Math 150A Midterm 1

(Dated: September 29, 2011)

Name: MAMA

SID:

Write clearly and box all your answers. Simplify all formulas to the very end. No calculators allowed. Do not work out of memory, rather think before starting. Use the back for more space. Show all steps you are performing and state all theorems you are using, if any.

1) Differentiate using the definition of derivative $y = \dfrac{x^2-1}{2x-3}$

$$\frac{(x+h)^2-1}{(2x+2h-3)} - \frac{x^2-1}{2x-3}$$

$$\frac{(x^2+h^2+2hx)(2x-3) - (x^2-1)(2x+2h-3)}{h\,(2x+2h-3)(2x-3)}$$

$$\frac{2x(2x-3) - 2(x^2-1)}{(2x-3)^2}$$

$$4x^2 - 6x - 2x^2 + 2$$

problem 1

2) Differentiate using any method of your choice $y = \dfrac{x^2+4x+3}{\sqrt{x}}$

$$= \cancel{2x^3} - \cancel{3x^2} + 2xh^2 - 3h^2 + 4hx^2 - 6hx - \cancel{2x} + \cancel{3} - \cancel{2x^3} - 2hx^2 + \cancel{3x^2} + \cancel{2x} + 2h - \cancel{3} \;/\; h\,(2x-3)^2$$

$$= \frac{1}{(2x-3)^2}\left(2xh - 3h - 6x + 2h + 4x^2 - 2x^2\right) = \frac{4x^2 - 6x + 2 - 2x^2}{(2x-3)^2} \quad \text{ok}$$

$$= \frac{2x^2-6x+2}{(2x-3)^2}$$

$\frac{3}{2}x^{+1/2} + \frac{4}{2\sqrt{x}} + 3\left(-\frac{1}{2}\right)x^{-3/2}$ problem 2

3) Find the equation of the tangent line to the curve $y = \dfrac{1}{1+x^2}$ at the point $(-1, \frac{1}{2})$.

$\frac{3}{2}\sqrt{x} + \frac{2}{\sqrt{x}} - \frac{3}{2}x^{-3/2} = y'$

$$y' = \frac{-2x}{(1+x^2)^2}$$

$$m = \frac{+2}{4} = +\frac{1}{2}$$

$$y = +\frac{1}{2}x + b$$

$$\frac{1}{2} = -\frac{1}{2} + b \qquad b = 1$$

$$y = +\frac{1}{2}x + 1$$

3

$\frac{14}{8}$

$\frac{11}{2}$

4) Consider the function: $g(x) = \dfrac{x^2 + x - 6}{|x - 2|}$. Find its limits as $x \to 2^+$ and $x \to 2^-$. Does the limit as $x \to 2$ exist?

$\lim_{x \to 2^+} \dfrac{x^2 + x - 6}{|x - 2|} = \lim_{x \to 2^+} \dfrac{(x+3)(x-2)}{(x-2)} = 5$

$\lim_{x \to 2^-} = = \dfrac{(x+3)(x-2)}{(2-x)} = -5$

the limit does not exist b/c the above are different

5) Evaluate the limit, if it exists $\lim_{x \to -4} \dfrac{\sqrt{x^2 + 9} - 5}{x + 4}$

$\lim_{x \to -4} \dfrac{x^2 + 9 - 25}{(x+4)(\sqrt{x^2+9} + 5)} = \dfrac{(x+4)(x-4)}{(x+4)(\quad)} = \dfrac{-8}{10} = \dfrac{-4}{5}$

6) Find the domain of the function $y = \sqrt{3 - x} - \sqrt{2 + x}$

$x \leq 3$ and $x \geq -2$

$-2 \leq x \leq 3$

$f \circ f = x + \frac{1}{x} + \frac{1}{x+\frac{1}{x}} = x + \frac{1}{x} + \frac{x}{x^2+1}$

$= \frac{x^4+x^2+x+1+x^2}{x(x^2+1)} = \boxed{\frac{x^4+3x^2+1}{x(x^2+1)}}$

7) Find the functions $y = f \circ g$, $g \circ f$, $f \circ f$ and $g \circ g$ where $f(x) = x + \frac{1}{x}$ and $g(x) = \frac{x+1}{x+2}$. Simplify to the very end.

