

Chapter 5

Discounted Cash Flow Valuation

Key Concepts and Skills

- Be able to compute the future value of multiple cash flows
- Be able to compute the present value of multiple cash flows
- Be able to compute loan payments
- Be able to find the interest rate on a loan
- Understand how loans are amortized, or “paid off”
- Understand how interest rates are quoted

Chapter Outline

- Future and Present Values of Multiple Cash Flows
- Valuing Level Cash Flows: Annuities and Perpetuities
- Comparing Rates: The Effect of Compounding Periods
- Loan Types and Loan Amortization

Multiple Cash Flows – FV Example 5.1

- Find the value at year 3 of each cash flow and add them together.
 - Today (year 0): $FV = \$7,000(1.08)^3 = \$8,817.98$
 - Year 1: $FV = \$4,000(1.08)^2 = \$4,665.60$
 - Year 2: $FV = \$4,000(1.08) = \$4,320$
 - Year 3: value = \$4,000
 - Total value in 3 years = $\$8,817.98 + 4,665.60 + 4,320 + 4,000 = \$21,803.58$
- Value at year 4 = $\$21,803.58(1.08) = \$23,547.87$

Multiple Cash Flows – FV Example 2

- Suppose you invest \$500 in a mutual fund today and \$600 in one year. If the fund pays 9% annually, how much will you have in two years?
 - $FV = \$500(1.09)^2 + \$600(1.09) = \$1,248.05$

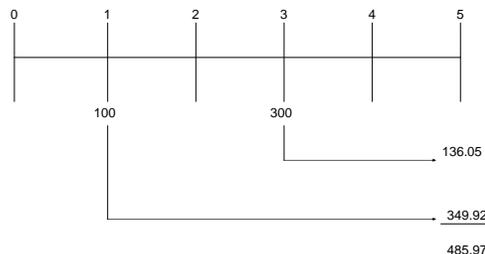
Example 2 Continued

- How much will you have in 5 years if you make no further deposits?
- First way:
 - $FV = \$500(1.09)^5 + \$600(1.09)^4 = \$1,616.26$
- Second way – use value at year 2:
 - $FV = \$1,248.05(1.09)^3 = \$1,616.26$

Multiple Cash Flows – FV Example 3

- Suppose you plan to deposit \$100 into an account in one year and \$300 into the account in three years. How much will be in the account in five years if the interest rate is 8%?
 - $FV = \$100(1.08)^4 + \$300(1.08)^2 = \$136.05 + \$349.92 = \$485.97$

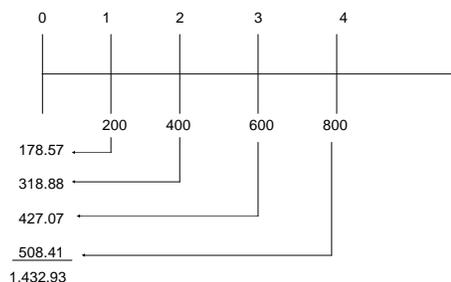
Example 3 Time Line



Multiple Cash Flows – PV Example 5.3

- Find the PV of each cash flow and add them
 - Year 1 CF: $\$200 / (1.12)^1 = \178.57
 - Year 2 CF: $\$400 / (1.12)^2 = \318.88
 - Year 3 CF: $\$600 / (1.12)^3 = \427.07
 - Year 4 CF: $\$800 / (1.12)^4 = \508.41
 - Total PV = $\$178.57 + 318.88 + 427.07 + 508.41 = \$1,432.93$

Example 5.3 Time Line



Multiple Cash Flows – PV Another Example

- You are considering an investment that will pay you \$1,000 in one year, \$2,000 in two years, and \$3,000 in three years. If you want to earn 10% on your money, how much would you be willing to pay?
 - $PV = \$1,000 / (1.1)^1 = \909.09
 - $PV = \$2,000 / (1.1)^2 = \$1,652.89$
 - $PV = \$3,000 / (1.1)^3 = \$2,253.94$
 - $PV = \$909.09 + 1,652.89 + 2,253.94 = \$4,815.92$

