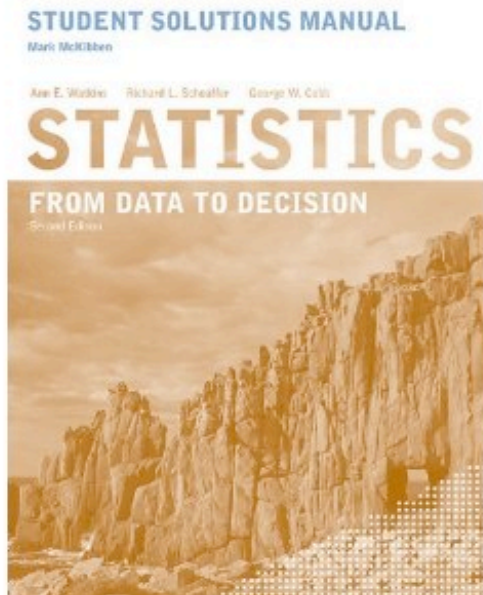


# Math 140

## Introductory Statistics



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# 5.1 Models of random behavior

**Outcome:** Result or answer obtained from a chance process.

**Event:** Collection of outcomes.

**Probability:**

Number between 0 and 1 (0% and 100%).

It tells how likely it is for an outcome or event to happen.

$P=0$  The event cannot happen.

$P=1$  The event is certain to happen.

# Where do probabilities come from?

Observed data (long-run relative frequencies)

For example, observation of thousands of births has shown that about 51% of newborns are boys.

You can use these data to say that the probability of the next newborn being a boy is about 0.51.

# Where do probabilities come from?

Symmetry(equally likely outcomes)

If we flip a fair coin, both sides are equally likely to come up.

Relying on symmetry, it is reasonable to think that heads and tails are equally likely.

So the probability of heads is 0.5.

# Where do probabilities come from?

## Subjective estimates

What's the probability that you'll get an A in this statistics class?

That's a reasonable, everyday kind of question, and the use of probability is meaningful, but you can't gather data or list equally likely outcomes.

However you can make a subjective judgement

# Models of Random behavior

If the chance of getting rain is 30%,  
The chance of not getting rain is 70%

The corresponding probabilities are 0.3 and 0.7

If the probability of having rain is  $P(A)$   
What is the probability of NOT getting rain?

# Models of Random behavior

If the chance of getting rain is 30%,  
The chance of not getting rain is 70%

The corresponding probabilities are 0.3 and 0.7

If the probability of having rain is  $P(A)$   
What is the probability of NOT getting rain?

$$1 - P(A)$$

$$A = \text{getting rain} \quad P(A) = 0.3$$

$$P \text{ of not getting rain} = 1 - P(A) = 0.7$$

# Models of Random behavior

A is the event (getting rain)

$P(A)$  is its associated probability (0.3)

$1 - P(A)$  is the probability of event A NOT HAPPENING (0.7)

Sometimes we call the latter  $P(\text{not } A)$

$$P(\text{not } A) = 1 - P(A)$$

The event of A not happening is called the complement of A



# Equally likely outcomes

If I roll a fair die, what is the probability that I will get the value 3?

There are six possibilities, that upon rolling I get  
1, 2, 3, 4, 5, 6

All are equally likely, so the probability I get 3 is just ONE out of those SIX.

$$A = ?$$

$$P(A) = ?$$

$$P(\text{not } A) = ?$$

# Equally likely outcomes

If I roll a fair die, what is the probability that I will get the value 3?

There are six possibilities, that upon rolling I get  
1, 2, 3, 4, 5, 6

All are equally likely, so the probability I get 3 is just ONE out of those SIX.

A = getting a 3

$$P(A) = 1/6$$

$$P(\text{not } A) = 5/6$$

# Equally likely outcomes

If we have a list of all possible outcomes  
and all of them are equally likely

$P(\text{specific outcome}) = 1 / \text{number of outcomes}$

For the die the number of outcomes is 6.

For a coin it is 2.

# Equally likely outcomes

If an event consists of more outcomes then

$$P(\text{event}) = \frac{\text{number of outcomes in that event}}{\text{number of outcomes}}$$

For example, is I want the probability of getting 3 or 4  
My event consists of **TWO** outcomes and

$$P(\text{event}) = 2/6 = 1/3$$

# A dispute

Starbucks and McDonald's coffee: can people tell the difference?

