

Quick Quiz: Part 2

- You know the payment amount for a loan and you want to know how much was borrowed. Do you compute a present value or a future value?
- You want to receive \$5,000 per month in retirement. If you can earn .75% per month and you expect to need the income for 25 years, how much do you need to have in your account at retirement?

Finding the Payment

- Suppose you want to borrow \$20,000 for a new car. You can borrow at 8% per year, compounded monthly (8/12 = .666666667% per month). If you take a 4-year loan, what is your monthly payment?
 - $\$20,000 = C[1 - 1 / 1.00666667^{48}] / .00666667$
 - $C = \$488.26$

Finding the Number of Payments – Example 5.6

- Start with the equation and remember your logs.
 - $\$1,000 = \$20(1 - 1/1.015^t) / .015$
 - $.75 = 1 - 1 / 1.015^t$
 - $1 / 1.015^t = .25$
 - $1 / .25 = 1.015^t$
 - $t = \ln(1/.25) / \ln(1.015) = 93.111 \text{ months} = 7.75 \text{ years}$
- And this is only true if you don't charge anything more on the card!

Finding the Number of Payments – Another Example

- Suppose you borrow \$2,000 at 5% and you are going to make annual payments of \$734.42. How long is it before you pay off the loan?
 - $\$2,000 = \$734.42(1 - 1/1.05^t) / .05$
 - $.136161869 = 1 - 1/1.05^t$
 - $1/1.05^t = .863838131$
 - $1.157624287 = 1.05^t$
 - $t = \ln(1.157624287) / \ln(1.05) = 3 \text{ years}$

Finding the Rate

- Suppose you borrow \$10,000 from your parents to buy a car. You agree to pay \$207.58 per month for 60 months. What is the monthly interest rate?

Annuity – Finding the Rate

- Trial and Error Process
 - Choose an interest rate and compute the PV of the payments based on this rate
 - Compare the computed PV with the actual loan amount
 - If the computed PV $>$ loan amount, then the interest rate is too low
 - If the computed PV $<$ loan amount, then the interest rate is too high
 - Adjust the rate and repeat the process until the computed PV and the loan amount are equal

Quick Quiz: Part 3

- You want to receive \$5,000 per month for the next 5 years. How much would you need to deposit today if you can earn .75% per month?
- What monthly rate would you need to earn if you only have \$200,000 to deposit?
- Suppose you have \$200,000 to deposit and can earn .75% per month.
 - How many months could you receive the \$5,000 payment?
 - How much could you receive every month for 5 years?

Future Values for Annuities

- Suppose you begin saving for your retirement by depositing \$2,000 per year in an IRA. If the interest rate is 7.5%, how much will you have in 40 years?
 - $FV(\text{Ordinary}) = \$2,000(1.075^{40} - 1)/.075 = \$454,513.04$
 - $FV(\text{Due}) = \$454,513.04 \times 1.075 = \$488,601.52$

Annuity Due

- You are saving for a new house and you put \$10,000 per year in an account paying 8%. The first payment is made today. How much will you have at the end of 3 years?
 - $FV = \$10,000[(1.08^3 - 1) / .08](1.08) = \$35,061.12$

