Math 140
Introductory Statistics

Next test on March 27th

Adding and multiplying to X

Just like rescaling and recentering

So, adding C and multiplying by D>0 our entries for the variables X gives new SD and new means:

X is now C + D*X

μ_x turns into μ_{C+DX} = C+ D*μ_x

σ_X turns into σ_{C+DX} = D*σ_X

In general

Linear Transformation Rule: The Effect of a Linear Transformation of *X* on μ_X and σ_X

Suppose you have a probability distribution for random variable X, with mean μ_X and standard deviation σ_X. If you transform each value by multiplying it by d and then adding c, where c and d are constants, then the mean and the standard deviation of the transformed values are given by

$$\mu_{c+dX} = c + d\mu_X$$
$$\sigma_{c+dX} = |d|\sigma_X$$

Question

Now, this was for TRIPLING the lottery

What if we kept the same lottery and bought 3 tickets?

What do you think?

If every time I play my average payout is \$0.6014
What do I get after buying 3 tickets?

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What do I get after buying 3 tickets?

Duh! 3 * 0.6014 = 1.804!

Just like before!

It does not matter if I triple the lottery or
if I buy three tickets, the result is the same.

My take-home on average is tripled.

What do you think?

We can conclude that when we select three items from the same distribution we find

$$\mu_{3,X} = 3\,\mu_x = \mu_x + \mu_x + \mu_x$$

In general, for different distributions if we are adding we get

$$\mu_{X,Y} = \mu_x + \mu_y$$

Two tickets from two lotteries

Let's buy a ticket from the lottery of California and of Texas

California $\mu_x = \$0.50$
Texas $\mu_Y = \$0.75$

What are the expected total winnings?

$\mu_{CA,TX} = \mu_{CA} + \mu_{TX} = \$0.50 + \$0.75 = \$1.25$

One roll of die

What is the expected roll value?

What is the variance?

What is the SD?

One roll of die

What is the expected roll value?

What is the variance?

What is the SD?

$\mu_X = 3.5$

$\sigma^2_X = 2.917$

$\sigma_X = 1.708$

What is the expected value for the total rolling outcome of two dice?

Sum of Two Dice, x	Probability, P
2	1/36
3	2/36
4	3/36
5	4/36
6	5/36
7	6/36
8	5/36
9	4/36
10	3/36
11	2/36
12	1/36
Total	1

But we could have used what we know

Rolling two dice?

This is the same as buying two tickets!

$$\mu_x = (1+2+3+4+5+6)/6 = 3.5$$

$$\mu_{x} + \mu_x = 3.5+3.5 = 7$$

And what do you think the expected
value for the difference is?

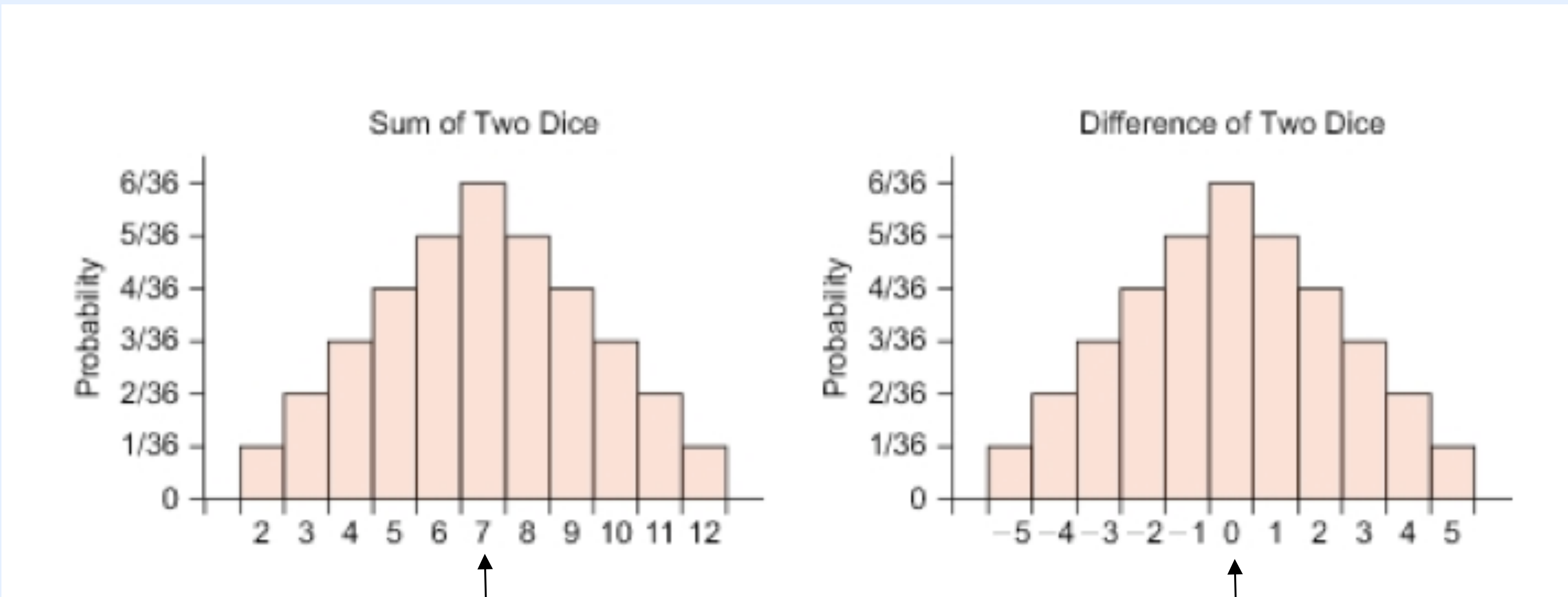
X = 1st die - 2nd die

		Second Die					
		1	2	3	4	5	6
First Die	1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
	2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
	3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
	4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
	5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
	6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

-5	1 /36
-4	2 /36
etc	

1 - 6 = -5 smallest value, one one way
1-5, 2-6 = - 4 two ways

What is the expected value for the total rolling outcome of two dice?



