

Price Theory  
Lecture 3: Theory of the Consumer

**I. Introduction**

The purpose of this section is to delve deeper into the roots of the demand curve, to see exactly how it results from people's tastes, income, etc.

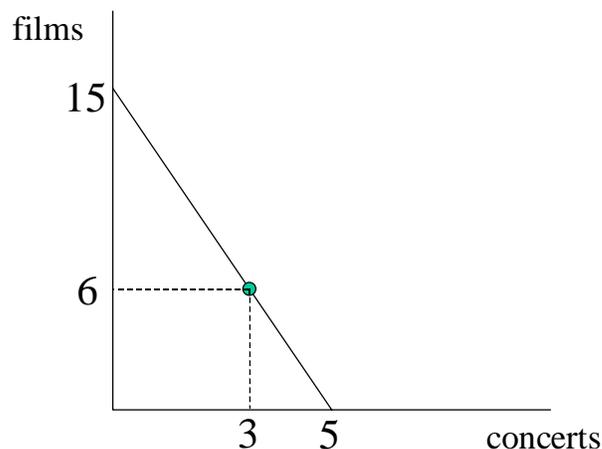
The analytical approach has two main features: preferences and constraints. These two features interact to yield choices. For the most part, the constraints a consumer faces are prices and income.

In much of what follows, we will be talking about bundles. A bundle is simply a collection of goods, with a specific amount for each good. For example, {3 apples and 6 oranges} is a bundle, as is {5 apples and 1 orange}.

**II. The Budget Constraint**

A consumer's budget constraint identifies the combinations of good and services the consumer can afford with a given income and given prices.

Example: Suppose you have \$150 to spend on only two goods, films and concerts. The price of a film is \$10, while the price of a concert is \$30. Here are some points within your budget constraint: By spending all your money on films, you could get (0, 15). By spending all your money on concerts, you could get (5, 0). By spending \$60 on films and the rest on concerts, you could get (3, 6). Graphing these points and connecting them, we get a straight line.



In most cases, a budget constraint will be a straight line. Why?

Example above: Your total spending is  $10f + 30c$ . If you spend your whole income, we conclude that  $10f + 30c = 150$ . That's the equation of a line. We could rewrite it in slope-intercept form:

$$10f = 150 - 30c$$

$$f = 15 - 3c$$

Notice that the vertical intercept of this line is 15. That's the number of films you could get if you spent all your income on films. We can find the horizontal intercept by setting  $f = 0$ , and we find that  $c = 5$ . So the horizontal intercept is the number of concerts you could get if you spent all your money on concerts.

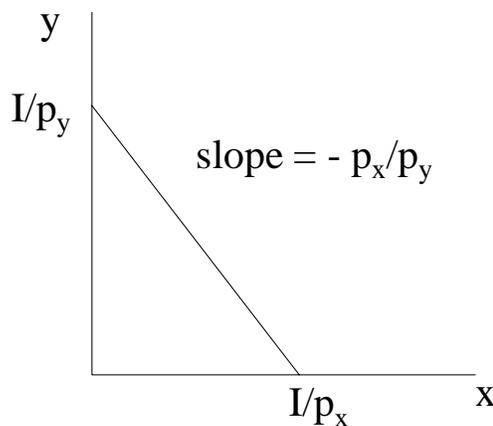
The general formula for a budget constraint is found like so: Let  $I$  = your income. Let  $P_x$  = the price of one good, and  $P_y$  = the price of the other good. Then your BC is

$$P_x x + P_y y = I$$

or alternatively, in slope-intercept form,

$$y = -(P_x/P_y)x + (I/P_y)$$

which is graphed like so:



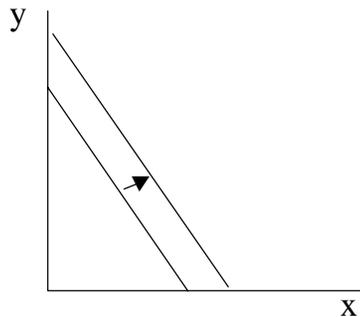
Note 1: The endpoints are the y-intercept and the x-intercept. They correspond to the maximum quantities of each good that could be bought.

