Math 140
Introductory Statistics

Extra hours at the tutoring center

Fri Dec 3rd 10-4pm, Sat Dec 4 11-2 pm

8.3 Making decisions

What are these P-values good for?

Quality control

5% of our windshields from a supplier don't meet standards.

We inspect a sample of 300 windshields from another company.

3% of their sample don't meet standards.
What should we do?
Switch or not switch?

8.3 Making decisions

What are these P-values good for?

Medicine

Tests can predict Down syndrome about 87% of the time. We want to make a new test that goes beyond that rate.

Our trial finds that our new test predicts Down syndrome 182 times out of a sample of 200 women.

What should we do? Develop the new test or not?

8.3 Making decisions

What are these P-values good for?

Politics

We want to spend money to advertise for Prop. 23 but only if polls show support falls below 55%.

A poll of 700 people shows that 53% support our proposition.

What should we do? Spend the money or wait?

The P value

The *P*-Value for a Test of Significance

The ***P*-value** for a test is the probability of seeing a result that is as extreme as or more extreme than the result you got from your sample *if the null hypothesis is true.*

The *P*-value measures the strength of the evidence against the null hypothesis. The closer the *P*-value is to 0, the stronger the evidence against the null hypothesis (and in favor of the alternative hypothesis). The closer the *P*-value is to 1, the weaker the evidence against the null hypothesis.

P - value: close to 1 - keep the null hypothesis

P-value close to 0 - switch to the alternative hypothesis

Driving in America

According to a CNN poll, about 61% of Americans think that 16 is too young to operate a car and that the driving age should be increased.

The poll was conducted on 1002 people.

If we had asked ALL Americans their opinion, is it plausible that 55% would have said yes to increasing the driving age in America?

Driving in America

1) What is the null hypothesis?

55% of the entire population agrees to raise the driving age.

We are asking whether this is plausible given our CNN sample results.

$p_0 = 0.55$

Driving in America

2) To answer this question we find the P value
Under the assumption that $p_0 = 0.55$.

Remember,

$p_0 = 0.55$ is an ASSUMPTION we are making
Poll results from DATA we cannot change.

So if the two things are incompatible according to our P value, it means we must change the ASSUMPTION.

Driving in America

3) Check if we can use results from the normal distribution to find the P value.

Is $n\ p_0 > 10$ and $n\ (1-p_0) > 10$?

For us $n=1022$

Are these conditions met?

Driving in America

3) Check if we even can use results from the normal distribution to find the P value.

Is $n\ p_0 > 10$ and $n\ (1-p_0) > 10$?

For us $n=1022$

Are these conditions met?

YES

Driving in America

4) What is our alternative hypothesis?
That the real value of p_0 is greater than 0.55, since we measured p = 0.61 in our sample.

So null hypothesis

$$p_0 = 0.55$$

Alternative hypothesis

$$p_0 > 0.55$$

Driving in America

5) Let's calculate the z-score for our SAMPLE assuming the null hypothesis is correct and $p_0 = 0.55$

For our sample is p = 0.61

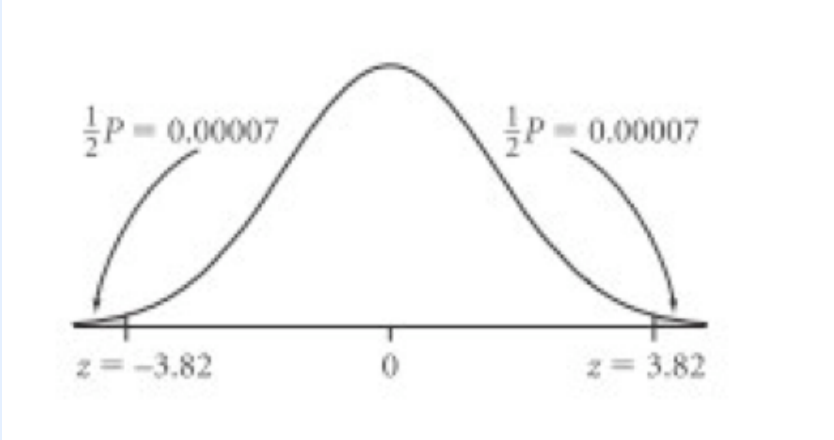
$$z = \frac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}} = \frac{0.61 - 0.55}{\sqrt{\dfrac{0.55 * (1-0.55)}{1022}}} = 3.82$$

Driving in America

6) If we look up page 759, we find the probability of finding (from a 1022 sample of people) the value p = 0.61 corresponds to 1 - 0.99993 = 0.00007

THIS IS YOUR P VALUE P = 0.00007

It is tiny!



Driving in America

You can't have your cake and eat it too!

