Math 140 Midterm 2

(Dated: March 27 2013)

Name: Solution

SID:

Do at least 8 problems out of 10. Write clearly and box all your answers. Do not work out of memory, rather think before starting. Use the back for more space. Show all steps you are performing.

1) Consider the sampling space for rolling two dice as shown below.

		First die					
		1	2	3	4	5	6
	1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
	2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
Second die	3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
	4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
	5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
	6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

Determine each of these probabilities

a) not getting doubles;

b) getting a sum of five;

c) getting a sum of 7 or 11;

d) a 5 occurring on the first die;

e) getting at least one 5;

f) a 5 occurring on both dice;

g) getting 5 as the absolute value of the difference of the numbers;

a) $\rightarrow 5/6$

b) $\rightarrow 1/9$

c) $\rightarrow 2/9$

d) $\rightarrow 1/6$

e) $\rightarrow \frac{1}{6} + \frac{1}{6} - \frac{1}{36} = \frac{11}{36}$

f) $\rightarrow 1/36$

g $\rightarrow \frac{1}{18}$

2) Suppose you roll two dice. Construct the probability distribution of the smaller of the two numbers. In the case of doubles, the larger and the smaller numbers are the same.

$P(1) = 11/36$

$P(2) = 9/36$

$P(3) = 7/36$

$P(4) = 5/36$

$P(5) = 3/36$

$P(6) = 1/36$

The main pump in a city works without failure for 1 month with probability 0.1, for 2 months with probability 0.3, and for 3 months with probability 0.6. The pump's backup works without failure for 1 month with probability 0.2 and for 2 months with probability 0.8. The main pump is used alone until it fails, then the backup pump kicks in and works alone until it fails. The two pumps are independent.

a) Find the expectation value of the working time of the main pump;

b) Find the expectation value of the working time of the backup pump;

c) Find the probability distribution of the total working time of the main pump followed by the backup pump.

d) Find the expectation value of the total working time of the main pump followed by the backup pump, using c). How does this value relate to a) and b)?

MAIN	
1	0.1
2	0.3
3	0.6

BACKUP	
1	0.2
2	0.8

a) μ_1 main $= 2.5 = \Sigma x P(x)$

b) μ_2 Back $= 1.8 = \Sigma x P(x)$

c) P(X) 2 Pumps

X	P(X)
2	0.02
3	0.14
4	0.36
5	0.48

d) $\mu_{Total} = 4.3$

=) SUM OF a) and b) !

4) Suppose you draw two cards from a 52-card deck. What is the probability that you draw

0.0059 = a) two aces if you replace the first card before drawing the second; $\rightarrow \frac{4}{52} \cdot \frac{4}{52} = \frac{4}{169}$

0.0045 = b) two aces if you don't replace the first card before drawing the second; $\rightarrow \frac{4}{52} \cdot \frac{3}{51} = \frac{6}{683}$

0.0060 = c) an ace followed by a king if you don't replace the first card before drawing the second; $\rightarrow \frac{4}{52} \cdot \frac{4}{51} =$

0.0059 = d) an ace followed by a king if you replace the first card before drawing the second; $\rightarrow \frac{4}{52} \cdot \frac{4}{52}$

0.25 = e) two cards of the same suit if you replace the first card before drawing the second; $\rightarrow \frac{13}{52}$

0.2353 = f) two cards of the same suit if you don't replace the first card before drawing the second one; $\rightarrow \frac{12}{51}$

5) There are two possible outcomes for newborns, boy or girl. About 51 % of newborns are boys.

a) List the possible outcomes for a family that has two boys (no twins).

b) Which of the outcomes is most likely? Which is least likely?

c) What is the probability that a two-child family has two boys? What are the assumptions?

a) BB · GG · BG GB

b) BB most likely
GG least "

c) $0.51 \cdot 0.51 = 0.2601$ / independent events

6) A committee of three students is to be formed from six juniors and five seniors. If selections are random, what is the probability that the committee will consist of all juniors? Of all seniors?

$$P(\text{all J}) = \frac{6}{11} \cdot \frac{5}{10} \cdot \frac{4}{9} = 0.12$$

$$P(\text{all S}) = \frac{5}{11} \cdot \frac{4}{10} \cdot \frac{3}{9} = 0.06$$

$$P(\text{all J or all S}) = 0.12 + 0.06 = 0.18$$

7) Five equally qualified students have applied to serve as student representatives. Two of them live on-campus, the other two off-campus. We randomly select three of the five applicants to serve.

a) How many groups of 3 students can you form from this pool of 5? List all possible samples.

b) Each of your samples is equally likely to occur. Construct the probability distribution of the random variable X, defined as the number of student representatives who live on-campus.

c) What is the probability that the committee has at least one student who lives on-campus?