Note 2: The slope of the BC is  $-(P_x/P_y)$ . We usually ignore the negative sign, so we have  $P_x/P_y$ . This expression is called the price ratio. It turns out that the price ratio is extremely important. Why? Because it turns out to represent the trade-off or opportunity cost of good x in terms of good y. Suppose you're at the right endpoint, where you're only buying good x. Then you decide not to buy one of those units of x. Then you have  $P_x$  more dollars to spend on good y. Divide by  $P_y$  to find the number of units of y you could get. Thus, by giving up one unit of x, you get  $P_x/P_y$  units of y. In other words, the price ratio is the opportunity cost of a unit of x, in terms of y.

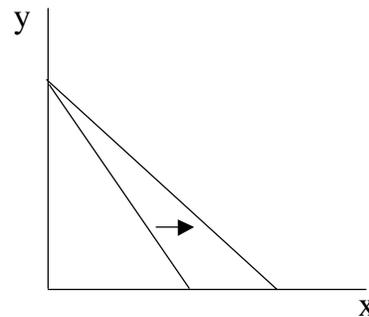
Continuing the example: Suppose you reduce your number of concerts from 5 to 4. That means you have \$30 to spend on films instead. Dividing \$30 by \$10 (the price of a film), we find that you can buy 3 more films. Thus, 3 is the opportunity cost of a concert in terms of films. It is also the slope of the budget constraint: the price of a concert divided by the price of a film.

What can cause a change in the BC?

- Changes in income. Any change in income will cause a parallel shift of the budget constraint. Why? Look at the equation:  $y = -(P_x/P_y)x + (I/P_y)$ . The slope is  $-(P_x/P_y)$ , which does not change. The vertical intercept is  $(I/P_y)$ , which increases because  $I$  increases.
- Changes in price. A change in the price of one good will change the slope of the line, rotating it from a fixed intercept. Say the price of  $x$  decreases. Your  $y$ -intercept remains the same. But you can now buy more  $x$  with the same income, so your  $x$ -intercept moves right, making the BC flatter. If the price of the other good,  $y$ , decreases, the  $x$ -intercept remains the same and the line becomes steeper.
- What if both prices change? What happens to the curve depends on the size of the changes. For example, suppose both prices exactly double. Notice that the slope remains the same, because the 2's cancel. But you can only buy half as much of each good, so it's as though your income had decreased, causing a parallel shift inward. However, if the prices don't change by the ratio, then the slope of the budget constraint will change.



BC shift from an increase in income



BC shift from a decrease in price of  $x$

### III. Unusual Budget Constraints

Sometimes there will be unusual circumstances that change the shape of the budget constraint.

Example 1: Suppose you already have 5 units of good  $x$ , and you can't exchange them. The BC turns out to have a flat (horizontal) section.

Example 2: Suppose the price of a good is \$3 for the first 10 units, and \$2 for all units thereafter. The BC is kinked inward at the point where you have 10 of the good.

There are many other examples. The technique for graphing them is like so: (1) Mark the endpoints, showing the maximum of each good that can be bought. (2) Mark the "weird" points on the constraint, such as the place where the price changes for a good, or the minimum quantity of a good that you can have. (3) Connect the dots. Make sure that at any point on the BC, all your income has been spent. Also, verify that within each section of the constraint, the slope on that section is equal to the price ratio in that range.

## IV. Preferences

### A. Ordinal versus Cardinal Utility

We assume that consumers have preferences that they are trying to satisfy, so as to maximize their personal satisfaction. “Utility” is a word we use to mean “the pleasure or satisfaction obtained from consuming goods and services.”

There are two different ways to think of utility, and they lead to two different models of consumer choice.

- Cardinal utility = countable utility. In this approach, we imagine that satisfaction is an actual quantity that could, in theory, be measured. We call the units of cardinal utility “utils.”
- Ordinal utility = ranking of bundles. Here, we don’t assume utility is measurable, but merely that consumers can compare bundles of goods and say which one is “better.”

