

Math 140

Introductory Statistics

First midterm

September 23 2010

A problem from yr homework E21, page 18

young

22, 25, 33, 35, 38, 48, 53, 55, 55, 55, 55, 56, 59, 64

a) How many ways to select 10 workers from 14?

A problem from yr homework E21, page 18

young

22, 25, 33, 35, 38, 48, 53, 55, 55, 55, 55, 56, 59, 64

a) How many ways to select 10 workers from 14?

$$\binom{14}{10} = 1001$$

A problem from yr homework E21, page 18

young

22, 25, 33, 35, 38, 48, 53, 55, 55, 55, 55, 56, 59, 64

b) If 10 workers are laid off, how many older ones might be among them?

A problem from yr homework E21, page 18

young

22, 25, 33, 35, 38, 48, 53, 55, 55, 55, 55, 56, 59, 64

b) If 10 workers are laid off, how many older ones might be among them?

Start laying off the younger ones

A problem from yr homework E21, page 18

young

22, 25, 33, 35, 38, 48, 53, 55, 55, 55, 55, 56, 59, 64

The best we can do is lay off only 5 older people

We need AT LEAST 5 older people

The answer is from 5 to 9

A problem from yr homework E21, page 18

young

22, 25, 33, 35, 38, 48, 53, 55, 55, 55, 55, 56, 59, 64

3) How many ways can we lay off
Seven old people & three young ones?

A problem from yr homework E21, page 18

young

22, 25, 33, 35, 38, 48, 53, 55, 55, 55, 55, 56, 59, 64

3) How many ways can we lay off

Seven old people & three young ones?

Since there are 9 old people
We pick 7 from the group

$$\binom{9}{7} = 36$$

A problem from yr homework

E21, page 18

young

22, 25, 33, 35, 38, 48, 53, 55, 55, 55, 55, 56, 59, 64

3) How many ways can we lay off

Seven old people & three young ones?

Since there are 5 young people

We pick 3 from the group

$$\binom{5}{3} = 10$$

A problem from yr homework E21, page 18

young

22, 25, 33, 35, 38, 48, 53, 55, 55, 55, 55, 56, 59, 64

For each way of making a 7-group of old people
we have 10 ways of making a 3-group of young people

$$\binom{9}{7} = 36 \quad \text{times} \quad \binom{5}{3} = 10 \quad = 360$$

A problem from yr homework E21, page 18

young

22, 25, 33, 35, 38, 48, 53, 55, 55, 55, 55, 56, 59, 64

Repeat for 8 old & 2 young and 9 old & 1 young

90 5

A problem from yr homework E21, page 18

young

22, 25, 33, 35, 38, 48, 53, 55, 55, 55, 55, 56, 59, 64

d) What is the probability that we will get 7 or more older people if we select 10 workers at random?

A problem from yr homework

E21, page 18

young

22, 25, 33, 35, 38, 48, 53, 55, 55, 55, 55, 56, 59, 64

d) What is the probability that we will get 7 or more older people if we select 10 workers at random?

$$360 + 90 + 5 = 455 \text{ out of } 1001$$

360 ways exactly 7 90 ways exactly 8
5 ways exactly 9

A problem from yr homework E21, page 18

young

22, 25, 33, 35, 38, 48, 53, 55, 55, 55, 55, 56, 59, 64

d) What is the probability that we will get 7 or more older people if we select 10 workers at random?