Experiment: Give each person both kinds of coffee, in random order, and ask which they prefer.

Easy start: What is the probability that **two** tasters will prefer McCafe?



# A dispute

What is your opinion?

Take a few seconds to think and discuss



# Claim A

There are three possible outcomes:

Neither person chooses McCafe,  
only one chooses McCafe,  
both choose McCafe.

These three outcomes are equally likely,  
so each outcome has probability  $1/3$ .

In particular, the  
probability that both choose McCafe is  $1/3$ .

# Claim B

There are four equally likely outcomes:

Both choose McCafe (MM);

First chooses McCafe - Second chooses Starbucks (MS);

First chooses Starbucks - Second chooses McCafe (SM);

Both choose Starbucks (SS).

Because these four outcomes are equally likely, each has probability  $1/4$ .

So the probability of having MM is  $1/4$



# Claims

People that  
choose  
McCafe

Claim A  
probability

Claim B  
probability

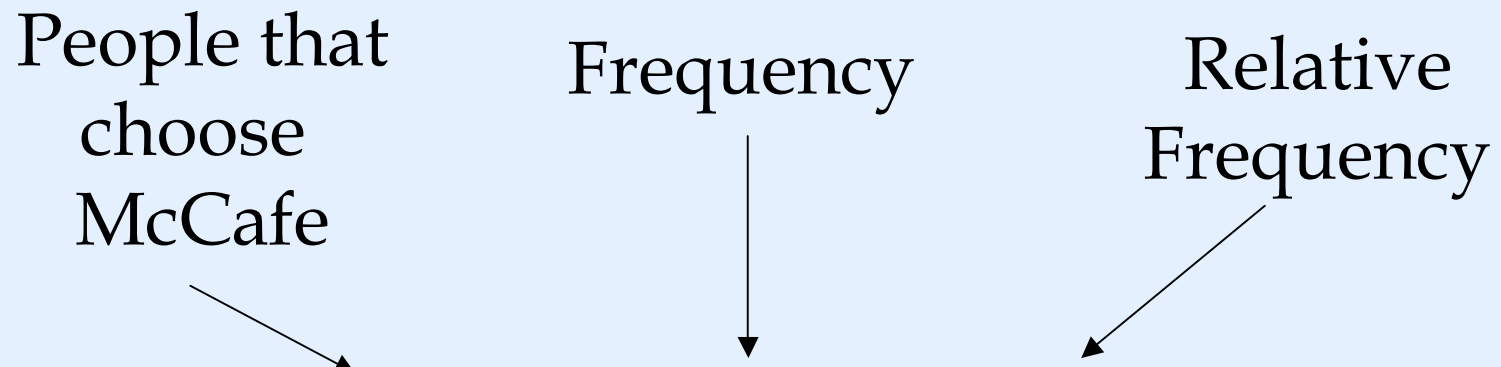
0	1/3	1/4
1	1/3	1/2
2	1/3	1/4

# Let's do an experiment

People that  
choose  
McCafe

Frequency

Relative  
Frequency



0	782	0.26
1	1493	0.50
2	725	0.24
<b>Total</b>	<b>3000</b>	<b>1.00</b>

A sample of 3000 people

Who is right?

# Let's do an experiment

People that  
choose  
McCafe

Frequency

Relative  
Frequency

0	782	0.26
1	1493	0.50
2	725	0.24
<b>Total</b>	<b>3000</b>	<b>1.00</b>

$\sim 1/4$   
 $\sim 1/2$   
 $\sim 1/4$

Claim B!

# Law of large numbers

In a random sampling,

the larger the sample,

the closer the proportion of successes in the sample tends to be the proportion in the population.

Example, simulation of flipping a coin

Number of Flips	10	100	1000	10000	100000
Heads	2	45	525	4990	50246
Tails	8	55	475	5010	49754

# Sample space

Sample Space for a chance process is a **complete** list of **disjoint outcomes** (all single possible results)

## Complete

no possible outcomes are left off the list.

## Disjoint (or mutually exclusive)

no two outcomes can occur at once.

*Often* by symmetry we can assume that the outcomes on a sample space are equally likely.

To be sure we need to collect data and see if indeed each of the outcomes occurs the same number of times (approximately).

