CHAPTER 7

Capital Asset Pricing and Arbitrage Pricing Theory

7.1 THE CAPITAL ASSET PRICING MODEL

Capital Asset Pricing Model (CAPM)

- Equilibrium model that underlies all modern financial theory
- Derived using principles of diversification with simplified assumptions
- Markowitz, Sharpe, Lintner and Mossin are researchers credited with its development

Assumptions

- Individual investors are price takers
- Single-period investment horizon
- Investments are limited to traded financial assets
- No taxes nor transaction costs

Assumptions (cont.)

- Information is costless and available to all investors
- Investors are rational mean-variance optimizers
- Homogeneous expectations

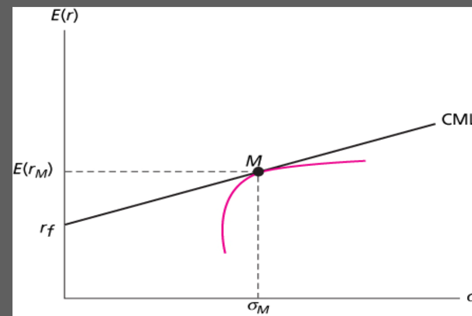
Resulting Equilibrium Conditions

- All investors will hold the same portfolio for risky assets – market portfolio
- Market portfolio contains all securities and the proportion of each security is its market value as a percentage of total market value

Resulting Equilibrium Conditions (cont.)

- Risk premium on the market depends on the average risk aversion of all market participants
- Risk premium on an individual security is a function of its covariance with the market

Figure 7.1 The Efficient Frontier and the Capital Market Line



The Risk Premium of the Market Portfolio

M = Market portfolio

r_f = Risk free rate

$E(r_M) - r_f$ = Market risk premium

$$\frac{E(r_M) - r_f}{\sigma_M} = \text{Market price of risk} = \text{Slope of the CAPM}$$

Expected Returns On Individual Securities

- The risk premium on individual securities is a function of the individual security's contribution to the risk of the market portfolio
- Individual security's risk premium is a function of the covariance of returns with the assets that make up the market portfolio

Expected Returns On Individual Securities: an Example

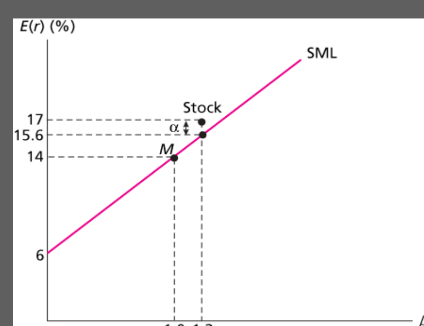
- Using the Dell example:

$$\frac{E(r_M) - r_f}{1} = \frac{E(r_D) - r_f}{\beta_D}$$

- Rearranging gives us the CAPM's expected return-beta relationship

$$E(r_D) = r_f + \beta_D \left[E(r_M) - r_f \right]$$

Figure 7.2 The Security Market Line and Positive Alpha Stock



SML Relationships

$\beta = [COV(r_i,r_m)] / \sigma_m^2$

$E(r_m) - r_f =$ market risk premium

$SML = r_f + \beta[E(r_m) - r_f]$

Sample Calculations for SML

$E(r_m) - r_f = .08 \quad r_f = .03$

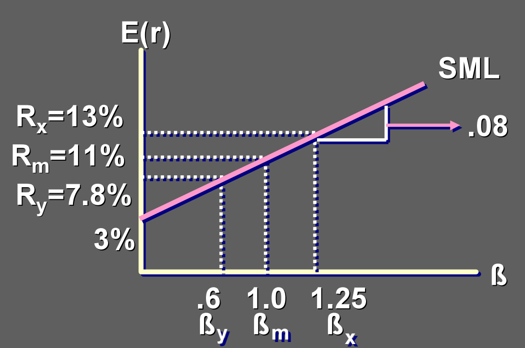
$\beta_x = 1.25$

$E(r_x) = .03 + 1.25(.08) = .13$ or 13%

$\beta_y = .6$

$e(r_y) = .03 + .6(.08) = .078$ or 7.8%

Graph of Sample Calculations



7.2 THE CAPM AND INDEX MODELS

Estimating the Index Model

- Using historical data on T-bills, S&P 500 and individual securities
- Regress risk premiums for individual stocks against the risk premiums for the S&P 500
- Slope is the beta for the individual stock

Table 7.1 Monthly Return Statistics for T-bills, S&P 500 and General Motors

table 7.1 Monthly return statistics: T-bills, S&P 500, and General Motors, January 1999–December 2003

	T-Bills	S&P 500	GM
Average excess return (%)	0.28	−0.33	0.49
Standard deviation (%)	0.16	4.96	11.24
Geometric average (%)	0.28	−0.17	0.15
Cumulative total 5-year return (%)	18.20	−9.54	9.10

