Math 140
Introductory Statistics

Next test on Oct 19th

Adding and multiplying to X

Just like rescaling and recentering

So, adding C and multiplying by D>0 our entries for the variables X gives new SD and new means:

X is now C + D*X

μ_x turns into $\mu_{C+DX} = C + D^*\mu_x$

σ_X turns into $\sigma_{C+DX} = D^*\sigma_X$

In general

Linear Transformation Rule: The Effect of a Linear Transformation of X on μ_X and σ_X

Suppose you have a probability distribution for random variable X, with mean μ_X and standard deviation σ_X. If you transform each value by multiplying it by d and then adding c, where c and d are constants, then the mean and the standard deviation of the transformed values are given by

$$\mu_{c+dX} = c + d\mu_X$$
$$\sigma_{c+dX} = |d|\sigma_X$$

Question

Now, this was for TRIPLING the lottery

What if we kept the same lottery and bought 3 tickets?

What do you think?

If every time I play my average payout is $0.6014
What do I get after buying 3 tickets?

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What do I get after buying 3 tickets?

Duh! 3 * 0.6014 = 1.804!

Just like before!

It does not matter if I triple the lottery or
if I buy three tickets, the result is the same.

My take-home on average is tripled.

What do you think?

We can conclude that when we select three items from the same distribution we find

$$\mu_{3,X} = 3\,\mu_x = \mu_x + \mu_x + \mu_x$$

In general, for different distributions if we are adding we get

$$\mu_{X,Y} = \mu_x + \mu_y$$

Two tickets from two lotteries

Let's buy a ticket from the lottery of California and of Texas

California $\mu_x = \$0.50$
Texas $\mu_Y = \$0.75$

What are the expected total winnings?

$\mu_{CA,TX} = \mu_{CA} + \mu_{TX} = \$0.50 + \$0.75 = \$1.25$

What is the expected value for the total rolling outcome of two dice?

Sum of Two Dice, x	Probability, P
2	1/36
3	2/36
4	3/36
5	4/36
6	5/36
7	6/36
8	5/36
9	4/36
10	3/36
11	2/36
12	1/36
Total	1

But we could have used what we know

This is the same as buying two tickets!

$\mu_x = (1+2+3+4+5+6)/6 = 3.5$

$\mu_{x\ +}\ \mu_x = 3.5+3.5 = 7$

And what do you think the expected value for the difference is?

But we could have used what we know

This is the same as buying two tickets!

$\mu_x = (1+2+3+4+5+6)/6 = 3.5$

$\mu_{x+}\mu_x = 3.5+3.5 = 7$

And what do you think the expected value for the difference is?

$\mu_{x-}\mu_x = 3.5 - 3.5 = 0$!

X = 1st die - 2nd die

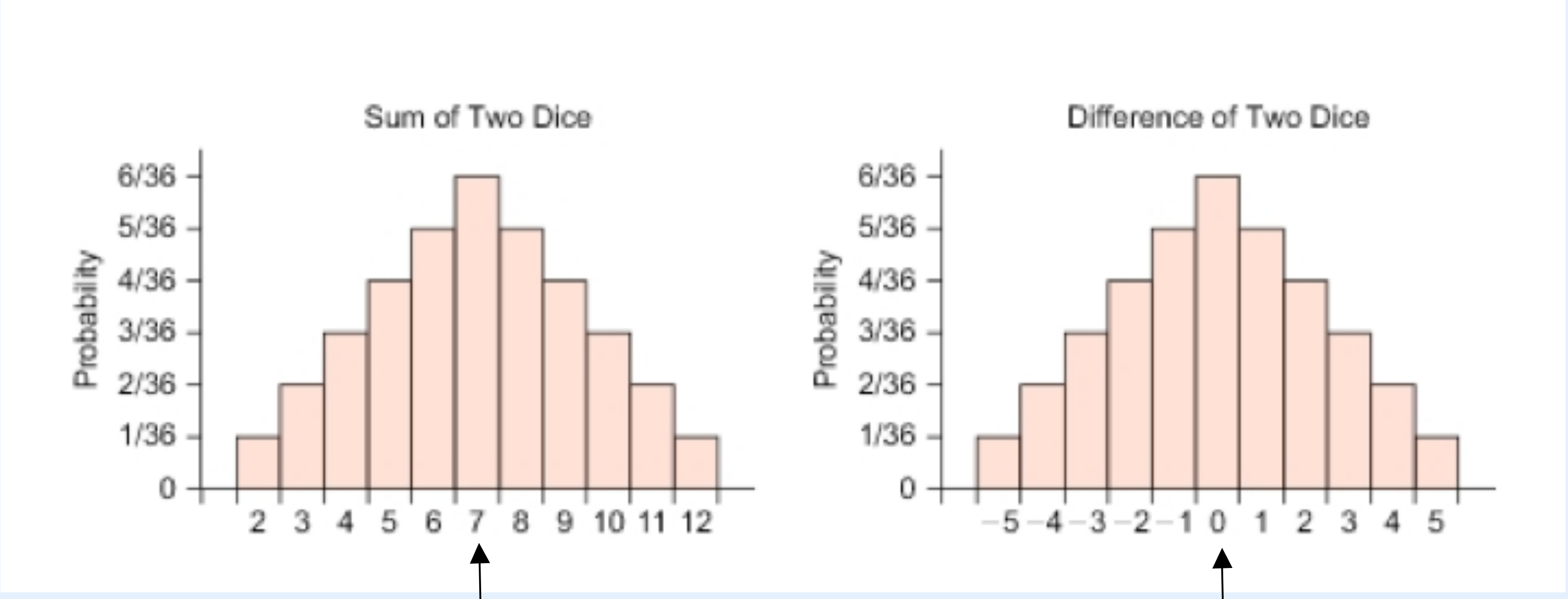
		Second Die					
		1	2	3	4	5	6
First Die	1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
	2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
	3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
	4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
	5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
	6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

-5	1 /36
-4	2 /36
etc	

1 - 6 = -5 smallest value, one one way

1-5, 2-6 = - 4 two ways

What is the expected value for the total rolling outcome of two dice?