# Examples

Rolling a fair die

Sample Space:  $\{1,2,3,4,5,6\}$

$$P(4) = 1/6$$

$$P(\text{number is even}) = 3/6 = 1/2$$

Selecting a card  
from a poker deck.

Sample Space:

$\{A \heartsuit, 2 \heartsuit, 3 \heartsuit, \dots, Q \heartsuit, K \heartsuit, A \diamondsuit, 2 \diamondsuit, 3 \diamondsuit, \dots, Q \diamondsuit, K \diamondsuit, A \clubsuit, 2 \clubsuit, 3 \clubsuit, \dots, Q \clubsuit, K \clubsuit, A \spadesuit, 2 \spadesuit, 3 \spadesuit, \dots, Q \spadesuit, K \spadesuit\}$

# Examples

Sample space

{A♥,2♥,3♥,...,Q♥,K♥, A♦,2♦,3♦,...,Q♦,K♦,  
A♣,2♣,3♣,...,Q♣,K♣, A♠,2♠,3♠,...,Q♠,K♠}

Select a card

$$P(3♥) = 1/52$$

$$P(\text{Ace}) = 4/52 = 1/13$$

$$P(♣) = 13/52 = 1/4$$

$$P(♣ \text{ or } ♥) = 26/52 = 1/2$$

# Random Process

Let's make a tree diagram every time we perform a random decision.

Example: we give three people McCafe and Strabucks and ask them which they prefer.

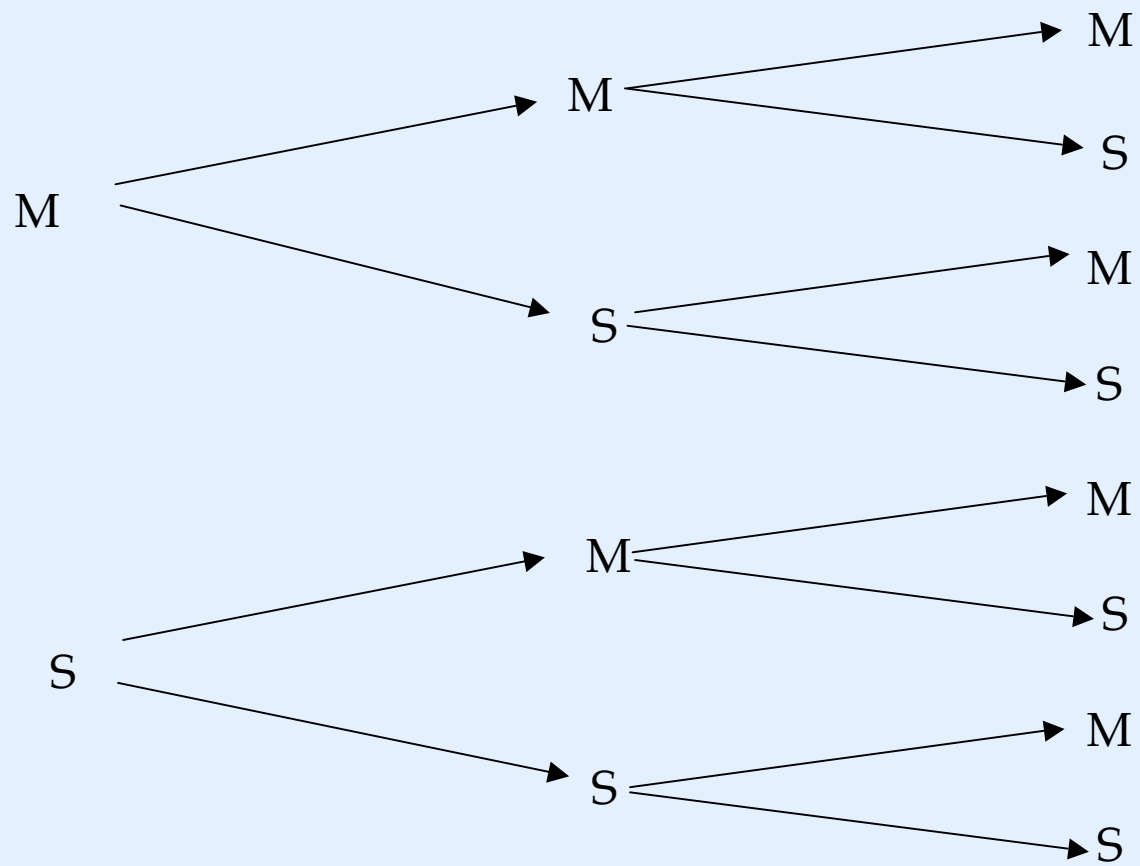
A random process is repeated several times



