Math 140B Midterm 2

(Dated: October 30th 2017)

Name: MAIA

SID:

Solutions

Write clearly and box all your numerical answers. Simplify all formulas to the very end. Round appropriately. Think before starting your calculations. Use the back for more space. Show all steps you are performing. No credit will be given for just giving the numerical answer without discussing the logic behind it.

Useful formulas:

Z-score for x from a normal distribution $N(\mu, \sigma)$ with mean μ and standard deviation σ

$$z = \frac{x - \mu}{\sigma} \qquad \text{mean } \bar{x} = \frac{\sum_{i=1}^{N} x_i}{N} \qquad \text{standard deviation } \sigma = \sqrt{\frac{\sum_{i=1}^{N} (x_i - \bar{x})^2}{N - 1}}$$

Standard deviation of a sampling distribution of mean p and sample size n:

$$\sigma = \sqrt{\frac{p(1-p)}{n}}$$

Standard error for a sample estimate $\hat{p}$ and sample size n:

$$\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Margin of error for a confidence interval corresponding to z^* and the above standard error:

$$z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Rule of thumb for applicability of normal distribution properties on a sample of size n

$$np > 10 \qquad n(1-p) > 10$$

Table C

Confidence Interval	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
z^*	0.674	0.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291

1) Three-hundred adults were aged 45 to 65 were recruited for an alcohol consumption experiment over the next five years. The test is to compare the effects on heart disease of moderate drinking of only red wine, only beer or only spirits. Outline the design of a completely randomized experiment using blocks of men and women. Be sure to specify how many subjects will be placed in each group.

150 M → 50 Red wine, 50 Beer, 50 Spirits

150 W → 50 Red wine, 50 Beer, 50 Spirits

→ compare

2) A couple plans to have three children. There are eight possible arrangements of boys (B) and girls (G). All arrangements are equally likely.

2a. Write down all eight arrangements and state the probability for each of these.

GGG GBG BBG BBB
GGB BGB
BGG GBB

P(any one configuration) = $\frac{1}{8}$

2b. What is the probability that the couple has two boys, in any order?

P(BBG) + P(GBB) + P(BGB) = $\frac{3}{8}$

2c. Find the distribution for the total number of boys. How many total numbers of boys can there be? What is the probability for each outcome?

P(0) = $\frac{1}{8}$ GGG

P(1) = $\frac{3}{8}$ BGG / GBG / GGB

P(2) = $\frac{3}{8}$ BBG / GBB / BGB

P(3) = $\frac{1}{8}$ BBB

3) A die is "loaded" so that the number 6 comes up too often and the number 1 comes up too seldom. The probability of 6 is modified to 0.2, the probabilities of 2, 3, 4, 5 are not affected with respect to a normal, not loaded die. Give the probability for all outcomes of the affected die

$P(1) = 1 - \frac{4}{6} - \frac{2}{10} = 1 - \frac{2}{3} - \frac{1}{5} = \frac{15-13}{15} = \frac{2}{15}$

$P(1) = \frac{2}{15}$ $P(2) = \frac{1}{6}$ $P(3) = \frac{1}{6}$ $P(4) = \frac{1}{6}$ $P(5) = \frac{1}{6}$ $P(6) = \frac{2}{10} = \frac{1}{5}$

4) In the U.S. 83% of Pinterest users are female. A simple random sample of 500 users is taken.

4a. What are the mean and standard deviation of the sampling distribution for the proportion of female Pinterest users?

$\sigma = \sqrt{\frac{0.83 \cdot 0.17}{500}} = \sigma = 0.0168$

$p = 0.83$

4b. What is the approximate probability that $\hat{p}$ is between 80% and 86%?

$\hat{p} > 0.86$ $z_1 = \frac{0.86 - 0.83}{\sigma} = \frac{0.03}{0.0168} = 1.79$; $z_2 = -1.79$

$z_1 \rightarrow$ cum area $= 0.9633$

$z_2 \rightarrow$ " $= 0.0367$

$P(\hat{p}$ between $80 \cdot 86\%) = 0.9266 =$ 92.66%

4c. What standard deviation must $\hat{p}$ have so that 95% of all samples gives a $\hat{p}$ within 3% of p?