Ordinal utility is the more sensible assumption, and it turns out that all the major results of consumer theory can be explained using only ordinal utility. But in your previous economics classes, you may have worked with cardinal utility, because it’s sometimes easier to think about.

Example: The concept of diminishing marginal utility says that the more you have of a good, the less additional satisfaction you get from having one more. This is really a cardinal utility concept. What does it mean to say that “my 3<sup>rd</sup> hamburger increases my satisfaction less than my 2<sup>nd</sup> hamburger”? You have to be able to measure the differences to make that comparison.

In this class, we will be working exclusively with ordinal utility. We will have an alternative assumption, consistent with ordinalism, that substitutes for diminishing marginal utility.

### B. Preference Symbols

We have some symbols that represent the idea of preferring one thing over another. Notice that they look similar to greater- and less-than signs, but they are not the same.

$A \succ B$  means “bundle A is preferred to bundle B.”

$A \succeq B$  means “bundle A is at least as good as bundle B.”

$A \sim B$  means “A and B are equally good.” Another way of saying this is: “You are indifferent between A and B.” This concept of indifference will prove very important later.

### C. Assumptions about Preferences

In order to create a model of consumption behavior, we have to make certain assumptions about people's preferences:

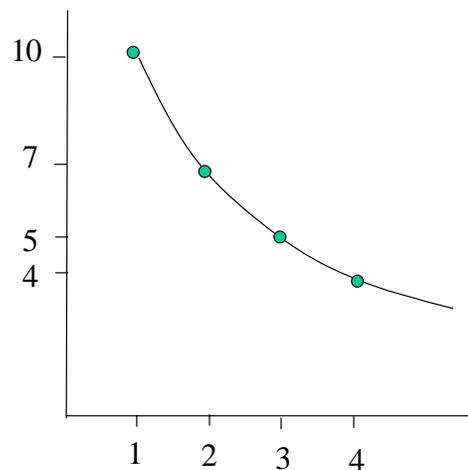
- Comparability (given any two bundles, you can say “better,” “worse,” or “indifferent”)
- More is better (if one bundle has more of a good than another, and no less of any other goods, then it's a preferable bundle)
- Transitivity (if A is better than B, and B is better than C, then A is better than C).

## V. Indifference Curves

Once we have these three assumptions about preferences, we can use them to find a way to draw a picture of people's preferences. Consider a two-dimensional graph with one good on each axis. Every point in this graph is a bundle that someone could consume. We want a way to show, in this picture, which bundles are preferred to which. The device we'll use is something called an indifference curve.

Definition of an indifference curve: a curve that shows all the bundles of goods that provide the same level of utility. Another way of saying this: an indifference curve is a collection of bundles that the consumer is indifferent between.

Here is a typical indifference curve:



Each one of the points (bundles) on this indifference curve, including those I haven't marked with a dot, give the consumer the same satisfaction or utility.

Things to note about this indifference curve:

- The curve is downward sloping. Why? Because the “more is better assumption” says that you get better off when you have more of something.

Say you start at A and get more one more of good x. Then you're at a better point. To make you indifferent instead of better off, you have to lose some of the other good. Taking away three of good y exactly compensates for the gain of one x (or vice versa).

We call the amount of good y necessary to compensate the consumer for losing one unit of good x the marginal rate of substitution of good y for good x, also known as the MRS.

In the picture, the MRS along the section from (1,10) to (2,7) is 3 (the ratio of the change in y to the change in x). Notice that the MRS is equal to the slope of the line through the points (1,10) and (2,7).

- The curve is actually curved – it's "convex," like a bowl. This means that MRS is decreasing as you get more of x.

In the picture, the MRS along the section from (1,10) to (2,7) is 3. The MRS along the section from (2,7) to (3,5) is 2. The MRS along the section from (3,5) to (4,4) is 1. Notice that the MRS is getting smaller as the amount of x increases.