Figure 7.3 Cumulative Returns for T-bills, S&P 500 and GM Stock

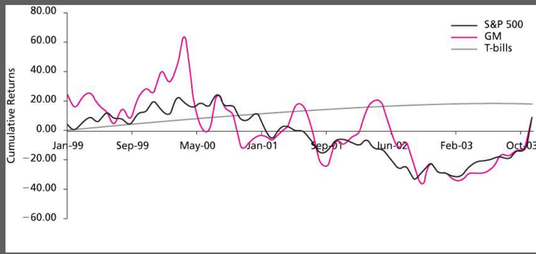


Figure 7.4 Characteristic Line for GM

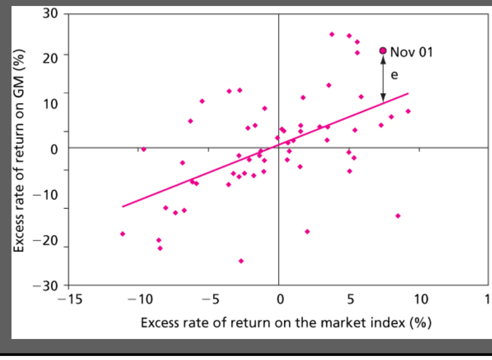


Table 7.2 Security Characteristic Line for GM: Summary Output

table 7.2

Security characteristic line for GM: Summary output

Regression Statistics	
Multiple R	0.5465
R-square	0.2987
Adjusted R-square	0.2866
Standard error	9.4909
Observations	60

ANOVA

	df	SS	MS	F	Significance F
Regression	1	2224.696	2224.696	24.698	0.000006
Residual	58	5224.451	90.077		
Total	59	7449.147			

	Coefficients	Standard Error	t-statistic	p-value	Lower 95%	Upper 95%
Intercept	0.8890	1.2279	0.7240	0.4720	−1.5690	3.3470
Slope	1.2384	0.2492	4.9697	0.0000	0.7396	1.7372

GM Regression: What We Can Learn

- GM is a cyclical stock
- Required Return:

$$r_f + \beta(r_M - r_f) = 2.75 + 1.24x5.5 = 9.57\%$$

- Next compute betas of other firms in the industry

Predicting Betas

- The beta from the regression equation is an estimate based on past history
- Betas exhibit a statistical property
  - Regression toward the mean

THE CAPM AND THE REAL WORLD

CAPM and the Real World

- The CAPM was first published by Sharpe in the *Journal of Finance* in 1964
- Many tests of the theory have since followed including Roll's critique in 1977 and the Fama and French study in 1992

7.4 MULTIFACTOR MODELS AND THE CAPM

Multifactor Models

- Limitations for CAPM
- Market Portfolio is not directly observable
- Research shows that other factors affect returns

Fama French Three-Factor Model

- Returns are related to factors other than market returns
- Size
- Book value relative to market value
- Three factor model better describes returns

Table 7.3 Summary Statistics for Rates of Return Series

TABLE 7.3

Summary statistics for rates of return series, 1999–2003

	Monthly Average (%)	Standard Deviation (%)	Geometric Average of Total Return (%)	Total Five-Year Return (%)
T-bill rate	0.28%	0.16%	.28%	18.20%
Broad index excess return	−0.10	5.19	0.05	2.99
SMB return	1.01	4.55	0.91	72.40
HML return	0.47	6.00	0.29	19.08
GM excess return	0.49	11.24	0.15	9.10

Table 7.4 Regression Statistics for the Single-index and FF Three-factor Model

table 7.4

Regression statistics for the single-index and the FF three-factor model

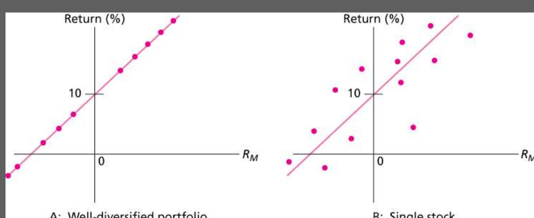
	Single-Index Regression (broad market index)	FF Three-Factor Model
Correlation coefficient	0.54	0.60
Adjusted R-square	0.27	0.32
Regression standard error	9.57	9.24
Intercept	0.60	0.30
Standard error	1.24	1.24
Market beta	1.16	1.26
Standard error	0.24	0.24
SMB beta	—	0.05
Standard error	—	0.29
HML beta	—	0.52
Standard error	—	0.22

7.5 FACTOR MODELS AND THE ARBITRAGE PRICING THEORY

Arbitrage Pricing Theory

- Arbitrage - arises if an investor can construct a zero beta investment portfolio with a return greater than the risk-free rate
- If two portfolios are mispriced, the investor could buy the low-priced portfolio and sell the high-priced portfolio
- In efficient markets, profitable arbitrage opportunities will quickly disappear

Figure 7.5 Security Line Characteristics



APT and CAPM Compared

- APT applies to well diversified portfolios and not necessarily to individual stocks
- With APT it is possible for some individual stocks to be mispriced - not lie on the SML
- APT is more general in that it gets to an expected return and beta relationship without the assumption of the market portfolio
- APT can be extended to multifactor models