## Math 140 <br> Introductory Statistics

Math 140 tutoring: LIVE OAK 1319

MW 11:30-4:30 TTh 3:30-5:30 F $10: 30-12: 30$
General Hours:
M - Th 10:00-5:30 F 10:00-3:00

Later: Saturday from 11 to 2.

## Last time

Westvaco laid off 3 people at round 2

$$
55,55,64
$$

Their average age was 58
The average age of all workers was 46.4
The average age of retained workers was 41.4

Was it by chance?

## Let's do a simulation - group of 3

Model of a chance process

Consider all ages of workers at Round 2 whose ages are
$\begin{array}{llllllllll}25 & 33 & 35 & 38 & 48 & 55 & 55 & 55 & 56 & 64\end{array}$

## Let's do a simulation - group of 3

Model of a chance process

## Person ONE will label them at random

\[

\]

## Let's do a simulation - group of 3

Model of a chance process
Persons TWO and THREE will simulate cases of of three people to be laid off from A to J

$$
\begin{array}{cc}
\text { example } 1 \text { - Layoffs: } & \text { B, F, A } \\
\text { example } 2 \text { - Layoffs: } & \text { C, A, I } \\
\ldots & \\
\text { example } 5 \text { - Layoffs: } & \text { F, G, A }
\end{array}
$$

5 per person, so that you have a sample of 10 simulation events

## Let's do a simulation - group of 3

Model of a chance process
Match layoff letters with ages Calculate average age of laid off people

You should end up with TEN estimates Put them in a dot plot

## Questions:

What is the largest possible average? What is the lowest possible average?

## What to do with this?

How many times did we find that the average layoff age was equal or greater than 58 ?

We should have roughly 400 estimates in total in this class

## In this case only $5 \%$ of outcomes were higher or equal to 58 years


some results were $42.7,48.0,42.7,37.0$

## Inference

Getting an average of 58 or more is not impossible It COULD HAVE BEEN BY CHANCE!

But it is extremely unlikely

The lay off being by chance is POSSIBLE but UNLIKELY

Other reasons?

## Martin vs. Westvaco

The statistician hired by Mr. Martin argued that the probability of getting an average layoff age larger than Westvaco's average layoff age, was only $5 \%$.

The layoffs were most likely not randomly distributed in age.

There could have been other reasons though.
The law sets the limit at $2.5 \%$

The case was settled out of court

## Simulation steps

## Define your random model

Calculate summary statistic (for us average age)

## Repeat many times

Display the distribution
Estimate the probability (for us about 5\%)
Come to some conclusion

## How do we make sense of it?

We compared the age of 58
to that of the rest of Westvaco's workers

The average number of hectares of the Earth we use in the USA per person for our basic needs is 9.7 .

Is that a lot or a little?

## Visualize the distribution



Number of hectares of earth used per person to satisfy our basic needs.

The red dot is us - the USA

# Shapes of distribution 

4 main possibilities:
Uniform or rectangular
Normal
Skewed
Bimodal or multimodal

## Uniform distribution

Each outcome occurs roughly the same number of times if we repeat measurements over and over

## Uniform distribution

Each outcome occurs roughly the same number of times if we repeat measurements over and over

Examples:
Number of people born per month Randomly distributed numbers
Number of times you get head/tail from coin toss
Number of times you get any number on the roll of dice

## Number

of births and deaths in the USA for the year 1997 (in thousands)

| Months | Births | Deaths |
| :--- | :--- | :--- |
| January | 305 | 218 |
| February | 289 | 191 |
| March | 313 | 198 |
| April | 342 | 189 |
| May | 311 | 195 |
| June | 323 | 182 |
| July | 345 | 192 |
| August | 341 | 178 |
| September | 353 | 176 |
| October | 329 | 193 |
| November | 304 | 189 |
| December | 324 | 192 |

## Plot the distributions



Uniform or rectangular distribution

## 1000 random numbers


generated by a computer
Where is center? What is the spread like? What does it mean?

## Other examples?

What should not be uniformly distributed?

## Normal distributions

Objects that are manufactured
(diameter of pennies, tennis balls)


Tennis ball diameter - what do you observe?

## Normal distributions



Bell shaped around a maximum Symmetric left and right

## Idealized normal distributions


mean

inflection points

## Idealized normal distributions




Center value is called the "mean".
The distribution is symmetric with respect to the mean.

The concavity changes at the "inflection points" Roughly $2 / 3$ of the area below the curve is between the inflection points.

## Idealized normal distributions



The distance between the mean and either of the inflection points is called

## Standard Deviation (SD)

and measures how spread the distribution is

## An example


center is at 47 , about $68 \%$ of ages fall between 43 and 51
We say the mean is 47 with SD of 4
The average age is 47 give or take 4 .

## Let's try



Weight of pennies (grams)

Diameter of tennis balls (mm)


Estimate mean and SD

## Skewed distributions



Not symmetric curves
Data is bunched on one end and a tail appears on the other side

## Skewed distributions



Skewed Left


Skewed Right

This happens because there is a 'wall'
A value of data you cannot go beyond.
Maybe 0 (for things like counts or measurements) or 100 (for percentiles)

## Skewed distributions



The weight of bears in pounds

## Skewed distributions



Your GPAs - can't go beyond 4!
We should look at quartiles and medians to investigate better

## Do in class and homework

Problems E13, E14, E17, E21 (page 16)
E1, E4, E5, E6, E10, E15 (page 42)
For problems E13 and E21
Number of ways to pick 3 objects from group of 4

$$
\binom{4}{3}=4
$$

$$
\binom{6}{2}=15
$$

Number of ways to pick 2 objects from group of 6