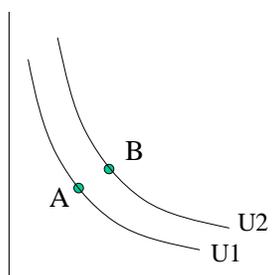
Intuitively, this is because the less of something you have, the less willing you are to give up the units you do have. It only takes one y to get you to give up an x when you have four. But it takes three y to get you to give up an x when you only have two of x.

The law of diminishing MRS says that MRS decreases as x increases. This "law" replaces the law of diminishing marginal utility.

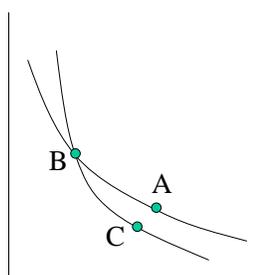
We will often want to find out the MRS for very, very small quantities, rather than the big jumps in this picture. Imagine if we take smaller and smaller intervals. As the intervals shrink to zero, the line through the endpoints becomes a tangent line. The slope of this line is the MRS at the point of tangency. So, to find the MRS at any point, just find the slope of the tangent line through the point.

There are several things you should know about indifference curves:

- The farther an indifference curve is from the origin, the more satisfaction the consumer gets from being on it. Why? Consider the diagram below. By "more is better," B is preferred to A. By transitivity, that means every point on B's indifference curve, U<sub>2</sub>, is preferred to A. And by transitivity again, that means every point on B's indifference curve is preferred to every point on A's indifference curve, U<sub>1</sub>.



- They cannot curve upward. Why? Again, the “more is better” assumption. If the curve slopes upward, that means there are points with more of both goods that are only regarded as equally good.
- They cannot cross. Why? Consider this diagram of crossing indifference curves:



By definition of indifference curves,  $A \sim B$  and  $B \sim C$ , therefore by transitivity  $A \sim C$ , but that contradicts “more is better,” which says A is preferred to C.

- They are ubiquitous, or dense. Pick out any point in the picture, and then find all the other bundles that the consumer would consider equally good to find the indifference curve going through the point you picked out. This characteristic does not actually hold true when we have non-divisible goods (what is half a car?), but we will usually (for simplicity) assume that we do have divisible goods.

Once we’ve characterized all preferences by means of indifference curves, we have a map of the consumer’s preferences. Now we can restate the consumer’s goal like so: attain the highest indifference curve possible.

Notice what we’ve done – we’ve managed to completely describe preferences without ever resorting to talking about measurable “utils.” All we’ve talked about is judgments of “better,” “worse,” and “indifferent.”

## VI. Unusual Indifference Curves

The diagram we’ve been looking at has a very typical indifference curve. But if we loosen some of our assumptions, we can get some special cases with unusual indifference curves.

Example 1: A radio station can be thought of as a bundle of songs. When the radio station increases the number of \*NSync songs, I get worse off. To compensate me, they'd have to increase the number of Eminem songs. Marking the starting point and the new point (where the increase in Eminem songs just barely compensates for the increase in \*NSync songs), we can draw in an indifference curve that is upward-sloping. This violates "more is better," because more \*NSync makes me worse off.

Example 2. I can't tell the difference between margarine and butter. I am always indifferent between a pound of butter and a pound of margarine. This case of perfect substitutes violates the law of diminishing MRS, because I'm always willing to substitute a pound of butter for a pound of margarine. This produces straight-line indifference curves. (Note that it doesn't have to be one-for-one. Consider Equal, which is twice as sweet as sugar. In that case, you'd have a one-for-two trade-off of Equal for sugar.)

Example 3. I have two feet. One left shoe does me no good, nor does one right shoe. But together they make me better off. This case of perfect complements violates the "more is better" assumption, and it produces right-angle indifference curves. (Note that it doesn't have to be one-to-one combinations. Consider someone who likes screwdrivers – three parts orange juice to one part vodka.)