AB | CDE
on | off

a) $\binom{5}{3} = \underline{\underline{10}}$

ABC
ABD
ABE

ACD
ADE
ACE

BDE
BCD
BCE
CDE

b)

c)

X	P(X)	
0	1/10	0.1
1	6/10	0.6
2	3/10	0.3

P(at least one) = $0.6 + 0.3 = 0.9$

8) The following table shows the percentage of US households with various numbers of color televisions. Suppose a duplex is occupied by two randomly selected households and you are interested in the random variable X, the total number of color TV sets in the duplex.

$(0,0) \rightarrow 0$ $P(0) = \cancel{0.012} = 0.012 \cdot 0.012 = 0.000144$

Number of Color TVs	Percentage of homes
0	1.2
1	27.4
2	35.9
3	21.8
4	9.5
5	4.2

a) Construct the probability distribution for X.

b) Find the expected value of X.

c) Compute the expected value of the number of color TVs in a single, randomly selected household. Compare with what you calculated in part b).

$2 \rightarrow (0,2)\ (1,1)\ (2,0)$

$3 \rightarrow (0\ 3)\ (1\ 2)\ (2\ 1)\ (30)$

$4 \rightarrow (0\ 4)\ (1,3)\ (2,2)\ (3,1)\ (4,0)$

$5 \rightarrow (0\ 5)\ (1,4)\ (2,3)\ (3,2)\ (4,1)\ (5,0)$

$6 \rightarrow (1,5)\ (2,4)\ (3,3)\ (4,2)\ (5,1)$

$7 \rightarrow (2,5)\ (3,4)\ (4,3)\ (3,4)$

$8 \rightarrow (3,5)\ (4,4)\ (5,3)$

$9 \rightarrow (4,5)\ (4,5)$

$10 \rightarrow (5,5)$

X	P(X)
0	0.000144
1	0.006576
2	0.083692
3	0.201964
4	0.250625
5	0.209592
6	0.138750
7	0.084176
8	0.027337
9	0.00798
10	0.001764

$\bar{X}_{2 Homs} = 4.5602$

$\bar{X}_{1 Home} = 2.236$

9) The amount of tips per week varies as follows

Dollar amount	Percentage
200	0.1
300	0.3
400	0.4
500	0.2

a) What are your expected weekly tips? What is the standard deviation (SD)?

b) A base salary of $60 a week is added. What are the expected weekly earnings? And the SD?

c) Each week you share 20 % of tips with other employees. What are the expected weekly earnings? What is the SD?

a) $\sum x\,P(x) = \$370$ $\quad$ SD = \$90

b) $\sum x\,P(x) = 60 + 370 = \$430$ $\quad$ SD = \$90

c) $60 + \underbrace{(0.8)}_{\text{keep}} \cdot 370 = \$356$ $\quad$ SD = $0.8 \cdot \cancel{370}\ 90 = \$72$

10) For each million tickets sold, the New York lottery awards one prize worth $50,000, nine prizes worth $5,000, ninety prizes worth $500 and nine hundred $50 prizes.

a) What is the expected value for a return on a ticket?

b) Tickets cost fifty cents each. How much does NY state earn for every million tickets sold?

c) What percentage of the income from the lottery was returned in prizes?

a) $\mu_X = \dfrac{185{,}000}{1{,}000{,}000} = 0.185\$ = 18.5¢$

b) $1000{,}000\,(0.50 - 0.185) = \$\,315{,}000$

c) earnings 500,000
gave out 185,000 $\longrightarrow$ percent $= \underline{37\%}$
185000/500000