Math 150B Midterm 2

(Dated: March 22^{nd} 2012)

Name: MATMA SOLUTIONS

SID:

Write clearly and box all your answers. Simplify all formulas to the very end. No calculators allowed. Do not work out of memory, rather think before starting. Use the back for more space. Show all steps you are performing and state all theorems you are using.

1) Evaluate the integral $\int \frac{1}{(x+a)(x+b)} dx$

$$\left(\int \frac{dx}{x+a} - \int \frac{dx}{x+b}\right) \frac{1}{b-a} = \boxed{\frac{1}{b-a} \ln\left|\frac{x+a}{x+b}\right| + C}$$

2) Evaluate the integral $\int \frac{x}{(x^2+4x+13)} dx$

$$= \frac{1}{2}\int \frac{2x+4}{x^2+4x+13} dx - \frac{1}{2}\int \frac{4\,dx}{(x^2+4x+13)}$$

$u = x^2+4x+13$
$du = 2x+4$

$$= \frac{1}{2}\int \frac{du}{u} - 2\int \frac{dx}{(x+2)^2+9}$$

$$= \boxed{\frac{1}{2}\ln|(x^2+4x+13)| - 2\frac{1}{3}\arctan\left(\frac{x+2}{3}\right) + C}$$

3) Evaluate the integral using the proper trigonometry substitution $\int_0^3 \frac{x}{\sqrt{36-x^2}} dx$

$x = 6\sin y$
$dx = 6\cos y$

$$\int_0^{\pi/6} \frac{6\sin y}{\cancel{6\cos y}} \cancel{6\cos y} =$$

$y = \arcsin \frac{x}{6}$

$$-6\cos y \Big|_0^{\pi/6} = 6 - 6\frac{\sqrt{3}}{2} =$$

$$\boxed{6 - 3\sqrt{3}}$$

4) Integrate using integration by parts $\int_4^9 \frac{\ln x}{\sqrt{x}} dx$

$$2\sqrt{x}\ln x \Big|_4^9 - 2\int_4^9 \cancel{\sqrt{x}} \frac{1}{\cancel{x}\sqrt{x}} dx$$

$$= 6\ln 9 - 4\ln 4 - 2\cdot 2\sqrt{x}\Big|_4^9 =$$

$$12\ln 3 - 8\ln 2 - 4\cdot 3 + 4\cdot 2 = \boxed{12\ln 3 - 8\ln 2 - 4}$$

5) Integrate using integration by parts $\int_0^{1/2} x\cos\pi x dx$

$$\frac{x}{\pi}\sin\pi x\Big|_0^{1/2} - \frac{1}{\pi}\int_0^{1/2}\sin\pi x\, dx$$

$$\frac{1}{2\pi}\sin\frac{\pi}{2} + \frac{1}{\pi^2}\cos\pi x\Big|_0^{1/2}$$

$$= \boxed{\frac{1}{2\pi} - \frac{1}{\pi^2}}$$

6) Integrate using integration by parts $\int \cos x\ln(\sin x)dx$

$\sin x = u$

$\cos x\, dx = du$

$$\int du \ln u$$

$$= u\ln|u| - u + C = \boxed{\sin x\ln|\sin x| - \sin x + C}$$

By parts

$$\sin x\ln(\sin x) - \int \frac{\cancel{\sin x}}{\cancel{\sin x}}\cos x\, dx$$

$$= \sin x\ln(\sin x) - \sin x + C$$

7) Evaluate the integral $\int \frac{x^2+2x-1}{x^3-x}dx$

$x^3 - x = x(x-1)(x+1)$

$$\frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1} = \frac{x^2+2x-1}{x^3-x}$$

$A = -B - C$

$B - C = 2$

$A = 1 \quad B = -C$

$C = -1$

$B = 1$

$$Ax^2 - A + Bx^2 + Bx + Cx^2 - Cx = x^2 + 2x - 1$$

$$\int\left(\frac{1}{x} + \frac{1}{x-1} + \frac{-1}{x+1}\right)dx = \ln|x| + \ln|x-1| - \ln|x+1| + C = \boxed{\ln\left|\frac{x(x-1)}{x+1}\right| + C}$$

8) Evaluate $\int \frac{\sin^3(\sqrt{x})}{\sqrt{x}}dx$

$\sqrt{x} = u$

$\frac{1}{2\sqrt{x}}dx = du$

$$2\int \sin^3 u \, du$$

$$2\int \sin u(1-\cos^2 u)\, du$$

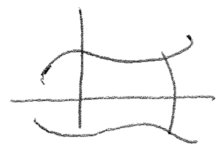
$$2\left[-\cos u - \int \sin u \cos^2 u\right] = -2\cos u + \frac{2}{3}\cos^3 u + C = \boxed{-2\cos\sqrt{x} + \frac{2}{3}\cos^3\sqrt{x} + C}$$

9) Evaluate $\int \sec^3 x \tan x dx$

$\sec x = u \qquad \sec x \tan x \, dx = du$

$$\int u^2 du = \boxed{\frac{1}{3}\sec^3 x + C}$$

10) Find the exact area of the surface obtained by rotating hte curve about the x axis $y(x) = \sqrt{1+4x}$ for $1 \le x \le 5$.



$y' = \frac{4}{2\sqrt{1+4x}} = \frac{2}{\sqrt{1+4x}}$

$S = \int 2\pi y(x)\,dx \sqrt{1+y'(x)^2}$

$= 2\pi \int_1^5 \sqrt{1+4x}\left(\sqrt{1+\frac{4}{1+4x}}\right) dx = 2\pi \int_1^5 \sqrt{5+4x}\; dx$

$= \cancel{2}\pi \; \frac{\cancel{2}}{3}(5+4x)^{3/2} \frac{1}{\cancel{4}} \Big|_1^5$

$= \frac{\pi}{3}\left(25^{3/2} - 9^{3/2}\right) =$

$\frac{\pi}{3}(125-27) = \boxed{\frac{98\pi}{3}}$

11) Find the exact length of the curve $y = \ln(\cos x)$ for $0 \le x \le \frac{\pi}{3}$

$\int_0^{\pi/3} \sqrt{1+\frac{\sin^2 x}{\cos^2 x}}\, dx = \int_0^{\pi/3} \sec x\, dx$

$= \ln|\sec x + \tan x| \Big|_0^{\pi/3} = \ln\left|2+\frac{\cancel{2}\sqrt{3}}{\cancel{2}}\right| - \cancel{\ln|1|} =$

$\boxed{\ln(2+\sqrt{3})}$

12) Evaluate the integral $\int \frac{10}{(x-1)(x^2+9)} dx$

$\frac{A}{x-1} + \frac{Bx+C}{x^2+9}$

$Ax^2 + 9A + Bx^2 - Bx + Cx - C = 10$

$A = -B$

$9A - C = 10$

$-10C = 10$

$B = C$

$A = -C$

$C = -1$

$B = -1$

$A = 1$

$= \ln|x-1| + \int \frac{-x}{x^2+9} dx + \int \frac{-1}{x^2+9} dx$

$= \boxed{\ln|x-1| - \frac{1}{3}\arctan\frac{x}{3} - \frac{1}{2}\ln|x^2+9| + C}$