Decisions, Decisions

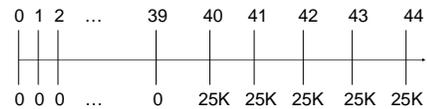
- Your broker calls you and tells you that he has this great investment opportunity. If you invest \$100 today, you will receive \$40 in one year and \$75 in two years. If you require a 15% return on investments of this risk, should you take the investment?
 - $PV = \$40/(1.15)^1 + \$75/(1.15)^2 = \$91.49$
 - No! The broker is charging more than you would be willing to pay.

Saving For Retirement

- You are offered the opportunity to put some money away for retirement. You will receive five annual payments of \$25,000 each beginning in 40 years. How much would you be willing to invest today if you desire an interest rate of 12%?

$$PV = \$25,000/(1.12)^{40} + \$25,000/(1.12)^{41} + \$25,000/(1.12)^{42} + \$25,000/(1.12)^{43} + \$25,000/(1.12)^{44} = \$1,084.71$$

Saving For Retirement Time Line



Quick Quiz: Part 1

- Suppose you are looking at the following possible cash flows: Year 1 CF = \$100; Years 2 and 3 CFs = \$200; Years 4 and 5 CFs = \$300. The required discount rate is 7%
- What is the value of the cash flows at year 5?
- What is the value of the cash flows today?
- What is the value of the cash flows at year 3?

Annuities and Perpetuities Defined

- Annuity – finite series of equal payments that occur at regular intervals
 - If the first payment occurs at the end of the period, it is called an ordinary annuity
 - If the first payment occurs at the beginning of the period, it is called an annuity due
- Perpetuity – infinite series of equal payments

Annuities and Perpetuities – Basic Formulas

- Perpetuity: $PV = C / r$
- Annuities:

$$PV = C \left[\frac{1 - \frac{1}{(1+r)^t}}{r} \right]$$

$$FV = C \left[\frac{(1+r)^t - 1}{r} \right]$$

Annuity – Example 5.5

- You borrow money TODAY so you need to compute the present value.
- Formula:

$$PV = 632 \left[\frac{1 - \frac{1}{(1.01)^{48}}}{.01} \right] = 23,999.54$$

Annuity – Sweepstakes Example

- Suppose you win the Publishers Clearinghouse \$10 million sweepstakes. The money is paid in equal annual installments of \$333,333.33 over 30 years. If the appropriate discount rate is 5%, how much is the sweepstakes actually worth today?

- $PV = \$333,333.33[1 - 1/1.05^{30}] / .05 = \$5,124,150.29$

Buying a House

- You are ready to buy a house and you have \$20,000 for a down payment and closing costs. Closing costs are estimated to be 4% of the loan value. You have an annual salary of \$36,000 and the bank is willing to allow your monthly mortgage payment to be equal to 28% of your monthly income. The interest rate on the loan is 6% per year with monthly compounding (.5% per month) for a 30-year fixed rate loan. How much money will the bank loan you? How much can you offer for the house?

Buying a House - Continued

- Bank loan
 - Monthly income = $\$36,000 / 12 = \$3,000$
 - Maximum payment = $.28(\$3,000) = \840
 - $PV = \$840[1 - 1/1.005^{360}] / .005 = \$140,105$
- Total Price
 - Closing costs = $.04(\$140,105) = \$5,604$
 - Down payment = $\$20,000 - 5,604 = \$14,396$
 - Total Price = $\$140,105 + 14,396 = \$154,501$

Quick Quiz: Part 2

- You know the payment amount for a loan and you want to know how much was borrowed. Do you compute a present value or a future value?
- You want to receive \$5,000 per month in retirement. If you can earn .75% per month and you expect to need the income for 25 years, how much do you need to have in your account at retirement?