The probability that $p_0 = 0.55$ and that from a sample $p = 0.61$ or more at the same time is

0.007%

Either the assumption is wrong or my data collection is wrong.

Driving in America

Since data cannot be changed, as we assume that we collected it properly,
the assumption must be wrong.

We can thus ABANDON the null hypothesis in favor of the alternate one that

$$P_0 > 0.55$$

Level of significance

The threshold we pick to abandon the null hypothesis and to move to the alternative one is that

$P < 0.05$

This value 0.05 is called the

LEVEL OF SIGNIFICANCE

And is denoted by the greek letter alpha α

Chocolate bars

Ads for two new chocolate bars

Is heavy or light branding better?

We expect heavy branding to be better

One ad shows light branding, the other heavy branding for a sample of customers

18 out of 27 people preferred the high branding commercial

Should we use light or heavy branding in our ad?

Chocolate bars

What is the null hypothesis?

Chocolate bars

What is the null hypothesis?

That it does not matter whether you do light or heavy branding

$p_0 = 0.5$

Chocolate bars

What is the alternative hypothesis?

Chocolate bars

What is the alternative hypothesis?

That people prefer the chocolate bar
with heavy branding

$p_0 > 0.5$

Chocolate bars

Can we use the normal distribution?

Chocolate bars

Can we use the normal distribution?

$p_0 n > 10$ and $(1 - p_0) n > 10$

YES

Chocolate bars

What is the value of $\hat{p}$
from our sample?

given that 18 out of 27 people
prefered heavy branding?

What is the value of the z score?

Chocolate bars

The value of $\hat{p}$ is

$$\hat{p} = 18/27 = 0.66$$

The z score is

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.66 - 0.5}{\sqrt{\frac{0.5*(1-0.5)}{27}}} = 1.73$$

Let's check on page 759

If $z = 1.73$, the probability that at the same time

$p_0 = 0.5$

and we can measure $p = 0.66$ or more
from our sample is

$1 - 0.9582 = 0.0418$ - about 4%

The P value is $P = 0.0418$

Null or alternative hypothesis?

If we set the threshold for the level of significance to be $\alpha = 0.05$ then since

$$P = 0.0418 < 0.05$$

we can discard the null hypothesis and choose the alternative one.

Null or alternative hypothesis?

It is then extremely unlikely that both

$p_0 = 0.5$ and that I can measure $p = 0.66$ or more
From my samples.

We have to change something

Since the only thing we can change is the hypothesis,
we conclude that $p_0 > 0.5$
and that it is better to do heavy branding

What if …

If we chose $\alpha = 0.01$?

Recall, our $P = 0.0418$

We would NOT reject the null hypothesis

The Level of Significance and Decision Making

If the P-value computed from your sample is less than the level of significance, α, that you have chosen, you have evidence against the null hypothesis. Reject the null hypothesis and say that the result is statistically significant.

If the P-value is greater than α, you do not reject the null hypothesis. The result is not statistically significant.

If a level of significance isn't specified, it is usually safe to assume that $\alpha = 0.05$.

If P value less than α, then reject the null hypothesis

If P value more than α, then keep the null hypothesis

A fixed level test

Is where we fix α in advance

Drugs in the workplace

5.3% of working Americans had used drugs
in the three days before a random drug test

The false positive rate is 2.5% for cocaine

Out of 100 positive tests,
about 2.5 are actually negative

We want to design a better test, with less false positives
And come up with a new method

Drugs in the workplace

We run some trials and find that for 500 positive values of the test, 9 were false positives.

We want our new test to perform better than A positive rate of 2.5%.

Use a 5% level of significance

You try to make sense of it

Standard: false positives = 2.5%

We want less false positives than that
with a level of significance of 5%

Our trial shows 9 out of 500 false positives

Shall we go ahead and commercialize
our new drug test?

Let’s do it

Null hypothesis $p_0 = 0.025$

Alternative hypothesis $p < 0.025$

W can use the normal distribution

$n\ p_0 = 12.5$ and $n\ (1-p_0) = 487.5$

$\hat{p}$ $=9/500 = 0.018$

Let's do it

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.018 - 0.025}{\sqrt{\frac{0.025 * (1-0.025)}{500}}} = -1.00$$

From table A, page 759 for -1.00
(we want to be LESS than 0.025)

P = 0.158

Let’s do it

Our fixed level for α was $5\% = 0.05$

Since $P = 0.158 > 0.05$

We cannot reject the null hypothesis

The medical text we invented
is not any better that the old one.

Let’s save our money and eat chocolate.

In summary

When we reject the null hypothesis we are basically saying that the assumption we made and the Results we found are too different and incompatible with each other.
These differences cannot be attributed to chance alone.

If we don't reject the null hypothesis we are basically saying that the assumptions we made and the results we found may be different, but are close enough that They can be attributed to chance.

Homework

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