Person A

Person B

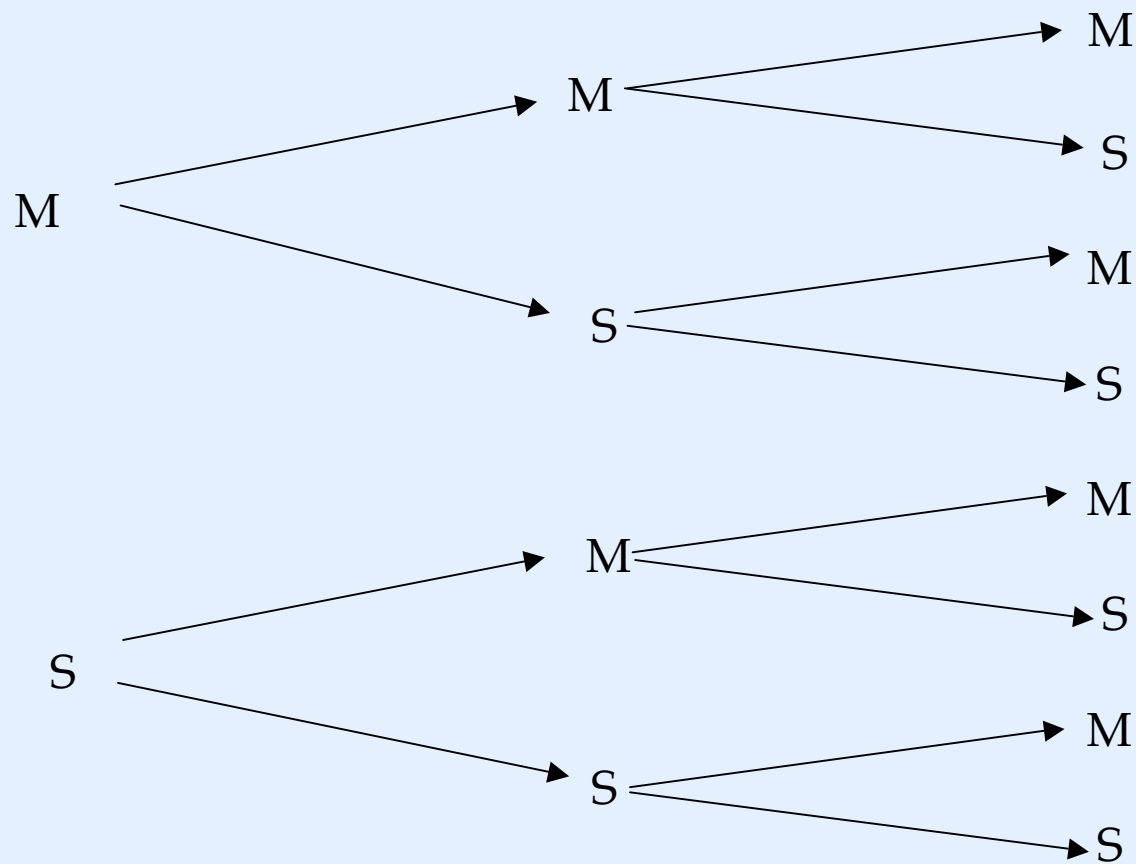
Person C



Person A

Person B

Person C

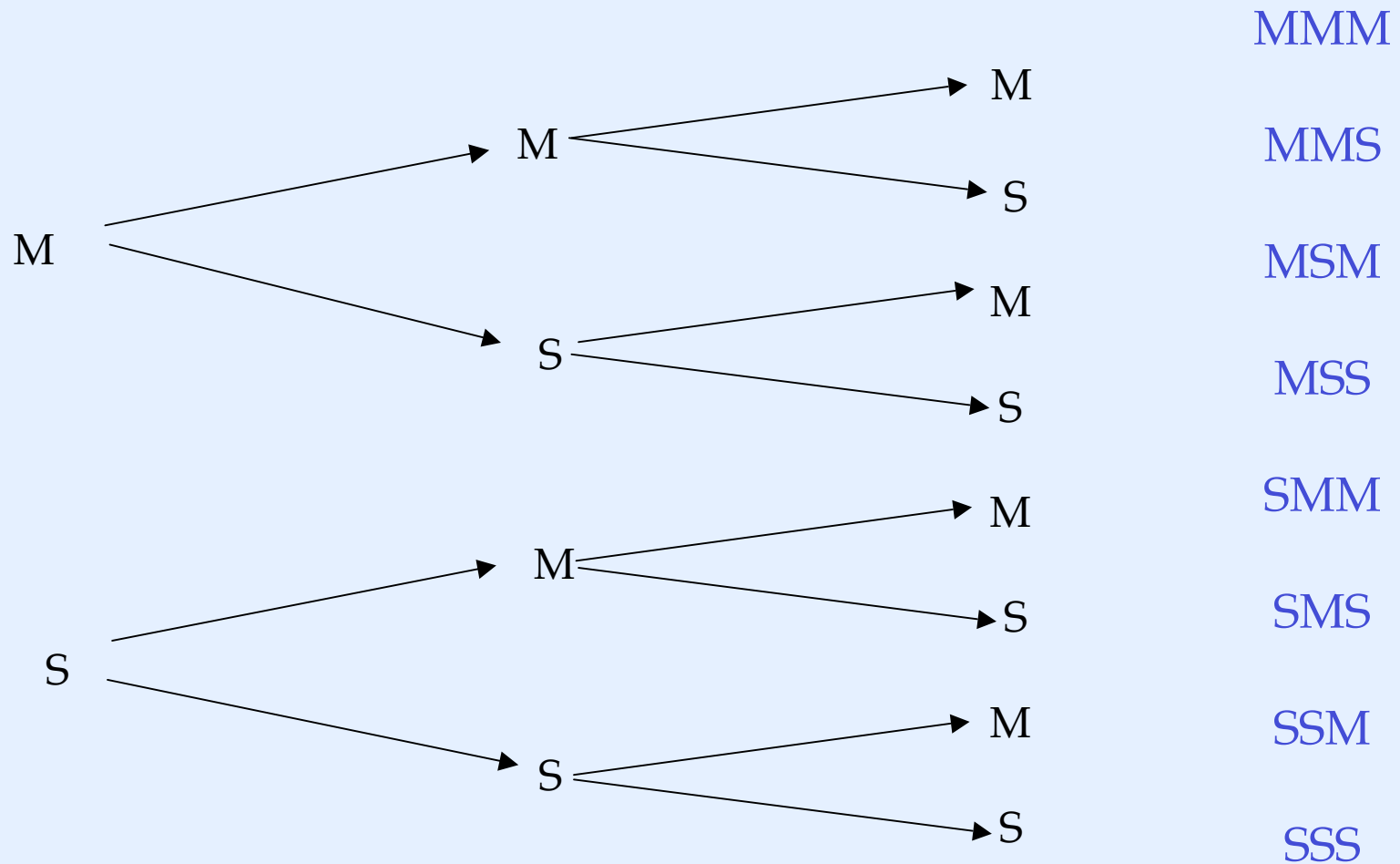


Outcome  
You write  
them

Person A

Person B

Person C



OUTCOMES

# The fundamental Counting Principle

For a two-stage process,  
with  $n_1$  possible outcomes for stage 1 and  
 $n_2$  possible outcomes for stage 2,  
the number of possible  
outcomes for the two stages together is  $n_1 * n_2$  .

More generally, if there are  $k$  stages,  
with  $n_i$  possible outcomes for stage  
 $i$ , then the number of possible outcomes for all  
 $k$  stages taken together is

$$n_1 * n_2 * n_3 * \dots * n_k$$

# The fundamental Counting Principle

Suppose you flip a fair coin five times.

- a. How many possible outcomes are there?
- b. What is the probability you get five heads?
- c. What is the probability you get four heads and one tail?

# Addition rule and disjoint events

“OR” in mathematics means one, the other, or both.

Two events A and B are called **disjoint** (mutually exclusive) if they have no outcomes in common.

If A and B are **disjoint** then  
 $P(A \text{ and } B) = 0$

Similarly if A, B are **mutually exclusive** then  
 $P(A \text{ or } B) = P(A) + P(B)$

# Let's go to Vegas

What is the probability that I roll a fair die  
and get 4 (event A) and 5 (event B)

$$P(A \text{ and } B) = ?$$

And

$$P(A \text{ or } B) = ?$$

# McCafe vs. Starbucks

What is the probability that one person out of two likes Starbucks more and the other likes McCafe more?

