Math 140 Midterm 3

(Dated: May 1 2013)

Maria

Name: SOLUTIONS

SID:

Do at least 8 problems out of 10. Write clearly and box all your answers. Do not work out of memory, rather think before starting. Use the back for more space. Show all steps you are performing.

please accept rounding errors by students on the last digits

MARIA

1) College admission scores are approximately normal with mean 500 and standard deviation 100.

a) if you select one score at random, what is the probability that it is 510 or more?

b) if you randomly select four scores, what is the probability that their mean is 510 or more?

c) if you randomly select 25 scores, what is the probability that their mean is 510 or more?

d) if you randomly select 25 scores, what is the probability that their mean is between 490 and 510?

a) $z \rightarrow \frac{510-500}{100} = 0.10$ P(510 or more) = 0.4602

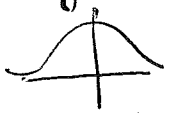
b) $z \rightarrow \frac{510-500}{\frac{100}{\sqrt{4}}} = 0.20$ P(" ") = 0.4207

c) $z = \frac{510-500}{\frac{100}{\sqrt{25}}} = 0.50$ P(" ") = 0.3085

d) $z = 490-500/100/\sqrt{25} = -0.50$ P(between 490 and 510) = 0.6915 − 0.3085 = 0.383

2) According to the US Census, about 22.3 percent of the Spanish surnames in the US are one of either a group of 12 (Garcia, Martinez, Rodrigues, Lopez, Hernandez, Ramirez, Torres, Gonzales, Perez, Sanchez, Gonzalez, Rivera). Suppose we take a group of 500 Spanish-surnamed people.

gaussian



0.223

a) Make a sketch of the sampling distribution in the sample that have one of these last names.

b) What is the probability of getting 20 percent or fewer people with one of these last names in your sample? → see back

c) What is the probability of getting 105 or more people with one of these last names?

d) What proportion of people with one of these last names would be considered a rare event?

a)

b) $z = \frac{0.2-0.223}{0.0186} = -1.24$ $\sigma = \frac{\sqrt{p(1-p)}}{\sqrt{n}} = \sqrt{\frac{0.223 \cdot 0.777}{500}} = 0.0186$

P(less than 20%) = 0.108

c) $z = \frac{\frac{105}{500} - \frac{0.223}{\hat{p}}}{0.0186} = 20.$ $\frac{0.21-0.223}{0.0186} = -0.6989$

P(105 or more) = 0.757 =

d) between 0.259 − 0.187 ↑ P(rare) =

$0.223 \pm 1.96 \cdot 0.0186$

4) The mean of the population of cars per family in the US is 1.7. standard deviation is 1.

a) What is the mean of the sampling distribution of size 4? Of size 10?

b) What is the standard error of the sampling distribution of size 4? Of size 10?

c) If three sample distributions were presented to you, each coming from a sample size of 1, 4 and 10, which would you expect to approximate best the normal distribution? The least?

d) In very simple words, what does the Central Limit Theorem say?

a) mean = 1.7 in all cases

b) S.E. = $\sigma_{n=4} = \frac{1}{\sqrt{4}} = \frac{1}{2}$ $\sigma_{n=10} = \frac{1}{\sqrt{10}} = 0.316$

c) Normal distr. is best approximated for large n, so BEST n=10
LEAST n=1

d) C.L.T says that as $n \to \infty$ (increases), the sampling distribution looks more and more normal.

5) Population scores for a college test are normal with mean 500 and standard deviation 100.

a) If we randomly picked 40 scores what is the probability that the mean will be within 10 points of the population mean? = # 490 / 510

b) How large a random sample of scores would you need to be 95 percent sure that the sample mean would be within 10 points of the population mean?

a) $z_1 = \frac{490 - 500}{\frac{100}{\sqrt{40}}} = -0.632$ $P(<490) = 0.2635$

$z_2 = \frac{510 - 500}{\frac{100}{\sqrt{40}}} = 0.632$ $P(>510) = 0.7365$

=1 between $0.7365 - 0.2635 = 0.473$

b) 95% sure we need to include 10 in the evaluation. 385

8) In 2008 38 percent of Americans considered themselves Democrats.

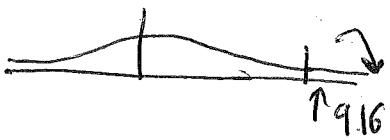
a) If 435 citizens are selected at random, what is the probability of getting 257 or more Democrats?

b) In the 2008 elections, 435 members were elected to the US House of Representatives. Of these, 257 were Democrats. Is this percentage higher than could reasonably occur by chance?

c) If so what are some possible reasons for this unusually large percentage?

$n = 435$ $\hat{p} = \frac{257}{435} = 0.59$ $p = 0.38$ $\sigma_{\hat{p}} = \sqrt{\frac{0.38(1-0.38)}{435}} = 0.023$

a) $z = \frac{0.59 - 0.38}{0.023} = 9.13$ ↘ large!



area almost zero!!

b) Yes! by chance the prob. is almost zero! (As from part a)

c) people did not vote along party lines/ independents voted democrat, etc

9) In a survey of 600 students, about 65 percent reponded that their school was helping them pursue their career choices.

a) Find the 95 % confidence interval for all students who would respond that their school is helping them pursue their career choices. What is the margin of error? (The factor to use is 1.96)

b) Find the 90 % confidence interval for all students who would respond that their school is helping them pursue their career choices. What is the margin of error? (The factor to use is 1.645)

c) Of the two you calculated, which confidence interval is larger? Why should that be the case?

$n = 600$, $\hat{p} = 0.65$

a) 95% c. interval = $0.65 \pm 1.96\sqrt{\frac{0.65 \cdot (1-0.65)}{600}} = \pm 0.69$ ↘ 0.61

margin error = 0.04 or 4%

b) 90% c. interval = $0.65 \pm 1.645\sqrt{\frac{0.65(1-0.65)}{600}} = \pm 0.68$ ↘ 0.62

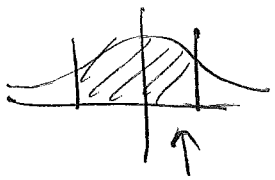
margin error = 0.03

c) 95% should be larger we need larger interval to capture more probability.

5b) 95% confidence interval:

$$\text{mean} \pm 1.96 \frac{\sigma}{\sqrt{n}}$$

$$500 \pm 1.96 \frac{100}{\sqrt{n}}$$



95% confidence interval

we need at least

$$1.96 \cdot \frac{100}{\sqrt{n}} = 10 \Rightarrow \sqrt{n} = \frac{1.96 \cdot 100}{10} \Rightarrow$$

$$n = (19.6)^2 = 384.16 \rightarrow 385$$

2d) rare event:

$$0.223 \pm 1.96 \cdot \sqrt{\frac{p(1-p)}{n}}$$

$$= 0.223 \pm 1.96 \cdot 0.0186 \begin{cases} + & 0.259 \\ - & 0.187 \end{cases}$$

rare events are outside the interval $[0.187, 0.259]$

i.e. for values > 0.259

< 0.187.

Appendix: Statistical Tables

Table entry for z is the probability lying below z.

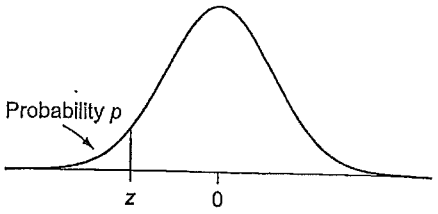


TABLE A Standard Normal Probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.8	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
−3.7	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
−3.6	.0002	.0002	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
−3.5	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641