$$360 + 90 + 5 = 455 \text{ out of } 1001$$

$$\text{Probability} = \frac{455}{1001}$$

3. Stemplots

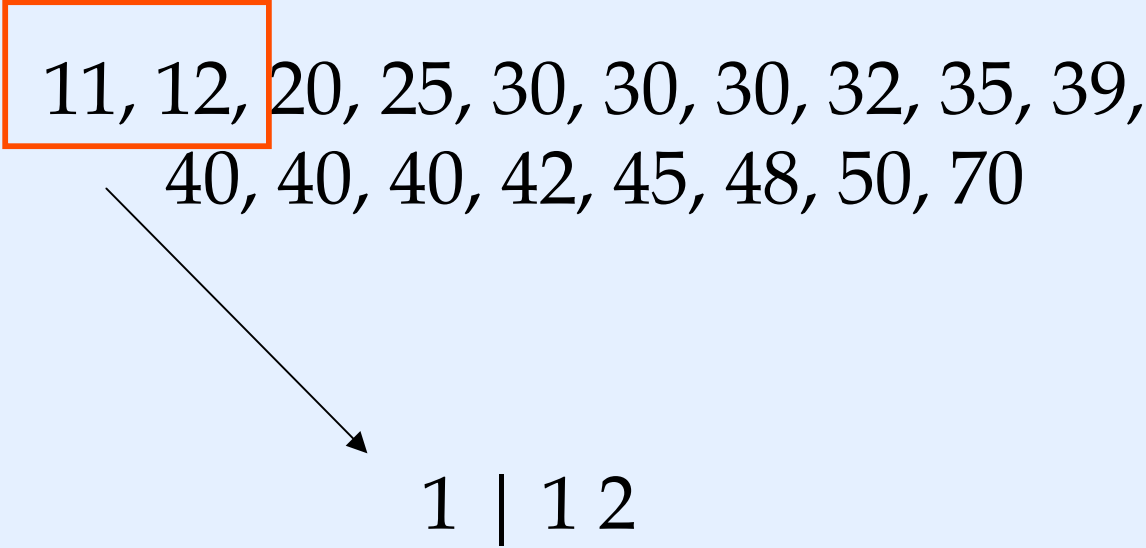
Speeds of mammals (mph)

11, 12, 20, 25, 30, 30, 30, 32, 35, 39,
40, 40, 40, 42, 45, 48, 50, 70

3. Stemplots

Speeds of mammals (mph)

11, 12, 20, 25, 30, 30, 30, 32, 35, 39,
40, 40, 40, 42, 45, 48, 50, 70



1 | 1 2

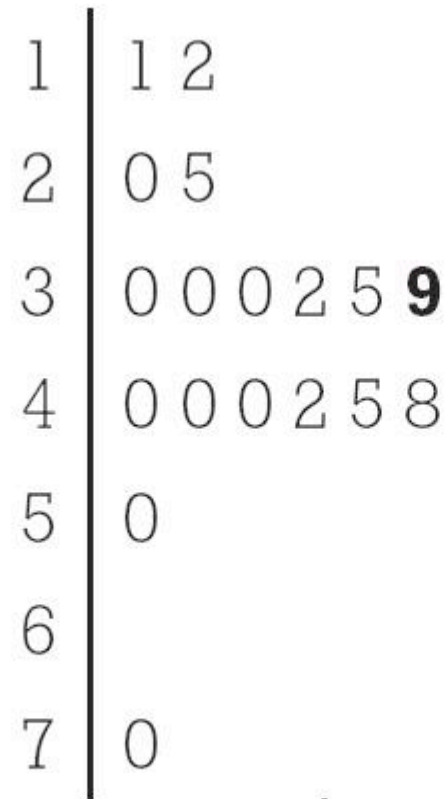
3. Stemplots

Speeds of mammals (mph)

11, 12, 20, 25, 30, 30, 30, 32, 35, 39,
40, 40, 40, 42, 45, 48, 50, 70

3 | 0 0 0 2 5 9

3. Stemplots



3 | 9 represents 39 mph

3. Stemplots

Or stem-and-leaf plots

Numbers on the left are called stems
(the first digits of the data value)

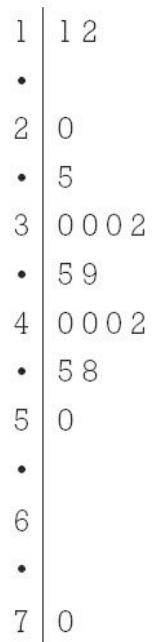
Numbers on the right are called leaves
(the last digit of the data value)

Split stemplots

```
1 | 1 2
•
2 | 0
• 5
3 | 0 0 0 2
• 5 9
4 | 0 0 0 2
• 5 8
5 | 0
•
6 |
•
7 | 0
```

