Math 140
Introductory Statistics

Extra hours at the tutoring center

Fri Dec 3rd 10-4pm, Sat Dec 4 11-2 pm

Final Dec 14th 5:30 - 7:30pm CH 5122

Last time: Making decisions

We have a null hypothesis

We have an alternative hypothesis

We select a level of significance
(the cutoff as to where to drop the null hypothesis)

We check if we can use the normal approximation

We calculate the test statistic z

We decide whether to discard the null hypothesis or not

8.4 Errors

		Null hypothesis is actually	
		True	**False**
Your decision	**Don't Reject H_0**	Correct	Type II error
	Reject H_0	Type I error	Correct

What if we reject the null hypothesis but it is true?
What if we don't reject the null hypothesis but it is false?

We have made a mistake

8.4 Errors

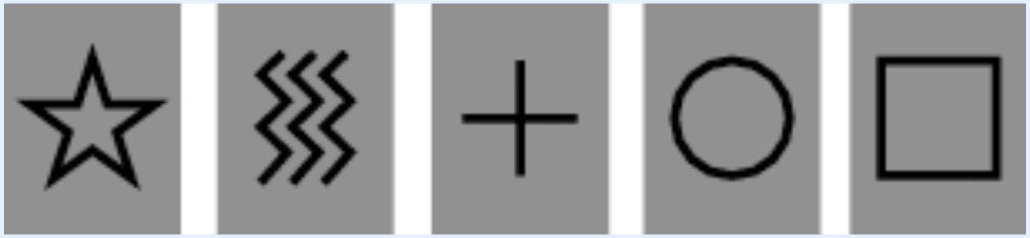
		Null hypothesis is actually	
		True	**False**
Your decision	**Don't Reject H_0**	Correct	**Type II error**
	Reject H_0	**Type I error**	Correct

TYPE 1 : we rejected the null hypothesis, but it was true!

TYPE 2 : we didn't reject it, but it was false

A problem

An extrasensory perception test (ESP)

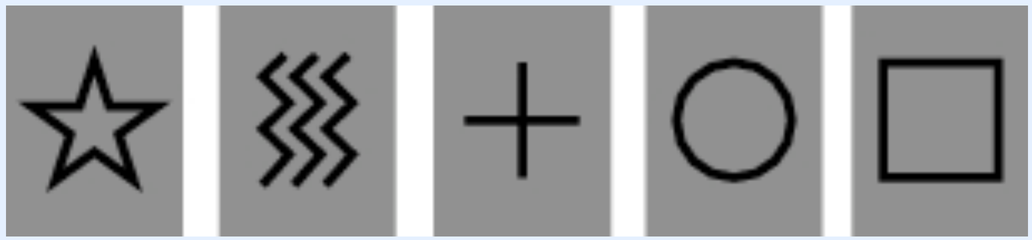


Can you tell which is which if they
are shown to you from the back?

Carla guessed right 29 out of 100 times.
Does she have extrasensory gifts?

Wonder woman

What is the null hypothesis? $p_0 = 1/5 = 0.2$



What is our alternative hypothesis? $p_0 > 0.2$

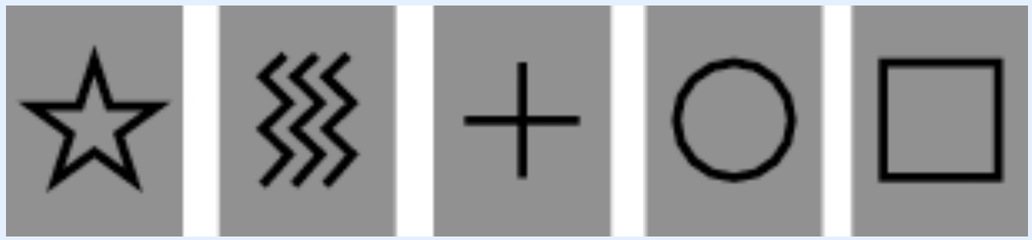
Can we use the normal distribution?

$N\ p_0 = 100 * 0.2 = 20$ OK!

$N\ (1-p_0) = 100 * 0.8 = 80$ OK!