95% $\rightarrow$ 2 SD

$2 = \frac{0.03}{\sigma}$ $\sigma = \frac{0.03}{2}$ 0.015

5) For each of the following, indicate whether it is a parameter (P) or a statistic (S):

5a. The fraction of all Americans who have never seen an ocean in person P

5b. The mean number of spots that a sample of 100 ladybugs have S

5c. The proportion of 100 randomly chosen homes in Milwaukee that have a swimming pool S

5d. The percent of all defective iPhones made by Apple P

5e. The mean height of all kindergarten children in California. P

6) A simple random sample of 2673 heterosexual adults is taken and 170 had more than one sexual partner in the last year. Calculate the following confidence intervals for the proportion of all heterosexual adults with multiple partners

$\hat{p} = \frac{170}{2673} = 0.064$

$SE = \sqrt{\frac{0.064 \cdot 0.936}{2673}} = 0.005$

6a) the 95% confidence interval

$0.064 \pm 2 \cdot 0.005 =$

0.064 ± 0.01

$(0.054; 0.074)$

6b) the 80% confidence interval

$0.064 \pm 1.282 \cdot 0.005$

$(0.058; 0.070)$

6c) the 90% confidence interval

$0.064 \pm 1.645 \cdot 0.005$ $(0.056; 0.072)$

6d) Interpret in words these results. Which one is largest? Why?

6a) is largest. The more confident we want to be, the more data we must include in our estimate

7) A simple random sample of 450 teens shows that 80% own an Ipad. Another simple random sample of 4500 teens reveals that 80% own an Ipad. For **both** samples calculate the 90% confidence interval for the percentage of the teen population that owns an Ipad. Which one is smaller? Why?

a) $0.8 \pm 1.645 \sqrt{\frac{0.8 \cdot 0.2}{450}} = 0.8 \pm 0.024$

$(0.776, 0.824)$

b) $0.8 \pm 1.645 \sqrt{\frac{0.8 \cdot 0.2}{4500}}$ 0.8 ± 0.0075

$(0.7925; 0.8075)$

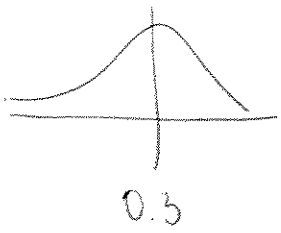
b) is smallest. Larger samples yield smaller error of margin.

8) Thirty percent of CSUN students wear contact lenses. We randomly pick 100 students. Let $\hat{p}$ represent the proportion of students who wear contacts.

8a. Check that the rule of thumb of applicability for a normal distribution is verified.

$0.3 \cdot 100 = 30 > 10$ (ok)
$0.7 \cdot 100 = 70 > 10$ (ok) !

8b. Draw the associated sampling distribution specifying the mean and the standard deviation



$SD = \sqrt{\frac{0.3 \cdot 0.7}{100}} = 0.046 = SD$

8c. Find the probability that less than 20% of this sample wear contacts.

$z = \frac{0.2 - 0.3}{0.046} = -2.17$ cum area = 0.015

$P(< 20\%) = 1.5\%$

8d. Find the probability that more than one third of this sample wear contacts.

$z = \frac{0.333 - 0.3}{0.046} = 0.724$

cum area = 0.7642

$P(> \frac{1}{3}) = 1 - 0.7642 = 0.2358$

$= 23.58\%$

Table entry for z is the area under the standard Normal curve to the left of z.

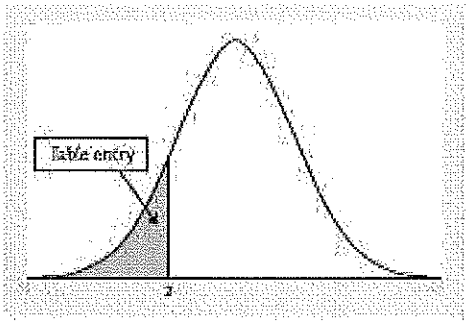


TABLE A STANDARD NORMAL CUMULATIVE PROPORTIONS

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Table entry for z is the area under the standard Normal curve to the left of z.

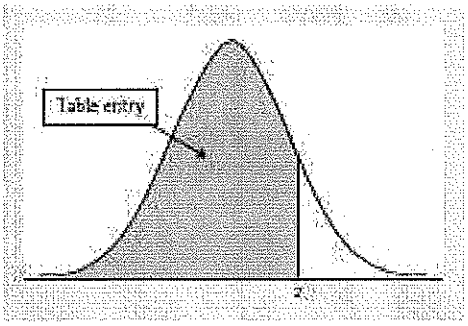


TABLE A STANDARD NORMAL CUMULATIVE PROPORTIONS (*CONTINUED*)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998