$f \circ g = \frac{x+1}{x+2} + \frac{x+2}{x+1} = \frac{x^2+2x+1+x^2+4x+4}{(x+2)(x+1)} = \boxed{\frac{2x^2+6x+5}{(x+2)(x+1)}}$

$g \circ f = \frac{x+\frac{1}{x}+1}{x+\frac{1}{x}+2} = \boxed{\frac{x^2+x+1}{x^2+2x+1}}$

$g \circ g = \frac{\frac{x+1}{x+2}+1}{\frac{x+1}{x+2}+2} = \frac{x+1+x+2}{x+1+2x+4} = \boxed{\frac{2x+3}{3x+5}}$

8) Find the values of a and b that make the function continuous everywhere

$$f(x) = \begin{cases} \frac{x^2-4}{x-2} & \text{if } x < 2 \\ ax^2 - bx + 3 & \text{if } 2 \le x < 3 \\ 2x - a + b & \text{if } x \ge 3 \end{cases} \quad (1)$$

$\lim_{x \to 2^-} = x+2 = 4$

$\lim_{x \to 2^+} = 4a - 2b + 3$

$\lim_{x \to 3^-} = 9a - 3b + 3$

$\lim_{x \to 3^+} = 6 - a + b$

$4a - 2b + 3 = 4$

$9a - 3b + 3 = 6a - a + b$

$(-2)\quad 4a - 2b = 1 \qquad 2b = 4a - 1$

$10a - 4b = 3$

$2a = 1$

$\boxed{a = 1/2,\ b = 1/2}$

9) Differentiate $x^2 \sin x \tan x$

$2x \sin x \tan x + x^2 \cos x \tan x + x^2 \sin x \sec^2 x$

$= \boxed{2x \sin x \tan x + x^2 \sin x + x^2 \sin x \sec^2 x}$

10) Find an equation of the tangent line to the curve $y = 2x \sin x$ for $x = \pi/2$.

$y' = 2\sin x + 2x\cos x$

$m = 2\sin\frac{\pi}{2} + 2\frac{\pi}{2}\cos\frac{\pi}{2} = 2$

$x = \frac{\pi}{2} \quad y = 2\frac{\pi}{2} = \pi$

$y = 2x + b$

$\pi = 2\frac{\pi}{2} + b$

$b = 0$

$\boxed{y = 2x}$

11) Differentiate $y = \dfrac{1+2x}{3-4x}$

$\dfrac{2(3-4x) + 4(1+2x)}{(3-4x)^2} = \dfrac{6 - 8x + 4 + 8x}{(\quad)^2}$

$= \dfrac{10}{(\quad)^2} = \boxed{\dfrac{10}{(3-4x)^2}}$

12) Differentiate $y = \dfrac{x \sin x}{1+x}$

$\dfrac{(x\cos x + \sin x)(1+x) - x\sin x}{(1+x)^2} =$

$\boxed{\dfrac{x\cos x + \sin x + x^2\cos x + x\sin x - x\sin x}{(1+x)^2}}$

13) Differentiate $y = \dfrac{\sec x}{1+\sec x}$

$$\frac{\sec\tan x(1+\sec x) - \sec x\tan x\cdot\sec x}{(1+\sec x)^2} =$$

$$= \boxed{\frac{\sec x\tan x}{(1+\sec x)^2}} \quad \text{or} \quad y = \frac{1}{\cos x+1}$$

$$y' = \boxed{\frac{+\sin x}{(\cos x+1)^2}}$$

$$= \frac{\sin x/\cancel{\cos^2 x}}{\cancel{\cos x}(1+\cos x)^2/\cancel{\cos^2 x}}$$

$$= \sin x/(1+\cos x)^2$$

14) Find the limit $\lim\limits_{x\to 0}\dfrac{x}{\sqrt{1+3x}-1}$

$$\frac{\cancel{x}\cdot(\sqrt{1+3x}+1)}{\cancel{1+3x}-1} = \boxed{\frac{2}{3}}$$