## Math 140 <br> Introductory Statistics



## Statistics, Student Solutions Manual: From Data to Decision [Paperback] Ann E. Watkins (Author), Richard L. Scheaffer (Author), George W. Cobb (Author) No customer reviews yet. Be the first. <br> Price: $\$ 35.00$ \& this item ships for FREE with Super Saver Shipping. Details <br> In Stock. <br> Ships from and sold by Amazon.com. Gift-wrap available. <br> Only 1 left in stock--order soon (more on the way). <br> Want it delivered Thursday, September 16? Order it in the next 23 hours and 30 minutes, and choose One-Day Shipping at checkout. Details <br> 6 new from $\$ 33.25 \quad 1$ used from $\$ 35.95$

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## Recentering

## Shift everything by a number c

The max will be shifted
The min will be shifted
Q1, Q3 will be shifted
The average and the median will be shifted
Will the range shift?

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Will the range shift?

## No!

## Recentering

Standard deviation is

$$
\sigma_{n-1}=\sqrt{\frac{\sum(x-\bar{x})^{2}}{n-1}}
$$

If we shift the data $x$, also $\bar{x}$ will be shifted $x+c$ and $\bar{x}+c$

Does their difference get shifted?

## Recentering

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Does their difference get shifted?

No!

## Recentering

Shift everything by a number c
The max will be shifted
The min will be shifted Q1, Q3 will be shifted
The average and the median will be shifted
The range WILL NOT shift The standard deviation WILL NOT shift

## Rescaling

Multiply everything by a number d
The max will be rescaled
The min will be rescaled Q1, Q3 will be rescaled
The average and the median will be rescaled
The range WILL rescale
The standard deviation WILL rescale

## Rescaling

# All number summaries will be rescaled by d 

## Recentering

Except for the range and the SD all number summaries will be shifted by c.

## An example

The distribution of the SAT scores for the University of Washington was roughly normal in shape, with mean 1055 and standard deviation 200.

1. What percentage of scores were 920 or below?
2. What SAT score separates the lowest $25 \%$ of the SAT scores from the rest?

## An example

The distribution of the SAT scores for the University of Washington was roughly normal in shape, with mean 1055 and standard deviation 200.

1. What percentage of scores were 920 or below?
2. What SAT score separates the lowest $25 \%$ of the SAT scores from the rest?

We already know that $68 \%$ of data is between 855 and 1255

## Unknown percentage problem

1. What percentage of scores were 920 or below?

Percentage $=$ ?

Given $x$ (a score), find the percentage

## Unknown value problem

2. What SAT score separates the lowest $25 \%$ of the scores from the rest?


Given the percentage $P$, find the score $z$

## Standard normal distribution

The normal distribution with mean $=0$ and $\mathrm{SD}=1$

The area under the curve equals 1 (or 100\%)


## Standard normal distribution

Any normal distribution can be rescaled or recentered to give you the normal distribution

STANDARDIZING or<br>CONVERTING TO STANDARD UNITS

## Given the score $z$ find $P$ Unknown percentage

Table A. Page 759

Use the units and the first decimal to locate the row and the closest hundredths digits to locate the column.

The number found is the percentage to the left of z .


## Symbols

Data prior to standardization is indicated with $x$
Data after standardization is indicated with z


SAT scores $x$


Standardized scores z

# How do we "standardize?" 

Recenter and Rescale
Recentering $=$ Make the mean zero

## How do we "standardize?"

Recenter and Rescale
Recentering $=$ Make the mean zero

1. Subtract the mean from all values

$$
x-\bar{x}
$$

first step towards new values z

## How do we "standardize?"

Recenter and Rescale
Rescale $=$ Make the SD one
2. Divide all previous values by the SD

$$
\mathrm{z}=(\mathrm{x}-\overline{\mathrm{x}}) / \mathrm{SD}
$$

Final step towards new values z
We say that $z$ is the $z$-score is the number of standard deviations above or below the mean

## How do we "standardize?"

$$
z=(x-\bar{x}) / S D
$$

## How do we "un standardize?"

Do the reverse operation

$$
x=\bar{x}+z^{*} S D
$$

## An example

The distribution of the SAT scores for the University of Washington was roughly normal in shape, with mean 1055 and standard deviation 200.

1. What percentage of scores were 920 or below?
2. What SAT score separates the lowest $25 \%$ of the SAT scores from the rest?

YOU TRY

## An example

$$
\text { Mean }=\bar{x}=1055 \mathrm{SD}=200 .
$$

1. What percentage of scores were 920 or below?

$$
z=(x-1055) / 200
$$

The $x$ we care about is 920

$$
z=(920-1055) / 200=-0.675
$$

Check on table $\sim$ about $25 \%$

## An example

Does this also answer question 2?
2. What SAT score separates the lowest $25 \%$ of the SAT scores from the rest?