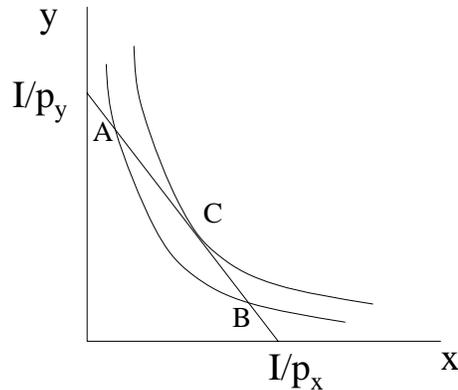
Example 4. Now I only have one leg (the left one). Right shoes are a worthless good to me; more of them make me neither better off nor worse off. Only left shoes matter. If we put left shoes on the horizontal axis, then I have vertical indifference curves.

## VII. Consumer Choice

Now that we've got both preferences and constraints, we can talk about choice.

The idea is for the consumer to reach the highest indifference curve he can, but whatever bundle he selects must be within his budget constraint. In the diagram below, we can see that an individual who has chosen a bundle where an indifference curve crosses the budget constraint (such as point A or B) is not doing as well as he can. He can move along his budget constraint, buying less of one good and more of another, to reach a point like C. This point has higher utility for the consumer, because a higher indifference curve goes through it.

The point of tangency is the point of utility maximization for this person; that is, it's the bundle of goods that gives the consumer his maximum possible utility subject to the budget constraint. Now, what does it mean for a line and a curve to be tangent? It means they have exactly the same slope at the point of tangency.



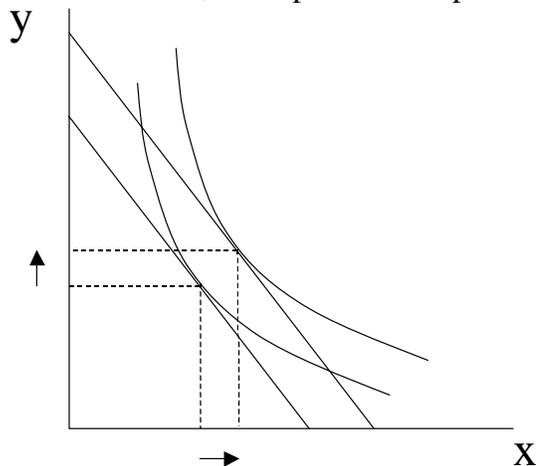
So, at the optimal consumption point (point C in the graph above), the BC and indifference curve must have the same slope. What is the slope of the BC? The price ratio. What is the slope of the indifference curve? The MRS. So we conclude that, at the optimal consumption point, it must be true that

$$MRS = P_x/P_y$$

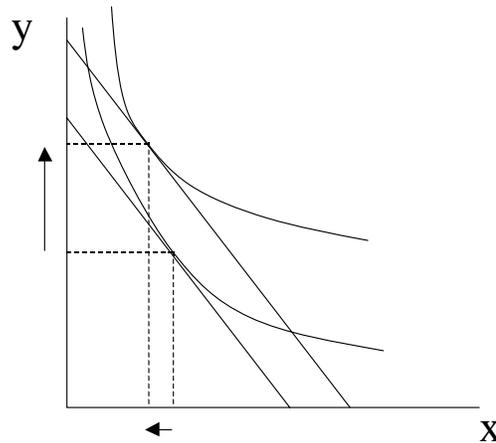
This mathematical condition has an intuitive meaning. Recall that the MRS is how much of good y you're willing to accept in return for one x. Meanwhile, the price ratio tells you how much of good y the market will give you in return for one x. (If you choose not to buy a unit of x, that's  $P_x$  dollars you can spend on y instead, so you can get  $P_x/P_y$  units of good y.) So, if the price ratio is greater than your MRS, the market will give you a good deal: it will give you more y for an x than you really need. So it makes sense to buy more y and less x. On the other hand, if the price ratio is less than your MRS, it makes sense (for the same reasons) to move in the other direction, buying less y and more x. So the only situation in which it makes sense to stay where you are is when the two are equal.