7

0

What about the standard deviation?

When we pick from more that one distribution,
The VARIANCE NOT THE SD gets added

In other words:

if we pick 3 tickets

$$\mu_x = \mu_x + \mu_x + \mu_x = 3\,\mu_x$$

$$\sigma^2_X = \sigma^2_X + \sigma^2_X + \sigma^2_X = 3\sigma^2_X$$

$$\sigma_X = \sqrt{3}\,\sigma_X$$

This is true for different distributions

If we pick tickets from two lotteries
and add their outcomes

$$\mu_x = \mu_x + \mu_y$$

$$\sigma^2_{X,Y} = \sigma^2_X + \sigma^2_Y$$

$$\sigma_{X,Y} = \sqrt{\sigma_X^2 + \sigma_Y^2}$$

This is true for different distributions

if we pick tickets from two lotteries
and subtract their outcomes

$$\mu_x = \mu_x - \mu_y$$

$$\sigma^2_{X,Y} = \sigma^2_X + \sigma^2_Y$$

$$\sigma_{X,Y} = \sqrt{\sigma_X^2 + \sigma_Y^2}$$

Summary

Addition and Subtraction Rules for Random Variables

If X and Y are random variables, then

$$\mu_{X+Y} = \mu_X + \mu_Y$$
$$\mu_{X-Y} = \mu_X - \mu_Y$$

If X and Y are independent, then

$$\sigma^2_{X+Y} = \sigma^2_X + \sigma^2_Y$$
$$\sigma^2_{X-Y} = \sigma^2_X + \sigma^2_Y$$

The Addition Rule generalizes in the obvious way when there are more than two random variables.

Calculate the SD for the sum of 2 dice

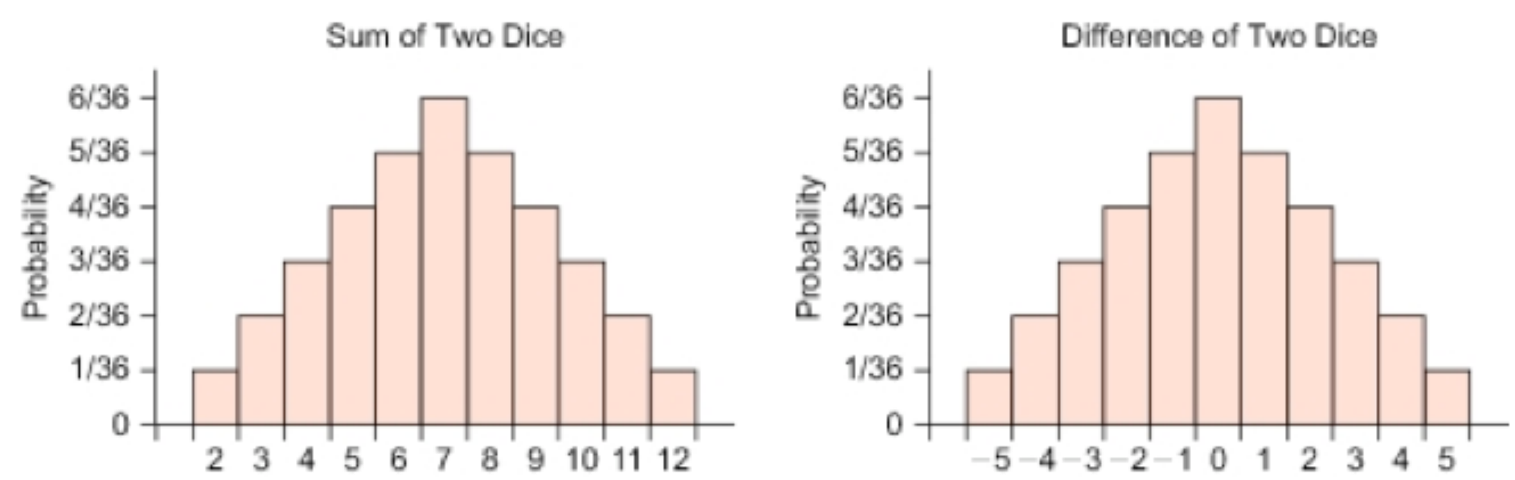
Sum of Two Dice, *x*	Probability, *P*
2	1/36
3	2/36
4	3/36
5	4/36
6	5/36
7	6/36
8	5/36
9	4/36
10	3/36
11	2/36
12	1/36
Total	1

$\mu_x = 7$

And verify
the formula
we just found

Then do the same
For the difference
of two dice

And yes, they are the same!



Summary

Shifting or multiplying the SAME DISTRIBUTION

$$\mu_{c+dX} = c + d\mu_X$$
$$\sigma_{c+dX} = |d|\sigma_X$$

Adding or subtracting DIFFERENT DISTRIBUTIONS

$$\mu_{X+Y} = \mu_X + \mu_Y$$
$$\mu_{X-Y} = \mu_X - \mu_Y$$

$$\sigma^2_{X+Y} = \sigma^2_X + \sigma^2_Y$$
$$\sigma^2_{X-Y} = \sigma^2_X + \sigma^2_Y$$

Practice

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