Math 150B Midterm 1

(Dated: February 23rd 2012)

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SID: Solutions

Write clearly and box all your answers. Simplify all formulas to the very end. No calculators allowed. Do not work out of memory, rather think before starting. Use the back for more space. Show all steps you are performing and state all theorems you are using.

1) Find an equation of the tangent line to the curve $xe^y + ye^x = 1$ at the point $(0, 1)$.

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$e^y + xe^y y' + y'e^x + ye^x = 0$

$y' = -\dfrac{e^y + ye^x}{xe^y + e^x} \quad @(0,1) = -\dfrac{e+1}{1} = -(1+e)$

$y = -(e+1)x + b \qquad @\ x=0,\ y=1 \qquad 1 = b$

$\boxed{y = -(e+1)x + 1}$

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2) Evaluate the integral $\int e^x \cos(e^x)dx$

$e^x = y$

$e^x dx = dy$

$\int \cos(y)\, dy = \boxed{\sin(y) + C}$

$\boxed{\sin(e^x) + C}$

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3) Use l'Hospital's rule to help sketch the following curve $y = xe^{-x}$

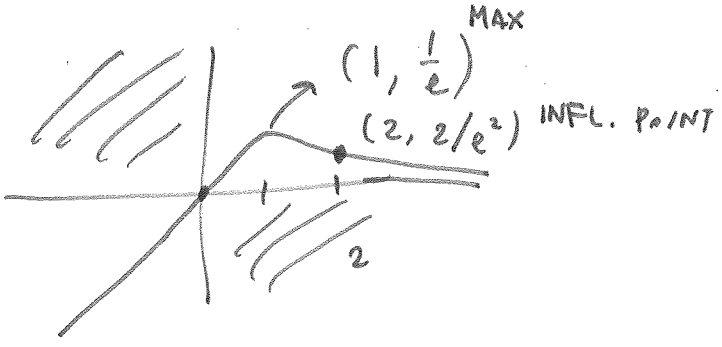
D: x all $\mathbb{R}$; $y > 0$ if $x > 0$; $y < 0$ " $x < 0$; $y = 0$ if $x = 0$

$\lim_{x\to\infty} \dfrac{x}{e^x} \overset{H}{=} \dfrac{1}{e^x} = 0$

$\lim_{x\to-\infty} xe^{-x} = -\infty$

$y' = e^{-x} - xe^{-x} = e^{-x}(1-x)$

max $x = 1$ $y = \frac{1}{e}$



$y'' = -e^{-x} + xe^{-x} - e^{-x}$

$= e^{-x}(x-2)$

4) Solve for x the following equation $e^{2x} - e^x - 6 = 0$

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$e^x = \frac{1 \pm \sqrt{1+24}}{2} = \frac{1 \pm 5}{2}$ $\begin{matrix} + \\ - \end{matrix}$ $\begin{matrix} 3 \\ -2 \end{matrix}$

$\boxed{x = \ln 3}$

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5) Differentiate the function $f(x) = x \sin(2^x)$.

$f' = \sin 2^x + x \cos 2^x \cdot 2^x \ln 2$

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6) Find the exact value of $\csc(\cos^{-1} \frac{3}{5})$

$\csc(x) = \frac{1}{\sin(x)} = \frac{1}{\sqrt{1 - 9/25}} = \frac{5}{4}$

7) Evaluate the integral $\int \frac{\sin 2x}{1+\cos^2 x}dx$

$1+\cos^2 x = y$

$2\cos x \sin x\, dx = -dy$

$\int \frac{-dy}{y}$

$= -\ln|y| + C = \boxed{-\ln(1+\cos^2 x) + C}$ ↓ always > 0

8) Find the limit $\lim_{x\to 0} \frac{\cos mx - \cos nx}{x^2}$

$\frac{0}{0} = H =$

$\lim_{x\to 0} = \frac{-m\sin mx + n\sin nx}{2x} = \frac{0}{0} = H = \lim_{x\to 0} \frac{-m^2\cos mx + n^2\cos nx}{2}$

$= \boxed{\frac{n^2-m^2}{2}}$

9) Prove the identity $\tanh(\ln x) = \frac{x^2-1}{x^2+1}$

$e^{\ln x} = x$

$e^{-\ln x} = 1/x$

$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{x - \frac{1}{x}}{x + \frac{1}{x}}$

$= \frac{x^2-1}{x^2+1}$ ✓

10) Differentiate and simplify $y = \tan^{-1}(\cos x)$

$$y' = \frac{1}{1+\cos^2 x}(-\sin x)$$

11) Using l'Hospital's rule find $\lim_{x\to 0}\left(\frac{1}{x} - \frac{1}{e^x - 1}\right)$

$$= \lim_{x\to 0} \frac{e^x - 1 - x}{x(e^x - 1)} = \frac{0}{0} = H = \lim_{x\to 0} \frac{e^x - 1}{e^x - 1 + xe^x} = H$$

$$\lim_{x\to 0} \frac{e^x}{e^x + e^x + xe^x} = \lim_{x\to 0} \frac{1}{2+x} = \boxed{\frac{1}{2}}$$

12) Find the limit $\lim x \to 0 \frac{\sinh x - x}{x^3}$ $= H =$

$$\lim_{x\to 0} \frac{\cosh x - 1}{3x^2} = \frac{0}{0} \overset{H}{=} \lim_{x\to 0} \frac{\sinh x}{6x} =$$

$$\frac{0}{0} = H = \lim_{x\to 0} \frac{\cosh x}{6} = \boxed{\frac{1}{6}}$$