3 | 9 represents 39 mph

Split stemplots

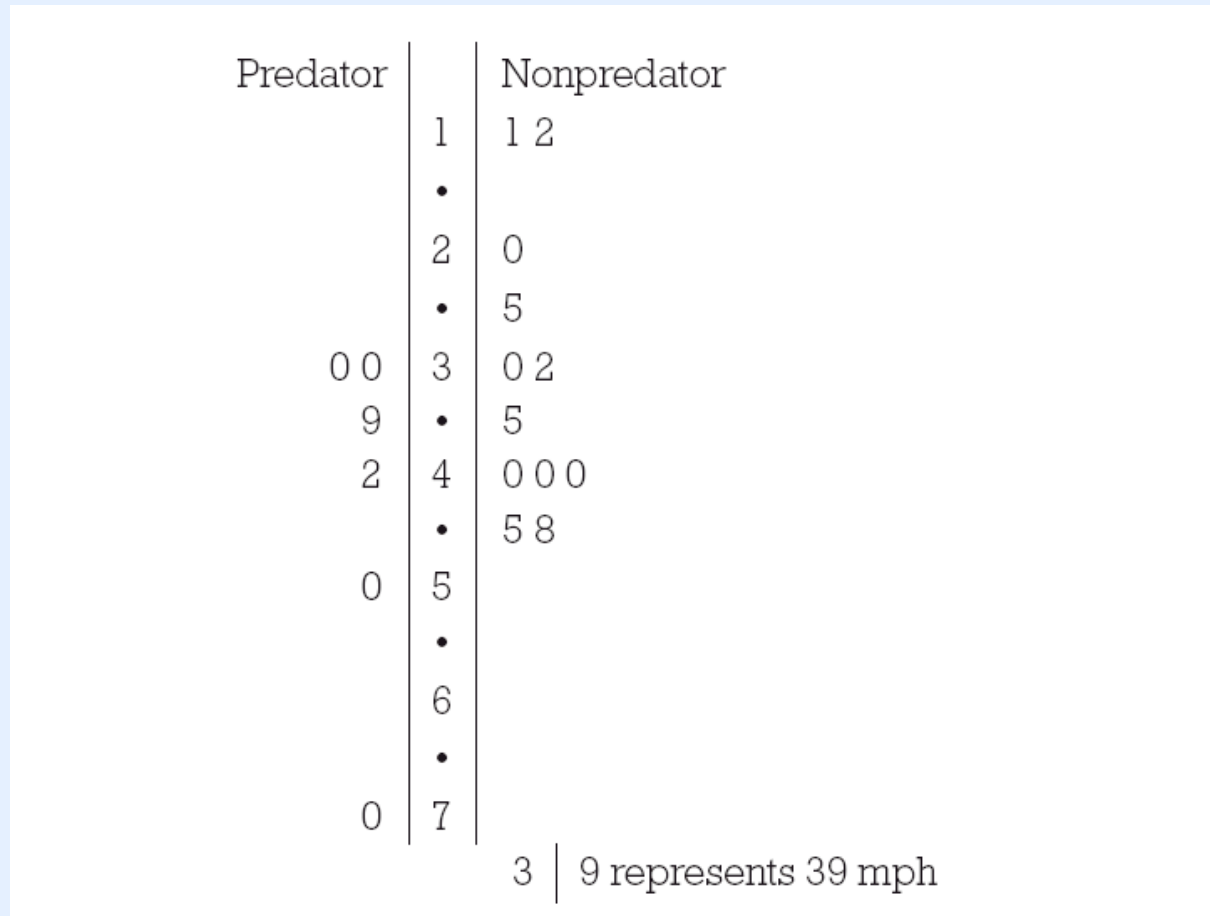


3 | 9 represents 39 mph

The unit digits
0,1,2,3,4 are associated
with the first stem and
they are placed on the first
line.

