-7}$

$= \boxed{\frac{5}{9}x^4 + \frac{2}{3\sqrt[3]{x}} + \frac{6}{x^7}}$

b $f(x) = x \cdot (\sin x)^2$

$$(\sin x)^2 + x \cdot 2\sin x \cdot \cos x$$

$$\sin^2 x + 2x \sin x \cos x$$

6c) $f(x) = \dfrac{x^2+1}{x-5}$ $\quad \dfrac{d}{dx}\left[\dfrac{f(x)}{g(x)}\right] = \dfrac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$

$$= \frac{(2x)(x-5) - (x^2+1)(1)}{(x-5)^2}$$

$$= \frac{2x^2 - 10x - x^2 - 1}{(x-5)^2}$$

$$= \boxed{\frac{x^2 - 10x - 1}{(x-5)^2}}$$

6d. $f(x) = \sqrt{a^2 + x^2} = (a^2 + x^2)^{1/2}$

$f'(x) = \frac{1}{2}(a^2 + x^2)^{-1/2}(2x)$

6e. $f(x) = \dfrac{x}{(1+2x)^9} = x(1+2x)^{-9}$

$f'(x) = (1+2x)^{-9} + x(-9)(1+2x)^{-10}(2)$

$= (1+2x)^{-9} - 18x(1+2x)^{-10}$

6

f) $f(x) = \tan^2(x^2+1)$

$$f'(x) = 2\tan(x^2+1) \cdot \frac{1}{\cos^2(x^2+1)} \cdot (2x)$$

$$f'(x) = \frac{4x\tan(x^2+1)}{\cos^2(x^2+1)}$$

7. equation of line tangent to curve $x^3 + 3x^2y^2 + y^3 = 3$

$$3x^2 + 6xy^2 + 3x^2 2y\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(3x^2 2y + 3y^2) = \frac{-3x^2 - 6xy^2}{3x^2 2y + 3y^2} = \frac{-3 + -6}{-6 + 3} = \frac{-9}{-3} = 3$$

(It is easier to plug in (1,–1) before solving for dy/dx above.)

Tangent line : $y = y_1 + m(x - x_1)$ gives $y = -1 + 3(x - 1)$,

or $y = 3x - 4$.

8. After the outside temperature reaches 50°F the daily cost to heat a certain house is decreasing at a rate of 0.15 dollars per degree F.

(So if the mean outside temperature increased from 50° to 51°, we would expect the daily cost to heat the house to drop by approximately $0.15.)

9. No solution provided.

10. Approx Area of a Triangle given

$A(\theta) = \frac{1}{2}\sin\theta\cos\theta \qquad \theta = \frac{\pi}{6} \pm 0.02$ Radians $(\Delta\theta)$

Find Area if $A(\frac{\pi}{6})$

$$A' = \frac{1}{2}\sin\theta\cos\theta \rightarrow \frac{1}{2}(\sin\theta)\left(\frac{d}{d\theta}\cos\theta\right) + \cos\theta\left(\frac{d}{d\theta}\frac{1}{2}\sin\theta\right) =$$

$$\frac{1}{2}(-\sin\theta\sin\theta + \cos\theta\cos\theta) = \frac{1}{2}(\cos^2\theta - \sin^2\theta)$$

$$\Delta A = \left(\frac{1}{2}\cos^2\theta - \sin^2\theta\right)\Delta\theta =$$

$$\frac{1}{2}\left(\left(\cos\left(\frac{\pi}{6}\right)\right)^2 - \left(\sin\left(\frac{\pi}{6}\right)\right)^2\right) 0.02 =$$

$$\frac{1}{2}(.75 - .25)\, 0.02$$

$$\Delta A = (.25)\, 0.02 = \boxed{.005}$$

11, 12. No solutions provided.

(13) (A) f is increasing from -2 to 1 because the graph of f' is positive (above the x-axis) on that intreval.

f is decreasing from 1 to 3 because the graph of f' is negative (below the x-axis) on that intreval.