Wonder woman

What is the null hypothesis? $p_0 = 1/5 = 0.2$



z score:

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.29 - 0.2}{\sqrt{\frac{0.2 * (1-0.2)}{100}}} = 2.25$$

Wonder woman

P value = 0.01

We use the significance level α = 0.05

Since P < α

We reject the null hypothesis

Carla has extrasensory powers!

But…

Carla know that she does not have any extrasensory powers.

She basically chose her answers at random!

The null hypothesis was correct,
We made a type 1 error.

Carla was just lucky.

In fact on her second attempt,
she got only 17 out of 100 right.

False positives

Our current HIV test has a 2.5% false positive rate.
On average, for every 100 positive results,
2.5 of them are wrong.

A new test showed 9 out of 500 specimens
were false positives (about 1.8%)

We calculated that $P = 0.1580$. We assumed $\alpha = 0.05$

Since $P > \alpha$ we don't reject the null hypothesis

We keep using the same HIV test.

But…

So we are keeping the null hypothesis

It could be that we are making a TYPE 2 error
Not rejecting the null hypothesis when it is false!

Maybe our HIV test is indeed better
than what currently available.
Had we increased the sample size, or had we
Taken a different set perhaps
we would have chosen to reject the null hypothesis

Rejecting the null hypothesis:

What to Consider When You Reject the Null Hypothesis

If you reject the null hypothesis, then there are several possibilities to consider:

- You are making the correct decision. The null hypothesis isn't true, and that's why the sample proportion, $\hat{p}$, was so far from p_0.
- You are making a Type I error—rejecting H_0 even though H_0 is actually true. It was just bad luck that resulted in $\hat{p}$ being so far from p_0.
- The sampling process or method of getting a response from the sample was biased in some way, so the value of $\hat{p}$ is itself suspicious.

Not rejecting the null hypothesis

What to Consider When You Don't Reject the Null Hypothesis

If you don't reject the null hypothesis, then there are also several possibilities to consider:

- You are making the correct decision, The null hypothesis is true, and you got just about what you would expect in the sample.
- You are making a Type II error—not rejecting H_0 even though H_0 is false. It was just by chance that $\hat{p}$ turned out to be close to p_0.
- The sampling process or method of getting a response from the sample was biased in some way, so the value of $\hat{p}$ is itself suspicious.

If the null hypothesis is true

How can we make a type one error?

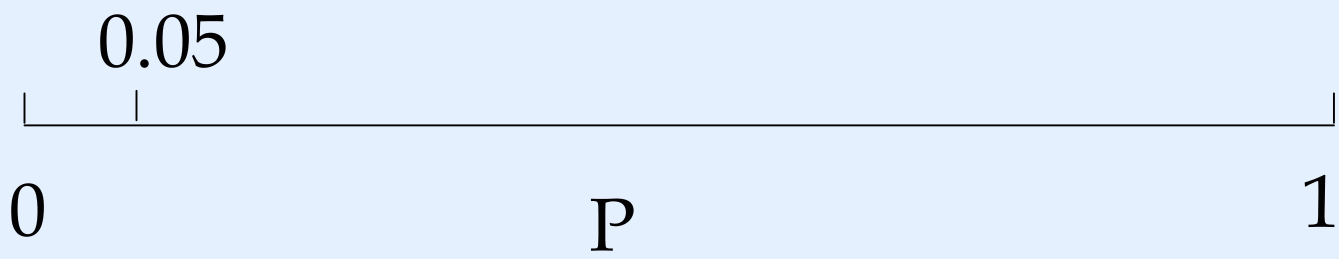
Only if, by chance, our estimate of $\hat{p}$ is very very different than p_0

Assume $\alpha = 0.05$.

To make a type one error $P < 0.05$

Since the P value is in between 0 and 1, it will be less than 0.05 only 5% of the time.