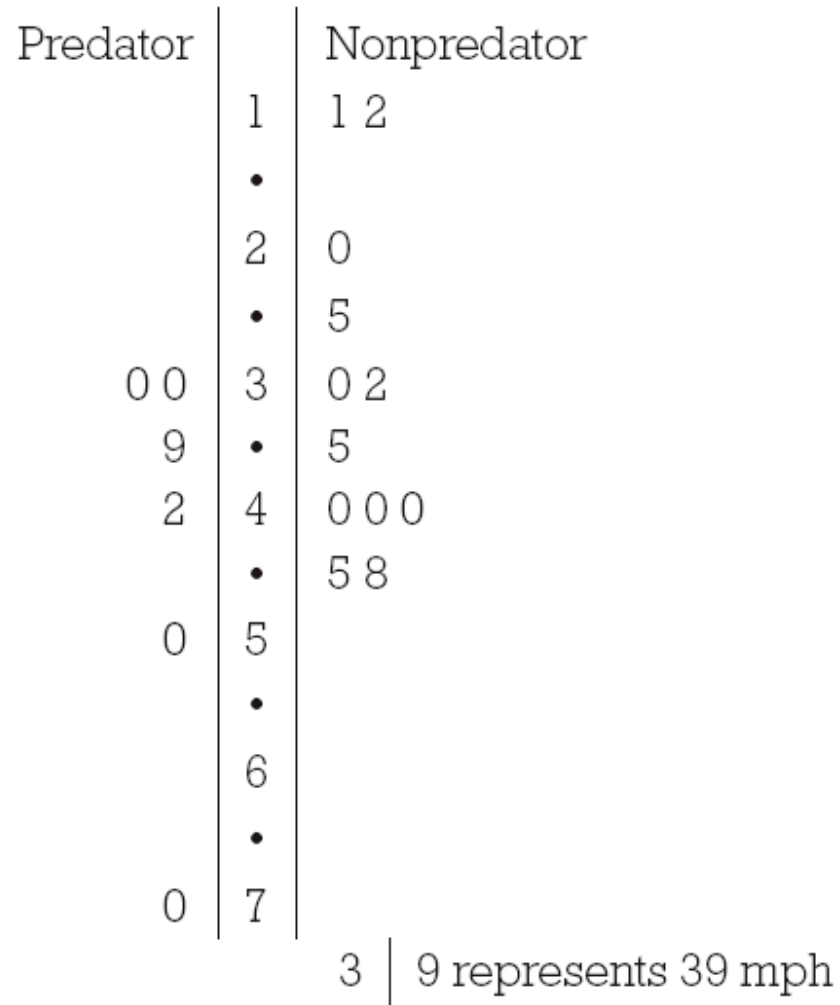
The unit digits 5,6,7,8,9 are
associated with the second
stem
and they are placed on the
second line.

Back to back stemplots



The data is differentiated on whether the mammals are predators or non-predators

Who has the faster speed?



Calculating medians and quartiles

Stem-and-leaf of Speeds N = 18
Leaf Unit = 1.0 N* = 21

2	1 12	
2	1	
3	2 0	
4	2 5	Lower quartile = 30
8	3 ①002	Median = 37
(2)	3 5 9	
8	4 000②	Upper quartile = 42
4	4 58	
2	5 0	
1	5	
1	6	
1	6	
1	7 0	

Stemplots work best when

Small number of values to plot

Want to keep track of individual values
(at least approximately)