Annuity Due Time Line

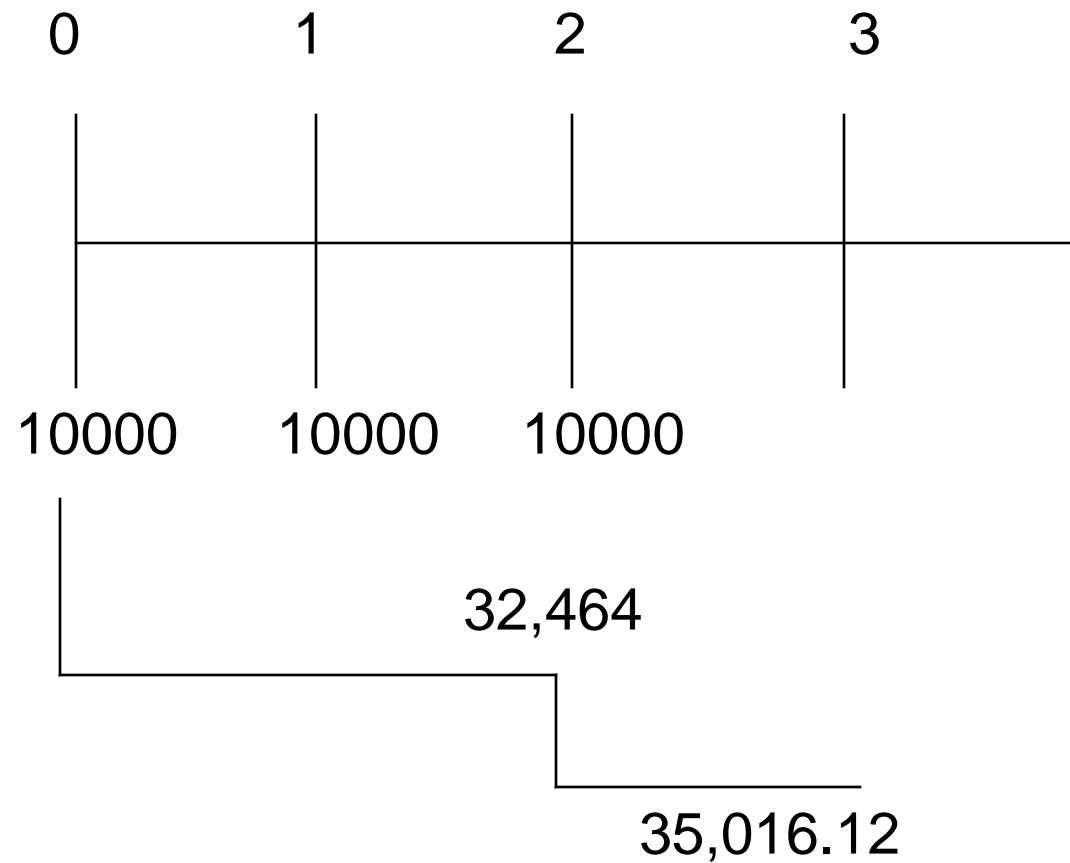


Table 5.2

I. Symbols

PV = Present value, what future cash flows are worth today

FV_t = Future value, what cash flows are worth in the future at time t

r = Interest rate, rate of return, or discount rate per period typically , but not always, one year

t = Number of periods typically , but not always, the number of years

C = Cash amount

II. Future value of C invested per period for t periods at r percent per period

$$FV_t = C \times [(1 + r)^t - 1]/r$$

A series of identical cash flows is called an annuity, and the term $[(1 + r)^t - 1]/r$ is called the *annuity future value factor*.

III. Present value of C per period for t periods at r percent per period

$$PV = C \times \{1 - [1/(1 + r)^t]\}/r$$

The term $\{1 - [1/(1 + r)^t]\}/r$ is called the *annuity present value factor*.

IV. Present value of a perpetuity of C per period

$$PV = C/r$$

A perpetuity has the same cash flow every year forever.

Example: Work the Web

- Another online financial calculator can be found at Moneychimp
- Click on the Web surfer and work the following example
 - Choose calculator and then annuity
 - You just inherited \$5 million. If you can earn 6% on your money, how much can you withdraw each year for the next 40 years?
 - Moneychimp assumes annuity due!!!
 - Payment = \$313,497.81



Perpetuity – Example 5.7

- Perpetuity formula: $PV = C / r$
- Current required return:
 - $\$40 = \$1 / r$
 - $r = .025$ or 2.5% per quarter
- Dividend for new preferred:
 - $\$100 = C / .025$
 - $C = \$2.50$ per quarter

Quick Quiz: Part 4

- You want to have \$1 million to use for retirement in 35 years. If you can earn 1% per month, how much do you need to deposit on a monthly basis if the first payment is made in one month?
- What if the first payment is made today?
- You are considering preferred stock that pays a quarterly dividend of \$1.50. If your desired return is 3% per quarter, how much would you be willing to pay?

Effective Annual Rate (EAR)

- This is the actual rate paid (or received) after accounting for compounding that occurs during the year
- If you want to compare two alternative investments with different compounding periods, you need to compute the EAR and use that for comparison.

Annual Percentage Rate

- This is the annual rate that is quoted by law
- By definition, $APR = \text{period rate} \times \text{number of periods per year}$
- Consequently, to get the period rate we rearrange the APR equation:
 - $\text{Period rate} = APR / \text{number of periods per year}$
- You should NEVER divide the effective rate by the number of periods per year – it will NOT give you the period rate

Computing APRs

- What is the APR if the monthly rate is .5%?
 - $.5\%(12) = 6\%$
- What is the APR if the semiannual rate is .5%?
 - $.5\%(2) = 1\%$
- What is the monthly rate if the APR is 12% with monthly compounding?
 - $12\% / 12 = 1\%$
 - Can you divide the above APR by 2 to get the semiannual rate? NO!!! You need an APR based on semiannual compounding to find the semiannual rate.

Things to Remember

- You ALWAYS need to make sure that the interest rate and the time period match.
 - If you are looking at annual periods, you need an annual rate.
 - If you are looking at monthly periods, you need a monthly rate.
- If you have an APR based on monthly compounding, you have to use monthly periods for lump sums, or adjust the interest rate appropriately if you have payments other than monthly

Computing EARs - Example

- Suppose you can earn 1% per month on \$1 invested today.
 - What is the APR? $1\%(12) = 12\%$
 - How much are you effectively earning?
 - $FV = 1(1.01)^{12} = 1.1268$
 - $\text{Rate} = (1.1268 - 1) / 1 = .1268 = 12.68\%$
- Suppose if you put it in another account, and you earn 3% per quarter.
 - What is the APR? $3\%(4) = 12\%$
 - How much are you effectively earning?
 - $FV = 1(1.03)^4 = 1.1255$
 - $\text{Rate} = (1.1255 - 1) / 1 = .1255 = 12.55\%$