## Z- scores

## moving from x to z

Example: Mean is 610 and the SD is 69
What is the z score for a value of 560 ?

## Z- scores

## Is the z we get from our initial x (moving from $x$ to $z$ )

Example: Mean is 610 and the SD is 69
What is the z score for a value of 560 ?

$$
\mathrm{z}=(\mathrm{x}-610) / 69
$$

The $x$ we care about is 560

$$
z=(560-610) / 69=-0.725
$$

## x- scores

Example: Mean is 610 and the SD is 69
What is the x score if the z -score is 1.6 ?

$$
x=610+z^{*} 69
$$

The z we care about is 1.6

$$
x=610+1.6^{*} 69=720
$$

## z-scores are useful for comparisons

|  | Mean | SD |
| :--- | :--- | :--- |
| Heart <br> disease | 219 | 46 |
| Cancer | 194 | 30 |

Death rates per 100,000 people in the entire US
Alaska: 94 deaths per 100,000 people - heart disease 110 deaths per 100,000 people - cancer

Which is more extreme with respect to the nation?

## You calculate the z-scores

|  | mean | SD |
| :--- | :--- | :--- |
| Heart <br> disease | 219 | 46 |
| Cancer | 194 | 30 |

Death rates per 100,000 people in the entire US
Alaska: 94 deaths per 100,000 people - heart disease 110 deaths per 100,000 people - cancer

Which is more extreme with respect to the nation?

## Let's calculate the z-scores

|  | mean | SD |
| :--- | :--- | :--- |
| Heart <br> disease | 219 | 46 |
| Cancer | 194 | 30 |

Heart disease $=(94-219) / 46=-2.72$

$$
\text { Cancer }=(110-194) / 30=-2.80
$$

They are both very distant from the mean but Death from cancer is slightly more extreme

## Cancer z-score in Alaska



## How tall are US men?

## Heights are normally distributed Mean $=70.4$ inches $S D=3.0$ inches

1. What percentage are more than 74 inches tall?


## How tall are US men?

## Heights are normally distributed Mean $=70.4$ inches $\mathrm{SD}=3.0$ inches

2. What percentage is between 72 and 74 inches tall?


Total area under curve is 1

## How tall are US men?

Heights are normally distributed
Mean $=70.4$ inches $\mathrm{SD}=3.0$ inches

$$
\begin{gathered}
\text { Above 74: } \\
z=(74-70.4) / 3=1.20
\end{gathered}
$$

Check table

Area to the left is about 0.885 .
Area to the right is $1-0.885=0.115$
The percentage is $11.5 \%$

## How tall are US men?

Heights are normally distributed Mean $=70.4$ inches $\mathrm{SD}=3.0$ inches

Above 72:

$$
z=(72-70.4) / 3=0.53
$$

Check table
Area to the left is about 0.702 .
Area to the right is $1-0.702=0.298$

## How tall are US men?



Area to the right of 74 is about 0.115 Area to the right is 72 is about 0.298

The difference is $0.298-0.115=0.183$

## How tall are US men?



The percentage of US men whose height is between 72 and 74 inches is
18.3\%

## How tall are US women?



Heights are normally distributed Mean $=65.1$ inches $S D=2.6$ inches

What value separates the lowest 75\% from the highest $25 \%$ ?

## How tall are US women?



Look up the chart for the z -score corresponding to 75\%

$$
\mathrm{z}=0.67 \text { approximately }
$$

## How tall are US women?


calculate x
$x=65.1+0.67$ * $2.6=66.8$ inches

## Central Intervals for Normal

 Distributions $68 \%$ of the values lie within 1 standard deviation of the mean.$90 \%$ of the values lie within 1.645 standard deviations of the mean.
$95 \%$ of the values lie within 1.96 (or about 2) standard deviations of the mean.
99.7\% (or almost all) of the values lie within 3 standard deviations of the mean.


## Where are the middle $90 \%$ of data?

## Cancer: mean $=194$, SD $=30$

$90 \%$ of the values lie within 1.645 standard deviations of the mean.


90\% of data for the standardized distribution are between -1.645 and 1.645

## Where are the middle $90 \%$ of data?

$$
\begin{gathered}
\text { From } \mathrm{z} \text { to } \mathrm{x} \\
\mathrm{x}=194+1.645 * 30=243 \\
x=194-1.645 * 30=145
\end{gathered}
$$

Cancer data


## Hk

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