Want to see shape of distribution

Have two or more groups that we want to compare

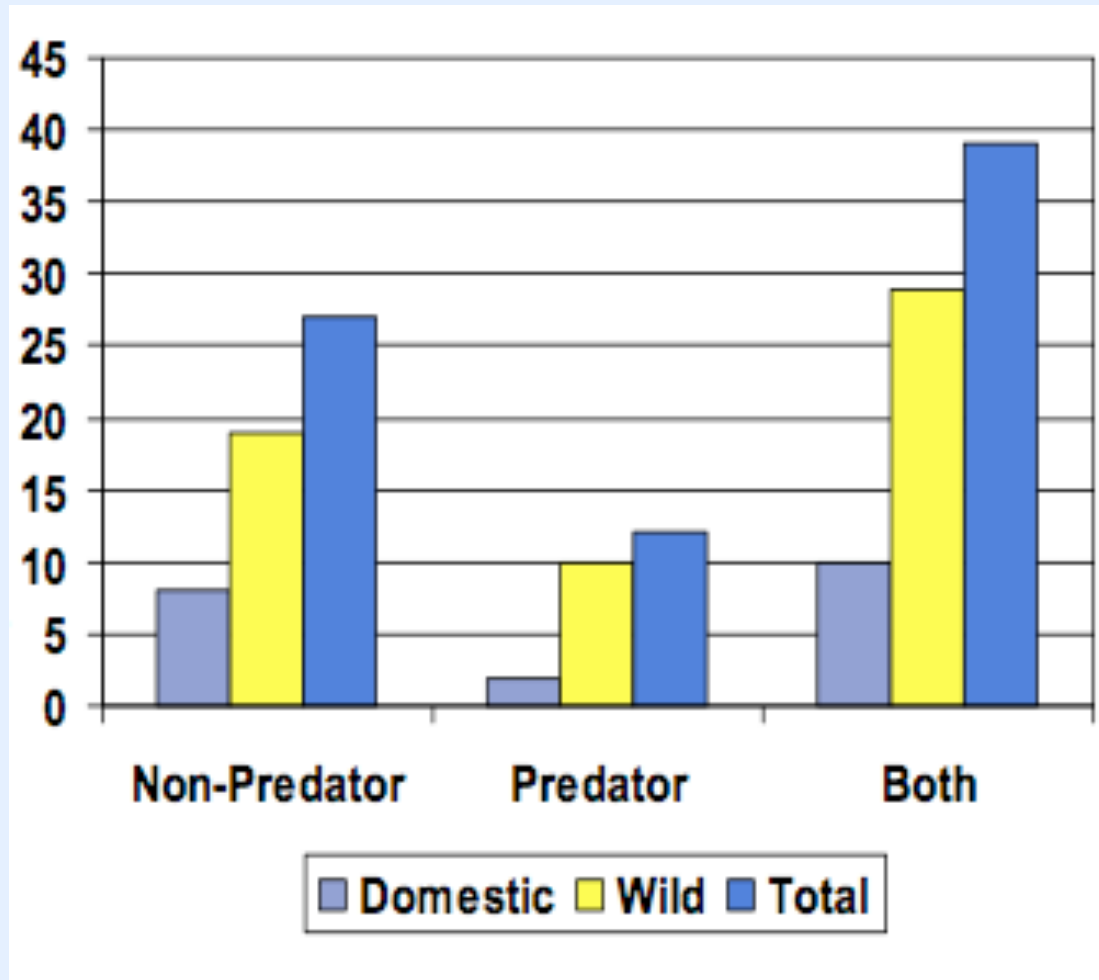
4. Bar graphs

One bar for each category

The height of the bar tells the frequency

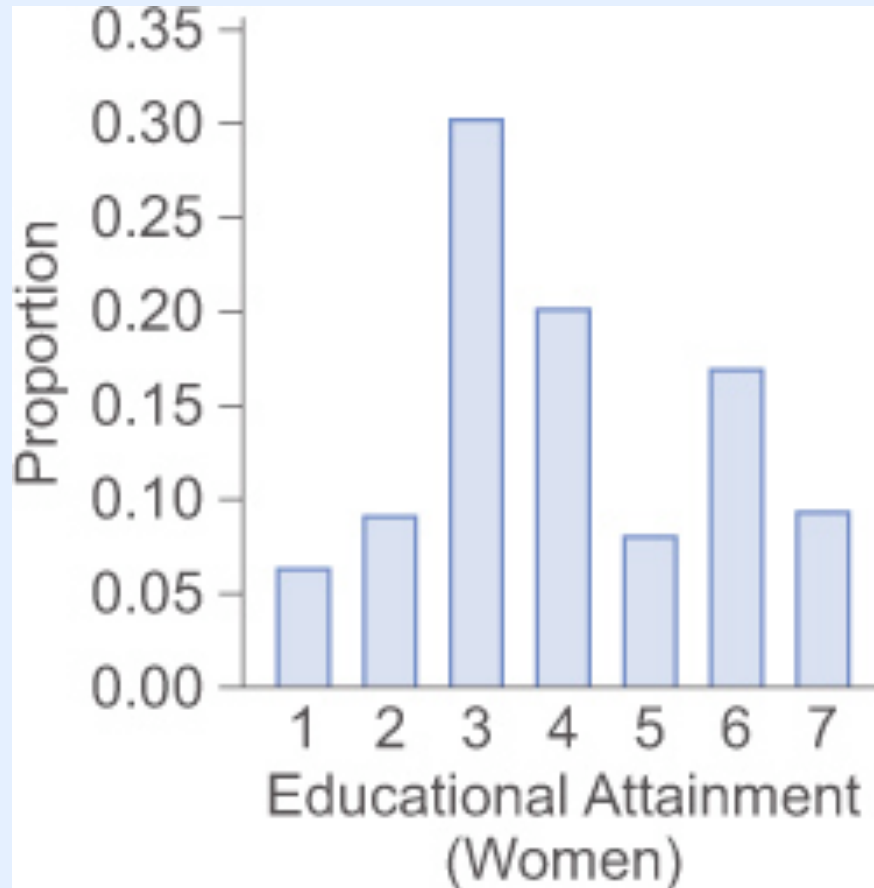
Bar graphs have categories in the horizontal axis, as opposed to histograms which have measurements.