The null hypothesis is true

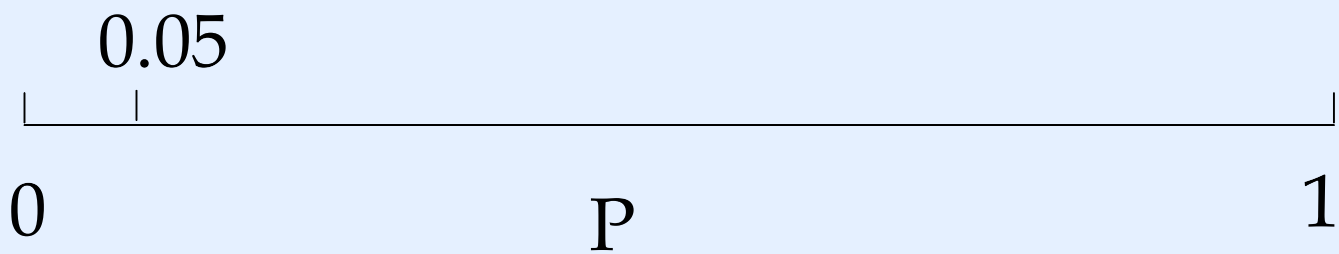


The probability that P is less than 0.05 is 5%
(obviously!)

So the probability we make a type one error is 5%

If we lower $\alpha = 0.01$, what is the probability we make a type one error?

The null hypothesis is true



The probability that P is less than 0.01 is 1%

Try this yourselves:

Problem P31
-the alternative hypothesis
is that the dice are not fair
double sided test

Americans and their pets

63% of Americans owned a pet in 2007 - 29,700 surveyed
56% of Americans owned a pet in 1994 - 6,786 surveyed

What is the change in percentage of pet ownership?

Of course the difference is 7%

Americans and their pets

But these are just from samples.
What can we say about the entire population?

Has pet ownership increased or not for the entire population?

9.1 A confidence interval for the difference of two proportions

The previous type of question is common:

We take two samples,
independently and from two different
populations with the goal of estimating
the size of the difference between
the proportion of success
in one of them and the other.

Intuitively

2007: p_1 = 63% = 0.63 1994: p_2 = 56% = 0.56

Of course the difference is p_1-p_2 = 7% = 0.07

Let's suppose we want to find the 95% confidence interval. From an intuitive point of view:

95 percent of all values of the difference between datasets will be found between

$(p_1 - p_2)$ - 1.96 * standard error of $(p_1 - p_2)$
$(p_1 - p_2)$ + 1.96 * standard error of $(p_1 - p_2)$

But what is the standard error?

If we have two separate samples of size n_1 and n_2 and estimates of $\hat{p}_1, \hat{p}_2$

The standard errors of the distributions of $\widehat{p_1}$ and $\widehat{p_2}$ can be estimated respectively as:

$$\sqrt{\frac{\widehat{p_1}(1-\widehat{p_1})}{n_1}} \text{ and } \sqrt{\frac{\widehat{p_2}(1-\widehat{p_2})}{n_2}}$$

But what is the standard error?

If we now assume that these
samples are independent,
The standard error of the
DIFFERENCE can be estimated

$$\sqrt{\frac{\widehat{p_1}(1-\widehat{p_1})}{n_1}+\frac{\widehat{p_2}(1-\widehat{p_2})}{n_2}}$$

The formula to use

This will give you the confidence interval

$$(\widehat{p_1} - \widehat{p_2}) \pm z^* \cdot \sqrt{\frac{\widehat{p_1}(1-\widehat{p_1})}{n_1} + \frac{\widehat{p_2}(1-\widehat{p_2})}{n_2}}$$

Here, z* is the factor associated to the percentile we specify,
For example, for us, since we selected 95% z* = 1.96.

How to use this?

Conditions that must be met to use this formula are

1) The two samples are taken randomly and independently from two populations

2) Each population is at least 10 times as large as its sample size

3) The following quantities must all be at least 5

$$n_1\hat{p}_1 \quad n_1(1-\hat{p}_1) \quad n_2\hat{p}_2 \quad n_2(1-\hat{p}_2)$$

Now you verify this for America's pet ownership

1994: 56% own a pet, out of 6786 homes
2007: 63% own a pet, out of 29,700 homes

1) Check the applicability

2) Find the 95% confidence interval

$$n_1\hat{p}_1 \quad n_1(1-\hat{p}_1) \quad n_2\hat{p}_2 \quad n_2(1-\hat{p}_2)$$

$$(\widehat{p_1}-\widehat{p_2}) \pm z^* \cdot \sqrt{\frac{\widehat{p_1}(1-\widehat{p_1})}{n_1}+\frac{\widehat{p_2}(1-\widehat{p_2})}{n_2}}$$

Pets in America:

For us the 95% confidence interval (with z* = 1.96) is

$$(\hat{p}_1 - \hat{p}_2) \pm z^* \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}} = 0.07 \pm 0.013$$

This means that it is 95% plausible that the TRUE difference for the ENTIRE US family population is anywhere between 0.057 and 0.083, that is 5.7% and 8.3%

Pets in America:

For us the 95% confidence interval (with $z^* = 1.96$) is

$$(\hat{p}_1 - \hat{p}_2) \pm z^* \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}} = 0.07 \pm 0.013$$

What can we say about the difference in a nutshell?