(B) f is concave up on -2 to 0 and 2 to 3 because f'' is positive on those intrevals. the graph of f' is increasing.

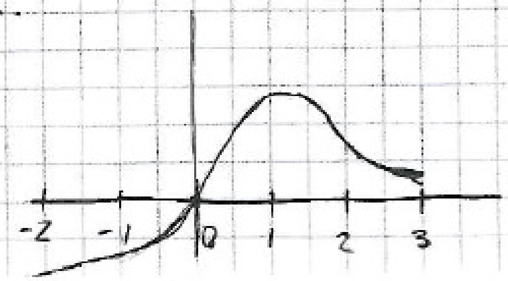
f is concave down on 0 to 2 because the graph of f' is decreasing on that intreval.

(C) $f(x)$ is greatest at $x=1$ because f' is incrasing up to that point.

$f'(x)$ is greatest at 0 because we can see it on the graph of f'.

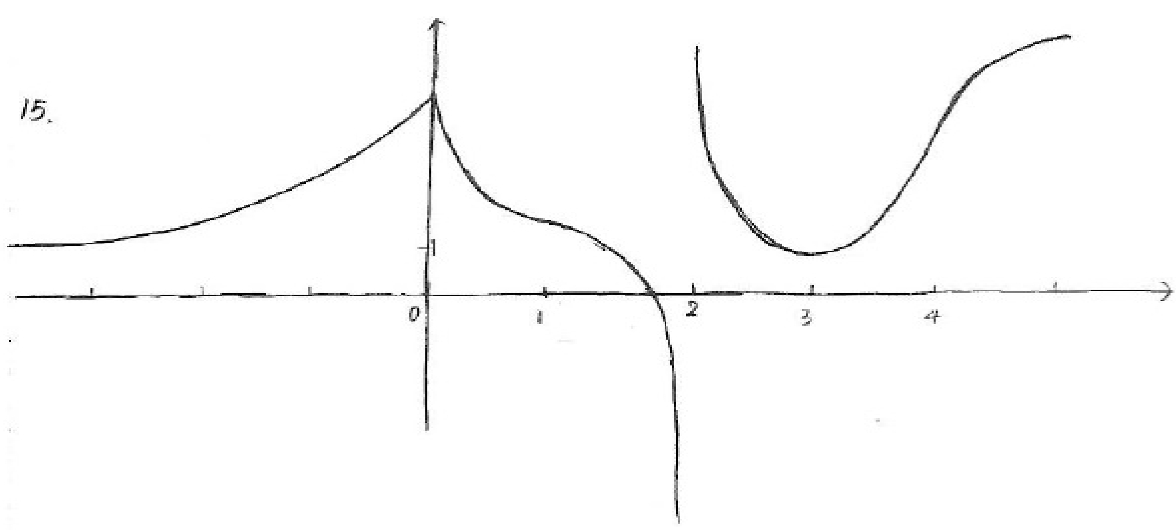
$f''(x)$ is greatest at -1 because the slope of f' is greatest at -1.