Bar graphs



Bars are separated so there is no confusion

US working women age 25 or older



1. Less than 9th grade
2. 9th to 12th grade, no diploma
3. High school grad
4. Some college, no degree
5. Associate degree
6. Bachelor degree
7. Phd or professional degree

Modal category: category with highest frequency

Measures of center: mean and median

Earlier we used visual estimates
to find out center and spread

Now we will learn how to calculate them exactly

Measures of Center

Mean

Median

Measures of Spread

Standard Deviation

Inter Quartile Range

Center: Mean (average)

Denoted as \bar{x}

$$\bar{x} = \frac{\text{sum of values}}{\text{number of values}} = \frac{\sum x}{n}$$

Example: 5, 12, 34, 18, 37, 11, 9, 21, 30, 6

$$\bar{x} = \frac{5 + 12 + 34 + 18 + 37 + 11 + 9 + 21 + 30 + 6}{10} = 18.3$$

Center: Median

Denoted as Q_2

Divides data into equal halves.

List all n values in increasing order
and find the middle one.

If n is odd the middle one is $(n+1)/2$

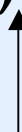
If n is even the median is the average of the two
values on each side of $(n+1)/2$

Center: Median

Example: 5, 6, 9, 11, 12, 18, 21, 30, 34, 37, 41

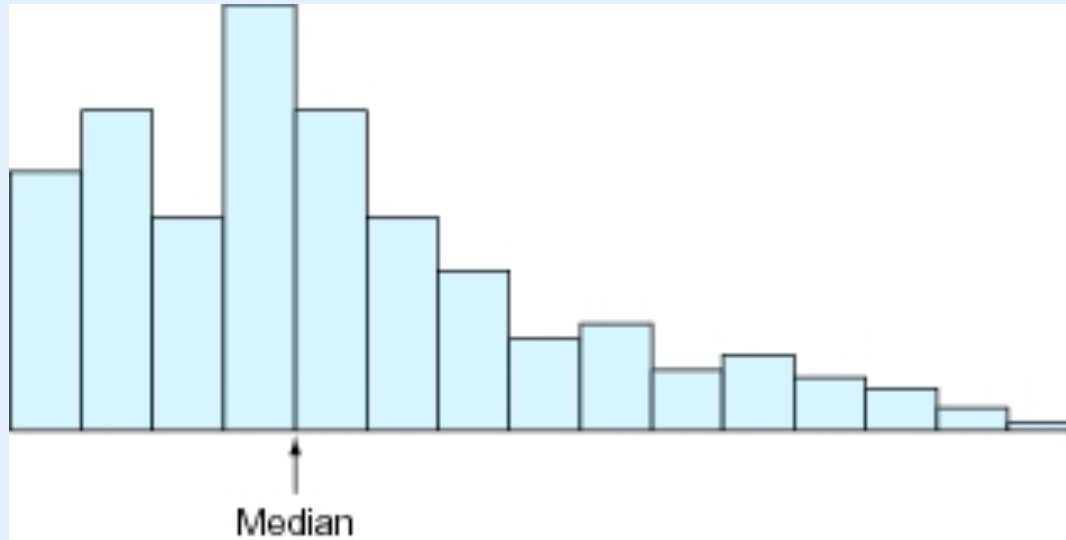
$n=11$ median is $(n+1)/2 = 18$

Example: 5, 6, 9, 11, 12, 18, 21, 30, 34, 37



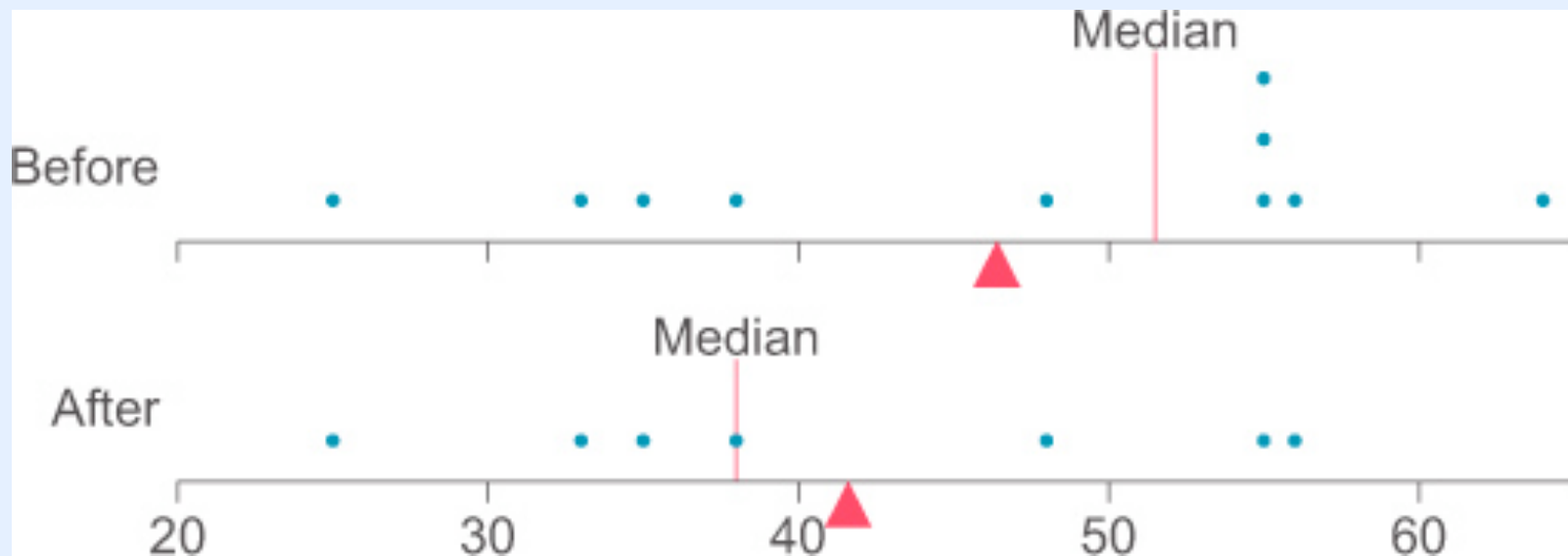
$n=10$ median is $(12 + 18)/2 = 15$

Center: Median



If placed in a histogram the median will divide the total area in two equal parts

Median



Calculate means and medians before and after Westvaco layoffs

25, 33, 35, 38, 48, 55, 56, 55, 55, 64

Spread - IQR

First Quartile or Lower Quartile Q1

Third Quartile or Upper Quartile Q3

Medians of left hand side of data and right hand side of
Data with respect to the median

Inter Quartile Range

$$\text{IQR} = Q3 - Q1$$

Five number summary

Q1, Q3, median, min, max

11, 12, 20, 25, 30, 30, 30, 32, 35, 39,
40, 40, 40, 42, 45, 48, 50, 70

These give the **five number summary**
From which to calculate

$$\text{IQR} = \text{Q3} - \text{Q1}$$
$$\text{range} = \text{max} - \text{min}$$