Pets in America

The 95% interval DOES NOT contain 0.

It is 95% plausible that pet ownership in America has increased.

Note: it does not matter who is p1 or p2, but usually we order them so that their difference is positive

Wireless vs. Wired

Do you prefer mobility or stationary internet will do?

DC= digital collaborator - use to work
High wireless use

MM = media movers - share info/photos across network
Lower wireless use

DV = desktop veterans - use internet desktop for emails
High wired use

IE = info encumbered = want to stay off internet
Lower wired use

Wireless vs. Wired

	Motivated by Mobility		Stationary Media Will Do	
	DC (273)	MM (228)	DV (442)	IE (399)
College grad	61	32	41	33
Get health information online	93	90	85	76
Buy a product online	95	81	82	52
Play a video game	48	44	28	14

Out of a sample of 273 digital collaborators and out of a sample of 228 media movers estimate the difference between the population proportions with regards to getting health info online in a 90% confidence interval

Wireless vs. Wired

	Motivated by Mobility		Stationary Media Will Do	
	DC (273)	MM (228)	DV (442)	IE (399)
College grad	61	32	41	33
Get health information online	93	90	85	76
Buy a product online	95	81	82	52
Play a video game	48	44	28	14

We only need to look at this section and use

$n1 = 273$, $p1 = 0.93$
$n2 = 228$, $p2 = 0.90$
$z^* = 1.645$

Wireless vs. Wired

	Motivated by Mobility		Stationary Media Will Do	
	DC (273)	MM (228)	DV (442)	IE (399)
College grad	61	32	41	33
Get health information online	93	90	85	76
Buy a product online	95	81	82	52
Play a video game	48	44	28	14

Check n1 * p1 = 273 * 0.93 = 253.89 > 5
and all the rest too.

Wireless vs. Wired

The 90% confidence interval is between

0.03 - 0.04 and 0.03 + 0.04

We are 90% confident that the difference between the percentage of DC people and MM people who download health info from the internet is between -0.01 and 0.07.

Because this interval CONTAINS 0, it is plausible that there is NO difference between their behaviors

Do the same for DV and IE

	Motivated by Mobility		Stationary Media Will Do	
	DC (273)	MM (228)	DV (442)	IE (399)
College grad	61	32	41	33
Get health information online	93	90	85	76
Buy a product online	95	81	82	52
Play a video game	48	44	28	14

Information encumbered
Desktop veterans

Get health info online, 95% confidence interval

Two AIDS treatments

We give patients AZT and also a cocktail of AZT and ACV

Treated with

Survived?		AZT	AZT + ACV	Total
	No	28	13	41
	Yes	41	49	90
	Total	69	62	131

Find:

n1, n2, $\hat{p}_1, \hat{p}_2$

What is the null hypothesis?

What is the alternative hypothesis?

Treated with

Survived?		AZT	AZT + ACV	Total
	No	28	13	41
	Yes	41	49	90
	Total	69	62	131

Hypotheses:

Null: the treatments
(AZT alone, or the cocktail AZT + ACV)
give the same survival rate

$p1 = p2$

Alternative: the AZT + ACV is better

$p2 > p1$

Find:

$n_1 = 69$	sample size of patients treated with AZT
$n_2 = 62$	sample size of patients treated with AZT+ACV
$\hat{p}_1 = \frac{41}{69}$	proportion of patients in the sample treated with AZT that survived
$\hat{p}_2 = \frac{49}{62}$	proportion of patients in the sample treated with AZT+ACV that survived
p_1	True proportion of survival if all patients were treated with AZT (unknown)
p_2	True proportion of survival if all patients were treated with AZT+ACV (unknown)

Check all requirements are satistified

Homework

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