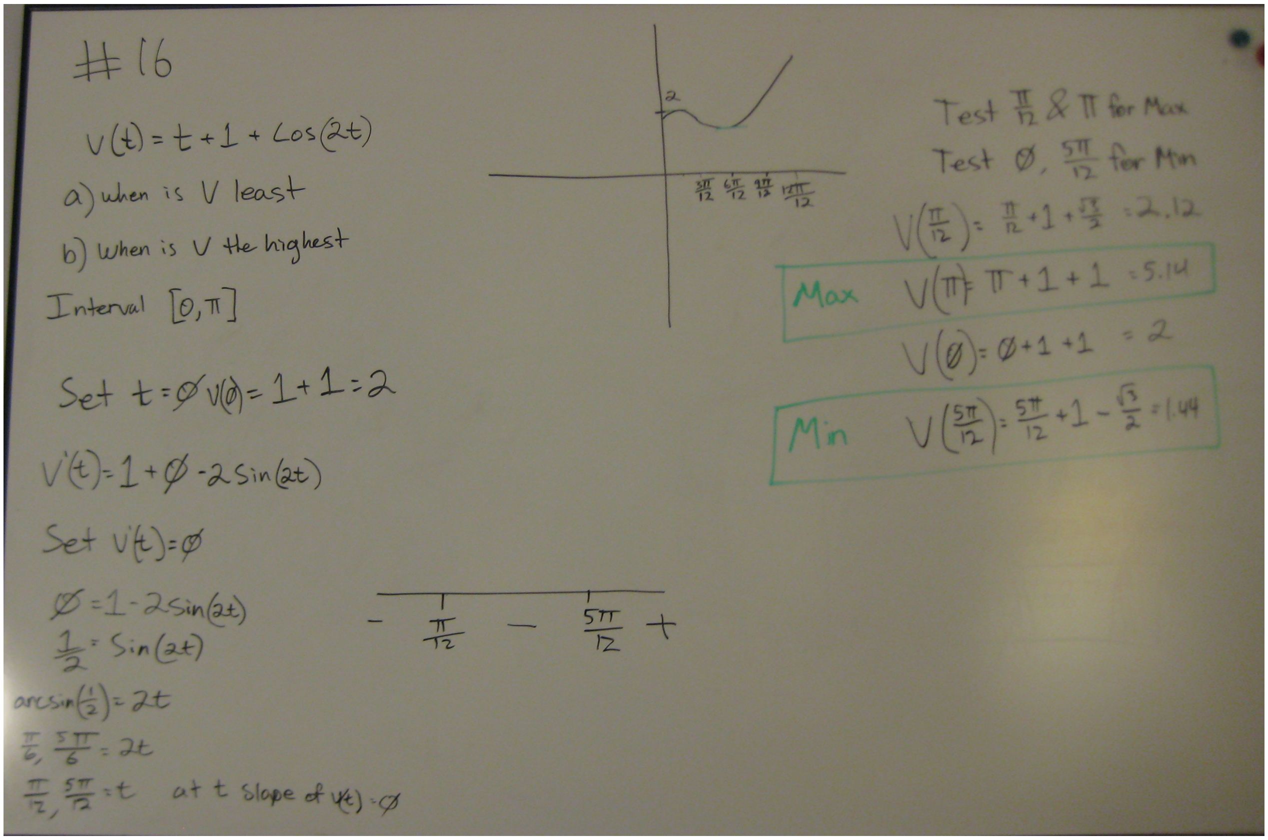
(d) possible graph of $f(x)$



14. Graph (c) is the derivative of graph (b) and graph (d) is the derivative of graph (a).

15.





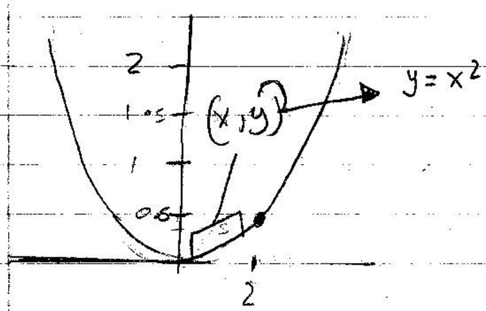
17. Find the inflection point between the points where the slope is zero.

1. Get the derivative: y’ = -3x^2 + 6x + 9 (Setting the derivative to zero will get you the points where the slope is zero)
2. Get the 2nd derivative: y’’ = -6x + 6. Setting it to zero, x = 1. The maximum rate of change occurs at $\boxed{x = 1}$, where y = -16. (Note that this gives the maximum of y’.)