### VIII. Changes in Income

As we observed earlier, changes in income lead to parallel shifts of the BC. How does this affect choice? Most of the time, we expect consumption of both goods to increase.



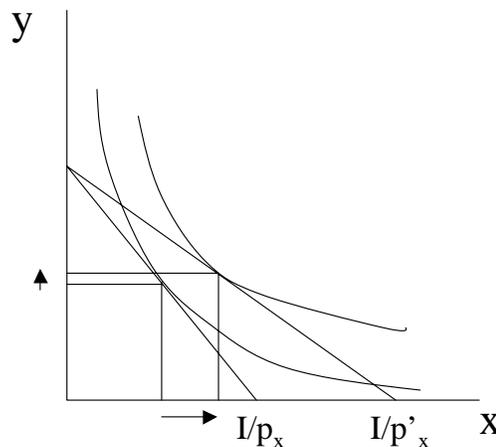
In this diagram, we've assumed that both goods are superior. A superior good is one that you buy more of when your income rises. But some goods are inferior – you buy less when your income rises. Examples include bus tickets (because you'll fly more), canned meat (because you'll buy more fresh meat), etc. Here is a diagram in which one good is superior and the other is inferior:



Note: The book uses the term “normal” to refer to what I call superior goods. I use the word normal for something else – any good for which the law of demand holds. Some normal goods are inferior, while others are superior.

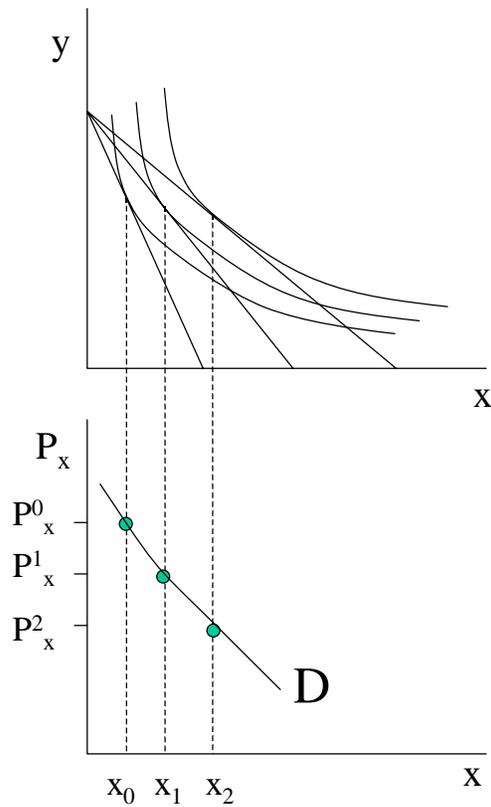
### IX. Changes in Price: Deriving the Demand Curve

As we observed earlier, a change in price causes the BC to rotate around one endpoint. How does this affect choice? Consider the diagram below:



We can use this analysis to derive a demand curve. Recall that a demand curve shows the quantity of a good that someone will buy at each price. So we can try many different prices, and find the different optimal levels of consumption. Drop the different consumption levels down to another graph, and for each consumption level mark the corresponding price. This results in a demand curve, showing the relationship between the price of x and the individual's consumption of x. Note that I have shown three budget

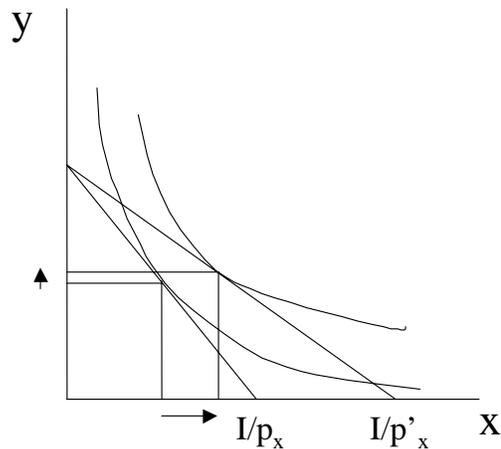
constraints. The innermost budget constraint corresponds to a high price for  $x$ ,  $P_x^0$ . The middle budget constraint corresponds to a lower price for  $x$ ,  $P_x^1$ . The outermost budget constraint corresponds to an even lower price for  $x$ ,  $P_x^2$ .