7

0

A different way of calculating these quantities

$$\mu_{x\,-}\,\mu_x = 3.5 - 3.5 = 0$$

$$\mu_{x\,+}\mu_x = 3.5 + 3.5 = 7$$

What about the standard deviation?

When we pick from more that one distribution,
The VARIANCE NOT THE SD gets added

In other words:

if we pick 3 tickets

$$\boxed{\begin{gathered}\mu_x = \mu_x + \mu_x + \mu_x = 3\,\mu_x \\ \sigma^2_X = \sigma^2_X + \sigma^2_X + \sigma^2_X = 3\sigma^2_X \\ \sigma_X = \sqrt{3}\,\sigma_X\end{gathered}}$$

This is true for different distributions

If we pick tickets from two lotteries
and add their outcomes

$$\mu_x = \mu_x + \mu_y$$

$$\sigma^2{}_{X,Y} = \sigma^2{}_X + \sigma^2{}_Y$$

$$\sigma_{X,Y} = \sqrt{\sigma_X^2 + \sigma_Y^2}$$

This is true for different distributions

if we pick tickets from two lotteries
and subtract their outcomes

$$\mu_x = \mu_x - \mu_y$$

$$\sigma^2{}_{X,Y} = \sigma^2{}_X + \sigma^2{}_Y$$

$$\sigma_{X,Y} = \sqrt{\sigma_X^2 + \sigma_Y^2}$$

Summary

Addition and Subtraction Rules for Random Variables

If X and Y are random variables, then

$$\mu_{X+Y} = \mu_X + \mu_Y$$
$$\mu_{X-Y} = \mu_X - \mu_Y$$

If X and Y are independent, then

$$\sigma^2_{X+Y} = \sigma^2_X + \sigma^2_Y$$
$$\sigma^2_{X-Y} = \sigma^2_X + \sigma^2_Y$$

The Addition Rule generalizes in the obvious way when there are more than two random variables.

Calculate the SD for the sum of 2 dice

Sum of Two Dice, x	Probability, P
2	1/36
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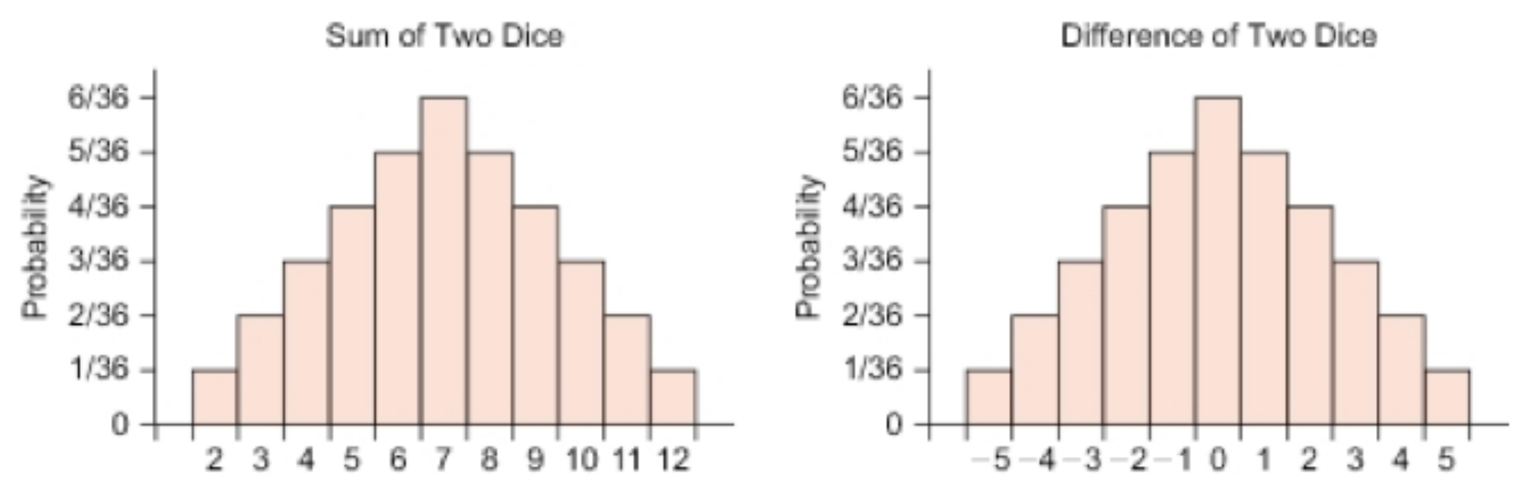
4	3/36
5	4/36
6	5/36
7	6/36
8	5/36
9	4/36
10	3/36
11	2/36
12	1/36
Total	1

$\mu_x = 7$

And verify
the formula
we just found

Then do the same
For the difference
of two dice

And yes, they are the same!



Summary

Shifting or multiplying the SAME DISTRIBUTION

$$\mu_{c+dX} = c + d\mu_X$$
$$\sigma_{c+dX} = |d|\sigma_X$$

Adding or subtracting DIFFERENT DISTRIBUTIONS

$$\mu_{X+Y} = \mu_X + \mu_Y$$
$$\mu_{X-Y} = \mu_X - \mu_Y$$

$$\sigma^2_{X+Y} = \sigma^2_X + \sigma^2_Y$$
$$\sigma^2_{X-Y} = \sigma^2_X + \sigma^2_Y$$

Practice

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