Is it  $1/3$  or  $1/4$ ?

THINK!



# Are these disjoint?

Labor  
force  
in the  
USA

Noninstitutional Population	Number of People (in millions)
Employees in nonagricultural industries	144
Employees in agricultural and related industries	2
Unemployed but seeking employment	7
Not in the labor force	79
Total	232

Activity	Percentage of U.S. Adults Who Engaged in Activity at Least Once in the Prior 12 Months
Dining out	49
Reading books	39
Computer games	20
Going to the beach	24

Our free time

# Some food for thought

**D11.** Suppose you select a person at random from your campus. Are these pairs of events mutually exclusive?

- a. has ridden a roller coaster; has ridden a Ferris wheel
- b. owns a classical music CD; owns a jazz CD
- c. is a senior; is a junior
- d. has brown hair; has brown eyes
- e. is left-handed; is right-handed
- f. has shoulder-length hair; is male

**D12.** Suppose there is a 20% chance of getting a mosquito bite each summer evening that you go outside. Can you use the Addition Rule for Disjoint Events to compute the probability that you will get bitten if you go outside on three summer evenings? If you go outside on six summer evenings?

# Obesity in America

		Male	Female	Total
Weight	Neither Overweight nor Obese ( $BMI < 25$ )	15.4	23.3	38.7
	Overweight ( $25 \leq BMI < 30$ )	21.9	14.9	36.8
	Obese ( $BMI \geq 30$ )	12.3	12.2	24.5
	Total	49.6	50.4	100

## Questions:

What is the probability of being overweight OR obese?

What is the probability of being overweight OR male?

# Obesity in America

		Male	Female	Total
Weight	Neither Overweight nor Obese ( $BMI < 25$ )	15.4	23.3	38.7
	Overweight ( $25 \leq BMI < 30$ )	21.9	14.9	36.8
	Obese ( $BMI \geq 30$ )	12.3	12.2	24.5
	Total	49.6	50.4	100

overweight OR obese =  $P(\text{over}) + P(\text{obese}) = 0.613$

Easy - why?

# Obesity in America

		Male	Female	Total
Weight	Neither Overweight nor Obese ( $BMI < 25$ )	15.4	23.3	38.7
	Overweight ( $25 \leq BMI < 30$ )	21.9	14.9	36.8
	Obese ( $BMI \geq 30$ )	12.3	12.2	24.5
	Total	49.6	50.4	100

overweight OR male =

$$P(\text{over}) + P(\text{male}) - P(\text{over and male}) =$$

$$0.368 + 0.496 - 0.219$$

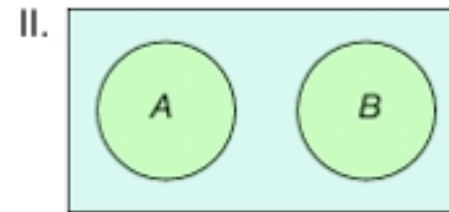
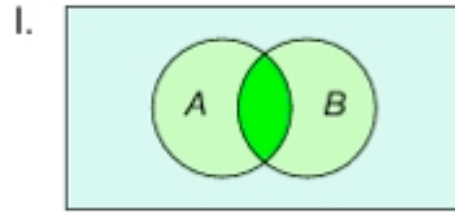
more thoughtful

# Addition rule

For any two events A and B

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

# Venn Diagrams



**Display 5.15**

Venn diagrams illustrating the two forms of the Addition Rule.

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Which of the two represents mutually exclusive events?

# A question

If I roll two dice, what is the probability that I get doubles or a sum of eight?

Identify  $A$ ,  $B$  and  $A$  and  $B$

And use the previous results



# A question

If I roll two dice, what is the probability that I get doubles or a sum of eight?

$$\begin{aligned} P(\text{doubles and sum8}) &= \\ P(\text{doubles}) + P(\text{sum8}) - P(4 \text{ and } 4) &= \\ \frac{6}{36} + \frac{5}{36} - \frac{1}{36} &= \\ \frac{10}{36} &= \frac{5}{18} \end{aligned}$$

Hk

Page 236/245

E2, E3, E4, E7, E8, E9, E11, E12, E13, E14,  
E15, E18, E28