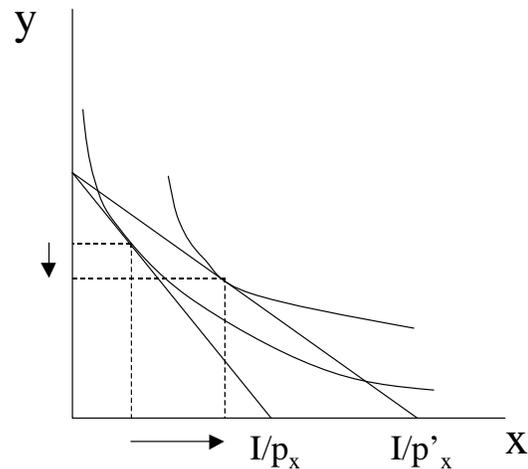
Note: This is an individual demand curve. To find the market demand curve, we'd have to add up the demand curves of all individuals.

### X. Complements and Substitutes

We've been examining the effect of a change in the price of a good on how much a consumer buys of that good. But what happens if the price of the other good changes?



In the picture above, the consumer has increased consumption of both goods in response to a decrease in the price of one. But we didn't have to draw it that way. We could have drawn it like so...



In a case like this, we say the goods are substitutes.

Ultimately, whether two goods are substitutes or complements depends entirely on the shape of the indifference curves. None of our assumptions about preferences rule out having either one.

Note: The two diagrams above look very similar to the diagrams showing "two superior goods" and "one inferior good and one superior good." But there is an important difference: in those diagrams, the budget constraint shifted in a parallel fashion in response to an increase in income. Here, the budget constraint is shifting in a non-parallel fashion in response to a price change.

The extreme cases of perfect complements and perfect substitutes show these results in stark form. Consider the case of perfect complements, which have right angle indifference curves. In this case, we cannot use the rule of "MRS equals price ratio," because the indifference curve goes from infinite to zero slope at the curve's elbow. Any price ratio will fall between these extremes. For that reason, consumption must always occur at the elbow. In other words, the consumer will always consume these two goods in the same ratio, regardless of their relative prices. When the price of one good decreases, the consumer will necessarily increase consumption of both goods.

Now consider the other extreme, perfect substitutes. The indifference curves are straight lines. Again, we find ourselves unable to use the "MRS equals price ratio" rule, because the MRS is constant. It's either always greater than the price ratio, or always less than the price ratio, or always equal to the price ratio. Whenever the MRS is greater than the price ratio, it makes sense for the consumer to buy only the good on the horizontal axis – and vice versa when the MRS is less than the price ratio. Now suppose the price ratio changes. If the change doesn't reverse MRS and the price ratio, the consumer continues to buy only one good. But if the change is large enough (and in the right direction) to

reverse MRS and price ratio, the consumer will shift his consumption entirely to the other good. This is an extreme case of the general result for substitutes, where a drop in the price of one good reduces consumption of the other. In this case, consumption of the other falls to zero.

## **XI. Application of Complements: Improving Education**

The notion of complementary goods is a specific example of a broader phenomenon: that the benefits from an improvement in one area may be felt in other areas. Here is another example of that phenomenon.

Suppose that new computer software allows faster learning in one subject, French. Will this new software lead to a measurable increase in French scores? Perhaps not. Here's why. Students have many different things they spend their time on, French being only one of them. Let's imagine that a student spends all his study time on Economics and French. We can think of this student spending time to buy scores in these two areas. The price, in time, for an exam point depends on the teaching technology. We can draw a time-BC to show how the student makes her choice. When the French technology improves, any given amount of time spent studying French will yield more exam points. This means that the time-BC shifts outward on the French-axis (just like for a fall in price on our usual BC). Now, what will be the effect? If Econ and French are complements (i.e., the student values doing better on both), then the positive effect of the new software is not only on the French exam score. Instead, the student reallocates her time so that both scores improve. The new software expanded the student's options by making her time more productive.