18. No solution provided.

19. $y = x^2$, find shortest pt $(2, 1/2)$

so... the two pts. we're working with are: $(2, 1/2)$, and (x, x^2)



Now in order to find the shortest distance from the pt $(2, 1/2)$ we use the distance formula: $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

$$= \sqrt{(2-x)^2+\left(\tfrac{1}{2}-x^2\right)^2} = \sqrt{(4-4x+x^2)+\left(\tfrac{1}{4}-x^2+x^4\right)}$$

$$= \sqrt{x^4-4x+\tfrac{17}{4}} = \left(x^4-4x+\tfrac{17}{4}\right)^{1/2}$$

now we take the derivative: $\frac{1}{2}\left(x^4-4x+\frac{17}{4}\right)^{-1/2} \cdot (4x^3-4)$
(using the chain rule)

$$= \frac{4x^3-4}{2\sqrt{x^4-4x+\frac{17}{4}}}$$

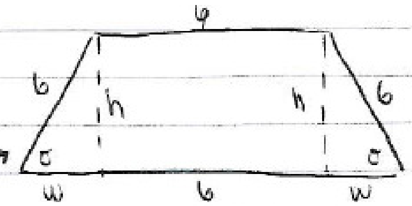
$4x^3-4=0 = 4(x^3-1)=0 \quad x=\pm 1$ (to find our x)

Since $y=x^2 = (1)^2 = 1$ so... $(1,1)$

So... the shortest distance from the point $(2, 1/2)$ will be at $(1,1)$:

$$\sqrt{(2-1)^2+(1/2-1)^2} = \boxed{\sqrt{5}/2}$$

(20)



Area of rectangle = $6h$

Area of triangle = $\frac{1}{2}wh$

2 triangles $2(\frac{1}{2}wh)$

$= wh$

sohcahtoa

$h =$

$\sin\theta = \frac{h}{6}$

$h = 6\sin\theta$

$w =$

$\cos\theta = \frac{w}{6}$

$w = 6\cos\theta$

$A = 6h + wh$

$A = 6(6\sin\theta) + (6\cos\theta)(6\sin\theta)$

$A = 36\sin\theta + 36(\cos\theta\sin\theta)$

$A' = 36\cos\theta + 36(-\sin^2\theta + \cos^2\theta)$

$36\cos\theta + 36\cos^2\theta - \underbrace{36\sin^2\theta}$

$\cos^2\theta + \sin^2\theta = 1$

$-\sin^2\theta = 1 - \cos^2\theta$

$A' = 36\cos\theta + 36\cos^2\theta - 36(1 - \cos^2\theta)$

$A' = 36\cos\theta + 36\cos^2\theta - 36 + 36\cos^2\theta$

$A' = 72\cos^2\theta + 36\cos\theta - 36$

$A' = 36(2\cos^2\theta + \cos\theta - 1)$

$\underbrace{(2\cos\theta - 1)}\underbrace{(\cos\theta + 1)}$

$\cos\theta = \frac{1}{2}$ $\quad$ $\cancel{\cos\theta = -1}$ Doesnt work

$\cancel{\theta = \pi}$

$\theta = 60°$ or $\frac{\pi}{3}$

21. No solution provided.

(22) $G(x) = \int_0^x \sqrt{t^4+1}\,dt$

(a) $G(0) = \int_0^0 \sqrt{t^4+1}\,dt = 0$

(b) $G'(x) = \sqrt{x^4+1}$

(c) $G''(x) = \frac{1}{2}(x^4+1)^{-1/2} \cdot (4x^3) = \frac{4x^3}{2\sqrt{x^4+1}} = \frac{2x^3}{\sqrt{x^4+1}}$