Five number summary

11, 12, 20, 25, 30, 30, 30, 32, 35, 39,
40, 40, 40, 42, 45, 48, 50, 70

$$\text{Min} = 11$$

$$\text{Max} = 70$$

$$Q1 = 30$$

$$\text{Median} = Q2 = 37$$

$$Q3 = 42$$

$$\text{Range} = \text{max} - \text{min} = 70 - 11 = 59$$

$$\text{IQR} = Q3 - Q1 = 42 - 30 = 12$$

Spread - Deviation

Deviation of a value x is how far it is from the mean

$$x - \bar{x}$$

This value is different for every data point x
and can be negative or positive

Standard deviation

$$\sigma_n = \sqrt{\frac{\sum (x - \bar{x})^2}{n}}$$

$$\sigma_{n-1} = \sqrt{\frac{\sum (x - \bar{x})^2}{n-1}}$$

The custom is to use σ_n

Standard deviation

Data 2, 7, 8, 12, 12, 19 $n=?$ average $\bar{x} = ?$

x	$x - \bar{x}$	$(x - \bar{x})^2$
2		
7		
8		
12		
12		
19		
total sum = 60		

Standard deviation

Example. Data: 2,7,8,12,12,19

$$n = 6, \bar{x} = (2 + 7 + 8 + 12 + 12 + 19) / 6 = 10$$

x	$x - \bar{x}$	$(x - \bar{x})^2$
2	-8	64
7	-3	9
8	-2	4
12	2	4
12	2	4
19	9	81

60	0	166
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Find σ_n and σ_{n-1}

Standard deviation

Example. Data: 2,7,8,12,12,19

$n = 6$, $\bar{x} = (2+7+8+12+12+19)/6 = 10$

x	$x - \bar{x}$	$(x - \bar{x})^2$
2	-8	64
7	-3	9
8	-2	4
12	2	4
12	2	4
19	9	81

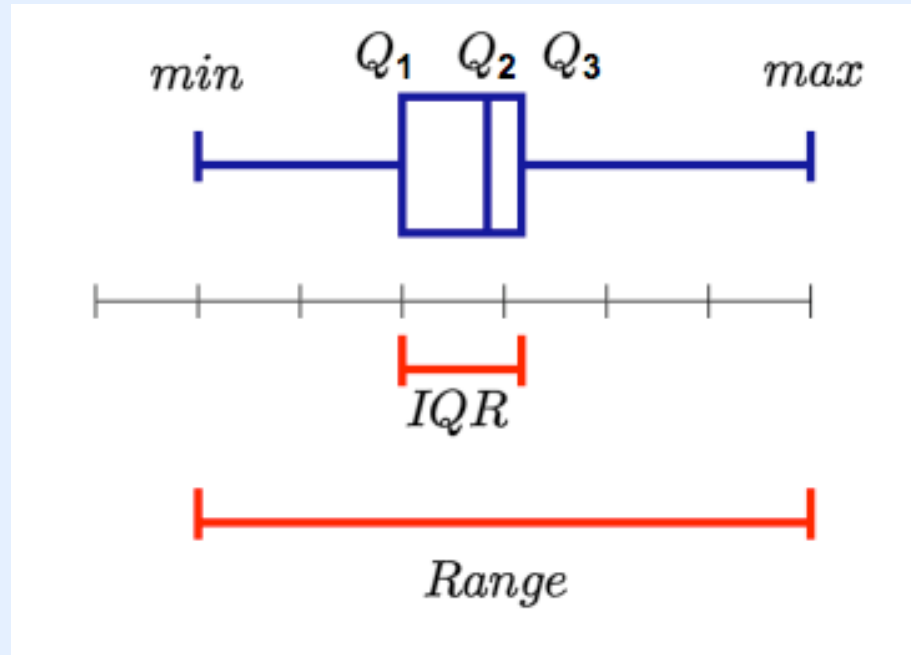
60	0	166
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$$\sigma_n = \sqrt{\frac{166}{6}} \approx 5.2599$$

$$\sigma_{n-1} = \sqrt{\frac{166}{5}} \approx 5.7619$$

Box Plots

Graphical display of 5 number summary
 Q_1 , Q_2 , Q_3 , \max , \min



Hk

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Outliers

If a value is more than 1.5 times the IQR from the nearest quartile it may be an outlier

Is the cheetah an outlier?

Is the pig an outlier?

Is the gazelle an outlier?

Is the lion an outlier?

Which animal is the largest non-outlier?