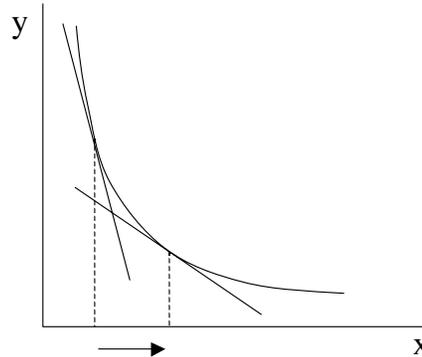
Now, will the improvement in French be measurable? Well, notice that we only considered one other activity (Econ) in this diagram. But there are actually many activities that compete for time with French, including other subjects, extracurricular events, dating, etc. To the extent that these activities are complementary (the student likes to have them all), the impact of the new software may be spread over them all. The student uses the time saved to do a little more of all these things. As a result, the impact on French alone may be too small to measure. This could explain why studies on the effects of computer technology on learning in a single subject rarely show any significant effect.

## **XII. Substitution & Income Effects**

When a fall in price causes a consumer to buy more of a good, there are two different reasons why that happens. They are:

- The substitution effect. The consumer buys more of the good because it's now relatively less expensive than the other good. In other words, the price ratio has changed.
- The income effect. The consumer becomes relatively richer, because the lower price means his dollars go further than they did before. His effective income has increased.

We can analyze these effects separately. Consider the substitution effect. In order to see this, we need to isolate the effect of having a new price ratio (in other words, a flatter BC), while setting aside the increase in utility attributable to a higher effective income. How do we do this? We look at the original indifference curve of the consumer, and find where it would be tangent to the new price ratio.



The budget constraints are not shown touching the axes here, because all we care about (for purposes of finding the substitution effect) is their slopes, which reflect the two different price ratios. The steeper straight line shows a higher price ratio, while the flatter straight line shows a lower price ratio. The change in consumption that we can see here is called the substitution effect. Note that it must be in the opposite direction of the price change (price goes down, consumption goes up).

We've already seen the income effect before, when we considered regular changes in income. What we already know about the income effect is that it can be positive or negative, depending on whether we have a superior or inferior good.

So, how do these two effects interact? If we put them together (in a diagram that's complex, and which I'm skipping), we'll get a total effect.

$$\text{Total Effect} = \text{Substitution Effect} + \text{Income Effect}$$

Suppose the price of a good goes down. For any superior good, the total effect must be positive. Why? Because the two effects are reinforcing. The substitution effect is always positive, as we saw from the diagram above. And the income effect is positive for a superior good. Therefore, the total effect is certainly positive. This means that all superior goods are normal (they have downward-sloping demand).

For an inferior good, the total effect is usually positive. The substitution effect is positive as always. The income effect is negative for an inferior good, which means the effects work in opposite directions. But the substitution effect is almost always bigger than the income effect. Therefore, the total effect is usually positive. This means that most inferior goods are also normal (they have downward-sloping demand).

But for some inferior goods, it is possible for the total effect to be negative. The income effect is large enough to swamp the substitution effect. These goods are called Giffen goods (which have upward-sloping demand). Giffen goods are extremely rare, and there is some debate as to whether they even exist. One example: Potatoes in Ireland during the potato famine of the 1840s. When the price of potatoes rose, that made the effective incomes of the Irish much smaller because potatoes were a staple of their diets. With their smaller incomes, the Irish cut back on their consumption of “luxury” foods like meat and bought more potatoes (or so said Samuel Giffen).

We can summarize these results like so:

	Superior	Inferior
Normal	Most goods (cars, houses, etc.). S & I same direction.	Some goods (bus tickets, low-grade meat, etc.) S & I opposite directions, $ S  >  I $ .
Giffen	impossible	Very few goods (potatoes in 1840s Ireland). S & I opposite directions, $ S  <  I $ .