EAR - Formula

$$\text{EAR} = \left[1 + \frac{\text{APR}}{m} \right]^m - 1$$

Remember that the APR is the quoted rate,
and m is the number of compounds per year

Decisions, Decisions II

- You are looking at two savings accounts. One pays 5.25%, with daily compounding. The other pays 5.3% with semiannual compounding. Which account should you use?
 - First account:
 - $\text{EAR} = (1 + .0525/365)^{365} - 1 = 5.39\%$
 - Second account:
 - $\text{EAR} = (1 + .053/2)^2 - 1 = 5.37\%$
- Which account should you choose and why?

Decisions, Decisions II Continued

- Let's verify the choice. Suppose you invest \$100 in each account. How much will you have in each account in one year?
 - First Account:
 - Daily rate = $.0525 / 365 = .00014383562$
 - FV = $\$100(1.00014383562)^{365} = \105.39
 - Second Account:
 - Semiannual rate = $.053 / 2 = .0265$
 - FV = $\$100(1.0265)^2 = \105.37
- You have more money in the first account.

Computing APRs from EARs

- If you have an effective rate, how can you compute the APR? Rearrange the EAR equation and you get:

$$\text{APR} = m \left[(1 + \text{EAR})^{\frac{1}{m}} - 1 \right]$$

APR - Example

- Suppose you want to earn an effective rate of 12% and you are looking at an account that compounds on a monthly basis. What APR must they pay?

$$APR = 12 \left[(1 + .12)^{1/12} - 1 \right] = .113865515$$

or 11.39%

Computing Payments with APRs

- Suppose you want to buy a new computer system. The store will allow you to make monthly payments. The entire computer system costs \$3,500. The loan period is for 2 years, and the interest rate is 16.9% with monthly compounding. What is your monthly payment?
 - Monthly rate = $.169 / 12 = .01408333333$
 - Number of months = $2(12) = 24$
 - $\$3,500 = C[1 - 1 / (1.01408333333)^{24}] / .01408333333$
 - $C = \$172.88$

Future Values with Monthly Compounding

- Suppose you deposit \$50 per month into an account that has an APR of 9%, based on monthly compounding. How much will you have in the account in 35 years?
 - Monthly rate = $.09 / 12 = .0075$
 - Number of months = $35(12) = 420$
 - $FV = \$50[1.0075^{420} - 1] / .0075 = \$147,089.22$

Present Value with Daily Compounding

- You need \$15,000 in 3 years for a new car. If you can deposit money into an account that pays an APR of 5.5% based on daily compounding, how much would you need to deposit?
 - Daily rate = $.055 / 365 = .00015068493$
 - Number of days = $3(365) = 1,095$
 - $FV = \$15,000 / (1.00015068493)^{1095} = \$12,718.56$

Quick Quiz: Part 5

- What is the definition of an APR?
- What is the effective annual rate?
- Which rate should you use to compare alternative investments or loans?
- Which rate do you need to use in the time value of money calculations?

Pure Discount Loans – Example

5.11

- Treasury bills are excellent examples of pure discount loans. The principal amount is repaid at some future date, without any periodic interest payments.
- If a T-bill promises to repay \$10,000 in one year and the market interest rate is 7 percent, how much will the bill sell for in the market?
 - $PV = \$10,000 / 1.07 = \$9,345.79$

Interest-Only Loan - Example

- Consider a 5-year, interest-only loan with a 7% interest rate. The principal amount is \$10,000. Interest is paid annually.
 - What would the stream of cash flows be?
 - Years 1 – 4: Interest payments of $.07(\$10,000) = \700
 - Year 5: Interest + principal = \$10,700
- This cash flow stream is similar to the cash flows on corporate bonds. We will talk about them in greater detail later.

Amortized Loan with Fixed Payment - Example

- Each payment covers the interest expense; plus, it reduces principal
- Consider a 4-year loan with annual payments. The interest rate is 8% and the principal amount is \$5,000.
 - What is the annual payment?
 - $\$5,000 = C[1 - 1 / 1.08^4] / .08$
 - $C = \$1,509.60$

Quick Quiz: Part 6

- What is a pure discount loan? What is a good example of a pure discount loan?
- What is an interest-only loan? What is a good example of an interest-only loan?
- What is an amortized loan? What is a good example of an amortized loan?

Comprehensive Problem

- An investment will provide you with \$100 at the end of each year for the next 10 years. What is the present value of that annuity if the discount rate is 8% annually?
- What is the present value of the above if the payments are received at the beginning of each year?
- If you deposit those payments into an account earning 8%, what will the future value be in 10 years?
- What will the future value be if you open the account with \$1,000 today, and then make the \$100 deposits at the end of each year?