(d) increasing from $[-\infty, \infty]$

decreasing never

23 (a) $\int (x+\frac{1}{x})^2 dx$

$= \int x^2 + x^{-2} + 2\, dx$

$= \frac{x^3}{3} + \frac{x^{-1}}{-1} + 2x + C$

$= \frac{x^3}{3} - x^{-1} + 2x + C$

(b) $\int \cos^5(3x)\sin(3x)\,dx$

$w = \cos(3x) \quad dw = -3\sin(3x)\,dx$

$-\frac{1}{3}\int -3\sin(3x)\cdot\cos^5(3x)\,dx$

$= -\frac{1}{3}\int w^5\,dw$

$= -\frac{1}{3}\cdot\frac{w^6}{6} + C$

$= -\frac{\cos^6(3x)}{18} + C$

(c) $\int \frac{1}{\sqrt{x}}(\sqrt{a}+\sqrt{x})^{\frac{5}{2}}\,dx$

$w = \sqrt{a}+\sqrt{x} \qquad dw = \frac{1}{2}x^{-\frac{1}{2}}\,dx$

$2\int \frac{1}{2}x^{-\frac{1}{2}}(\sqrt{a}+\sqrt{x})^{\frac{5}{2}}\,dx$

$= 2\int w^{\frac{5}{2}}\,dw$

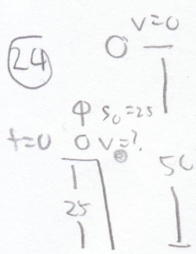
$= 2\cdot w^{\frac{7}{2}}\cdot\frac{2}{7} + C$

$= \frac{4}{7}\cdot(\sqrt{a}+\sqrt{x})^{\frac{7}{2}} + C$

d) $\int_0^t \frac{d}{dx}\left(\cos(x^3+1)\right)dx$

$= \cos(t^3+1)$

24



$f''(x) = A = \frac{dv}{dt} = -32$

At beginning

$f'(x) = v = \frac{ds}{dt} = -32t + C \rightarrow f'(0)$: $0 = C = v_0$

$f(x) = s = -16t^2 + Ct + C_2 \rightarrow f(0)$: $-16(0)^2 + v_0(0) + C_2 = 25$

$C_2 = 25 = s_0$

At maximum:

$0 = -32t + v_0$

$t = \frac{v_0}{32}$

$50 = -16\left(\frac{v_0}{32}\right)^2 + v_0\left(\frac{v_0}{32}\right) + C_2 \qquad s_0 = 25$

Solve for v_0: $25 = \frac{-16v_0^2}{32^2} + \frac{v_0^2}{32}$

$25 = \frac{-16v_0^2 + 32v_0^2}{32^2}$

$25 = \frac{16v_0^2}{1024}$

$v_0^2 = 1600$

$v_0 = 40$

∴ initial velocity is 40 ft/s

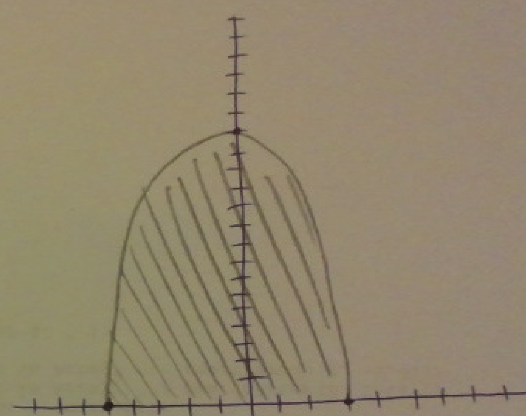
25. No solution provided.

(27) area above x-axis
below $y = 15-2x-x^2$

Set $y=0$ to find intercepts

$0=(5+x)(3-x)$

$x=-5,3$



$$\int_{-5}^{3} 15-2x-x^2\,dx$$

$$15x - \frac{\cancel{2}x^2}{\cancel{2}} - \frac{x^3}{3}\Big|_{-5}^{3}$$

$$15(3)-(3)^2-\frac{(3)^3}{3} - \left(15(-5)-(-5)^2-\frac{(-5)^3}{3}\right)$$

$$45-9-9 + \left(+75 + 25 \mp \frac{125}{3}\right) = \boxed{\frac{256}{3}}$$

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