1) Determine whether the series is absolutely convergent, conditionally convergent or divergent

$$\sum_{n=1}^{\infty} \frac{(-1)^n e^{1/n}}{n^3}$$

?

ABS. conv $\displaystyle\sum_{n=1}^{\infty} \frac{e^{1/n}}{n^3}$

consider $e^{1/n} < e$ so $\left|\dfrac{e^{1/n}}{n^3}\right| < \dfrac{e}{n^3}$ $\downarrow$ CONV by ptest

SERIES is ABS convergent

2) Test the series for convergence or divergence

$$\sum_{n=1}^{\infty} (-1)^n \arctan n$$

D

alt. series, arctan n

but also note $\lim_{n\to\infty} (-1)^n \arctan n \neq 0$

in fact DNE since

$\arctan n \to \dfrac{\pi}{2}$ !!

3) Determine whether the series is convergent or divergent

$$\sum_{n=1}^{\infty} \frac{1}{n^2 + 6n + 13}$$

C

has same character as $\dfrac{1}{n^2}$

CONV by comp. test

4) Find $\dfrac{dy}{dx}$ of the curve $x = 1 + t^2$, $y = t^2 + t$.

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} =$$

$$\boxed{y' = \frac{2t}{2t+1}}$$

5) Determine whether the following series is convergent or divergent and why

$$\sum_{n=1}^{\infty} \frac{n+2}{n+1}$$

$\lim_{n \to \infty} \frac{n+2}{n+1} = 1$ NOT CONVERG

DIV by div. test

6) Determine whether the following series is convergent or divergent and why

$$\sum_{n=1}^{\infty} \frac{n}{n^4+1}$$

same character as $\frac{1}{n^3}$ CONV by p test

7) Find a polar equation for the curve represented by the following Cartesian equation $xy = 4$

$x = r\cos\theta$

$y = r\sin\theta$

$\boxed{r^2 \cos\theta \sin\theta = 4}$ or

$2r^2 \sin\theta \cos\theta = 8$ or

$\boxed{r^2 \sin 2\theta = 8}$

8) Determine whether the following series are convergent or divergent, and the sum if possible

C $\sum_{n=1}^{\infty} \frac{1+2^n}{3^n}$, $\sum_{n=1}^{\infty} \frac{1+3^n}{2^n}$ D div by div. test

$\sum_{n=1}^{\infty} \left[\frac{1}{3^n} + \left(\frac{2}{3}\right)^n\right]$ conv to

$\frac{1}{2^n}$ conv but $\left(\frac{3}{2}\right)^n$ $\lim_{n \to \infty} \left(\frac{3}{2}\right)^n = \infty$

$= \sum_{n=1}^{\infty} \frac{1}{3^{n-1}}\left(\frac{1}{3}\right) + \sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^{n-1} \cdot \frac{2}{3}$

$= \frac{1/3}{1-1/3} + \frac{2/3}{1-2/3} = \frac{1}{3-1} + \frac{2}{3-2} = \frac{1}{3} + 2 = \frac{7}{2}$

9) Determine the value of the following sum by expressing the argument of the sum via partial fractions and by writing out the first few terms of the sum.

$$\sum_{n=2}^{\infty} \frac{2}{n^2-1} = \sum \left( \frac{\cancel{2}}{n-1} - \frac{\cancel{2}}{n+1} \right) \frac{1}{\cancel{2}} =$$

$$= \sum_{n=2}^{\infty} \left( \frac{1}{n-1} - \frac{1}{n+1} \right) =$$

telescoping

$$\left(1 - \cancel{\frac{1}{3}}\right) + \left(\frac{1}{2} - \cancel{\frac{1}{4}}\right) + \left(\cancel{\frac{1}{3}} - \frac{1}{5}\right) + \left(\cancel{\frac{1}{4}} - \frac{1}{6}\right) \dots = \frac{3}{2}$$

10) Determine whether the following series is convergent or divergent

$$\sum_{n=1}^{\infty} \frac{\cos(n)}{n^2}$$

CONV

(in fact is absol. conv)

$$0 < \left| \frac{\cos n}{n^2} \right| \leq \frac{1}{n^2}$$

$\frac{1}{n^2}$ CONV

CONV by comp. test

11) Give an example of a conditionally convergent series (convergent but its absolute value is not)

$\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ is conv, but $\sum_{n=1}^{\infty} \frac{1}{n}$ is not

12) Determine convergence or divergence of the following series

$$\sum_{n=1}^{\infty} \sin n$$

$\lim_{n \to \infty} \sin n$ DNE $\Rightarrow$ series is divergent

13) Find the exact length of the curve $x = 1 + 3t^2$, $y = 4 + 2t^3$ with $0 \le t \le 1$.

$$\frac{dx}{dt} = 6t \qquad \frac{dy}{dt} = 6t^2 \qquad ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$L = \int_0^1 6\sqrt{t^2 + t^4}\, dt = 6\int_0^1 t\sqrt{1+t^2}\, dt \qquad 1+t^2 = u \quad 2t\,dt = du$$

$$= \frac{6}{2}\int_0^2 du\,\sqrt{u} = \cancel{3}\frac{2}{\cancel{3}}u^{3/2}\Big|_0^2 = \boxed{2\cdot 2^{3/2}} = 2^{5/2} = 4\sqrt{2}$$

14) Eliminate the parameter to find a Cartesian equation of $x = \sin t$, $y = \csc t$ with $0 < t < \frac{\pi}{2}$.

$$x = \sin t \qquad y = \frac{1}{\sin t} \qquad \boxed{xy = 1}$$

$$0 < x < 1$$

15) Determine whether the sequence $\{n^2 e^{-n}\}$ converges or not.

$$\lim_{n\to\infty} n^2 e^{-n} =$$ use l'hospital on x

$$\lim_{x\to\infty} \frac{x^2}{e^x} = H \qquad \frac{\infty}{\infty}$$

$$= \lim_{x\to\infty} \frac{2x}{e^x} = H = \lim_{x\to\infty} \frac{2}{e^x} = 0$$

CONV

16) Determine whether the sequence $\{\tan\left(\frac{2n\pi}{1+8n}\right)\}$ converges or not.

$$\lim_{n\to\infty} \tan\frac{2n\pi}{1+8n} = \lim_{n\to\infty} \tan\frac{2\pi}{\cancel{8}4} = \frac{1}{\sqrt{2}}$$

the seq. is Conv.

CONV