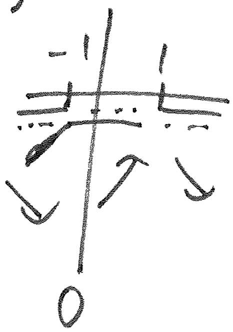
1) Find the local and absolute extreme values of $f(x) = \dfrac{x}{x^2 - x + 1}$ on the interval $[0, 3]$

$$f' = \frac{x^2 - x + 1 - (2x-1)x}{(\quad)^2}$$

$$= \frac{-x^2+1}{(\quad)^2}$$



min @ $x = -1$ $\quad \frac{-1}{3}$

max @ $x = 1$ $\quad \frac{1}{1}$

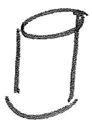
$\cancel{(-1 \quad -\tfrac{1}{3})}$ ~~LOCAL · ABS MIN~~

$(1 \quad 1)$ ABS MAX

$(0 \quad 0)$ ~~ABS MIN~~ ABS MIN

$(3 \quad 3/7)$

2) A cylindrical tank with radius 5 meters is being filled with water at a rate of 3 m^3/min. How fast is the height of the water increasing?



$$V = \pi r^2 h$$

$$\frac{dV}{dt} = \pi r^2 \frac{dh}{dt}$$

$$3 = \cancel{2\pi \cdot 5}\ 25\ \frac{dh}{dt}$$

$$\boxed{\frac{dh}{dt} = \frac{3}{25\pi}}$$

3) Find the derivative of $\cos\sqrt{\sin(\tan \pi x)}$

$$y' = -\sin\left(\sqrt{\sin(\tan \pi x)}\right) \cdot \frac{1}{2\sqrt{\sin(\tan \pi x)}} \cdot \cos(\tan \pi x) \cdot \sec^2 \pi x \cdot \pi$$

4) Find dy/dx by implicit differentiation for $y\cos x = 1 + \sin(xy)$

$$y'\cos x - y\sin x = \cos(xy)\,(y + xy')$$

$$y' = \frac{\cos(xy)\cdot y + y\sin x}{\cos x - x\cos(xy)}$$

5) Find the limit or show it does not exist $\lim_{x\to\infty} \sqrt{x^2+ax} - \sqrt{x^2+bx}$

$$\lim_{x\to\infty} \frac{\cancel{x^2} + ax - \cancel{x^2} - bx}{\sqrt{x^2+ax} + \sqrt{x^2+bx}} = \frac{(a-b)\,\cancel{x}}{\cancel{x}\left[\sqrt{1+\frac{a}{x^2}} + \sqrt{\ }\right)} = \frac{a-b}{2}.$$

6) Sketch $y(x) = \dfrac{x}{\sqrt{x^2+1}}$. Find slant asymptotes if they exist and evaluate the concavity.

$\lim_{x\to\infty} f(x) = 1$

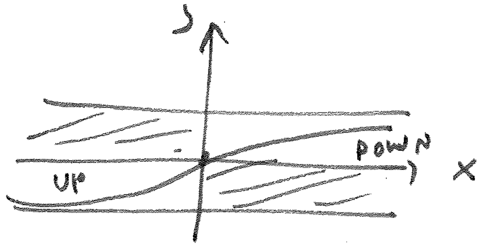
$\lim_{x\to-\infty} = -1$

$$f'(x) = \frac{\sqrt{x^2+1} - \frac{x^2}{\sqrt{x^2+1}}}{(x^2+1)} = \frac{1}{(x^2+1)^{3/2}} \geq 0 \text{ always}$$

$$f''(x) = -\frac{3}{2}(x^2+1)^{-3/2}\cdot 2x$$

0

∪ ∩



7) Does there exist a function f such that $f(0) = -1$, $f(2) = 4$ and $f'(x) \le 2$ for all x? Use the Mean Value Theorem to prove or disprove your answer.

MVT: $\exists c$: $f'(x) = \dfrac{f(2) - f(0)}{2} = \dfrac{4+1}{2} = \dfrac{5}{2} > 2$

no! f dos not exist

8) Find the intervals upon which $f(x) = 4x^3 + 3x^2 - 6x + 1$ is increasing or decreasing. Find its local maximum and minimum. Sketch the function. Find concavity and convexity and inflection points.

$\lim_{x \to \infty} f(x) = +\infty$

$\lim_{x \to -\infty} f(x) = -\infty$

$f'(x) = 12x^2 + 6x - 6 = 6(2x^2 + x - 1)$

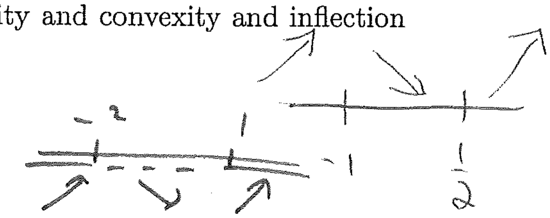
$x = \dfrac{-1 \pm \sqrt{1+8}}{4} = \dfrac{-1 \pm 3}{2}$ $\quad \frac{1}{2}$, -1

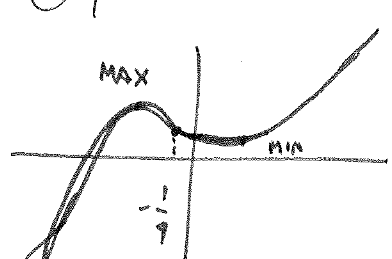
max @ $x = 2$ $\quad y = 32 + \cancel{12} - \cancel{12} + 1 = 33$

min @ $x = 1$ $\quad y = 2$

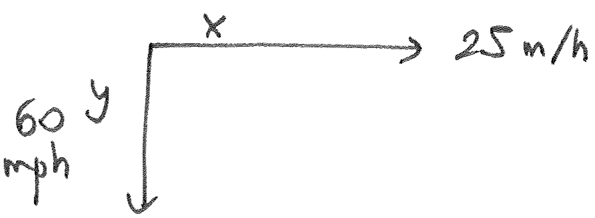
$f''(x) = 24x + 6 = 6(4x+1)$

$-1/4$





9) Two cars start moving from the same point. One travels south at 60 mi/hr and the other travels west at 25 mi/h. At what rate is the distance between them increasing two hours later?



$z^2 = x^2 + y^2$

$\cancel{2}z\dfrac{dz}{dt} = x\dfrac{dx}{dt} + y\dfrac{dy}{dt}$

$\dfrac{65}{11}$

after 2h $\quad x = 50$, $y = 120$ $\quad d^2 = 25 + 144 = 169$

$\dfrac{dz}{dt} = \dfrac{50 \cdot 25 + 120 \cdot 60}{\sqrt{16900}} = \dfrac{5 \cdot 25 + 60 \cdot 12}{13}$

10) Show that the equation $2x - 1 - \sin x = 0$ has exactly one real root. Use Rolle's theorem.

$f(x) = 2x - 1 - \sin x$

@ $x = 0$ $f(0) = -1$ < 0

@ $x = 2$ $f(2) = 4 - 1 - \sin 2 > 0$

$\exists$ a point where $f'(x) = 0$, $f'(c) = 0$

$0 \leq c \leq 2$

$f'(x) = 2 - \sin x \geq 0$

always

$f(x)$ always increasing

must cross at least once

and only once