(1) a. Draw a sketch which illustrates the relationship between square yards and square feet.

1b. Use your illustration for part a to show why carpet that costs $4 per square foot is not cheaper than carpet that costs $33 per square yard.

The illustration demonstrates that 1 yd$^2$ is 9 ft$^2$.
This is confirmed by the dimensional analysis:

\[
\frac{1 \text{ yd}^2}{1 \text{ yd} \cdot 1 \text{ yd}} = \frac{3 \text{ ft}}{1 \text{ yd}} \cdot \frac{3 \text{ ft}}{1 \text{ yd}} = 9 \text{ ft}^2
\]

\[
\frac{4 \text{ $/ft}^2}{1 \text{ yd}^2} = \frac{4 \text{ $/ft}^2}{9 \text{ ft}^2} \cdot \frac{1 \text{ yd}^2}{1 \text{ yd}^2} = \frac{36 \text{ $}}{1 \text{ yd}^2}
\]

So carpet at $4 per ft$^2$ costs more than $33$ per yd$^2$.

IT IS IMPORTANT TO RECOGNIZE THAT
A SQUARE YARD IS NOT THE SAME THING AS A YARD...
A SQUARE FOOT IS A QUITE DIFFERENT THING FROM A FOOT....
A CUBIC YARD IS NOT THE SAME THING AS A YARD (Hey, a cubic yard contains 27 cubic feet!) and so on....

(2) Convert each of the following units, showing your work.

a. 0.52 km = \[\text{__________ cm}\]

\[0.52 \text{ km} = \frac{0.52 \text{ km}}{1 \text{ km}} \cdot \frac{1000 \text{ m}}{1 \text{ m}} \cdot \frac{100 \text{ cm}}{1 \text{ m}} = 52000 \text{ cm}\]

b. 0.5 mi = \[\text{__________ ft}\]

\[0.5 \text{ mi} = \frac{0.5 \text{ mi}}{1 \text{ mi}} \cdot \frac{5280 \text{ ft}}{1 \text{ ft}} = 2640 \text{ ft}\]

c. 5.2 m$^2$ = \[\text{__________ cm}^2\]

\[5.2 \text{ m}^2 = \frac{5.2 \text{ m}^2}{1 \text{ m}^2} \cdot \frac{100 \text{ cm}}{1 \text{ m}} \cdot \frac{100 \text{ cm}}{1 \text{ m}} = 52000 \text{ cm}^2\]

The number of cm$^2$ in a m$^2$ is NOT 100. Draw a sketch (in the manner of question #1) and see!

d. 325000 m$^3$ = \[\text{__________ km}^3\]

\[325000 \text{ m}^3 = \frac{325000 \text{ m}^3}{1 \text{ km}^3} \cdot \frac{1 \text{ km}}{1000 \text{ m}} \cdot \frac{1 \text{ km}}{1000 \text{ m}} \cdot \frac{1 \text{ km}}{1000 \text{ m}} = .000325 \text{ km}^3\]

It takes m$^3$ to cancel m$^3$! (cubic meters cancel cubic meters.)
3. Find the volume of the pentagonal pyramid shown at right, given that the base of the pyramid has area 800 m² and the height (h) is 60 m. Show your work.

Volume of Cone or Pyramid is 1/3 the Volume of the corresponding cylinder.

\[
\text{Volume} = \frac{1}{3} \times \text{(Area of base)} \times \text{(height)} \\
= \frac{1}{3} \times 800 \text{ m}^2 \times 60 \text{ m} \\
= 16,000 \text{ m}^3
\]

4. Find the surface area of the right circular cylinder with measurements given:

Surface Area = Area of Base + Area of Top + Area of Side

(Base and Top are both circular regions with radius 3m...)
(Rectangle 6m high by C= 2π(3m) long)

\[
\text{SA} = \pi (3m)^2 + \pi (3m)^2 + (6m)(2\pi 3m) \\
= 18 \pi \text{ m}^2 + 36 \pi \text{ m}^2 \\
\text{SA} = 54 \pi \text{ m}^2
\]

Surface Area is not really different from “AREA” – the key is to make sure to include the area of each face of the object.

5. Find the area contained within this polygon. Find the perimeter.

Area = width \times height

\[
= 11 \text{ m} \times 4 \text{ m} \\
= 44 \text{ m}^2
\]

Perimeter = total length of boundary

\[
P = 11 \text{ m} + 7\text{ m} + 11 \text{ m} + 7\text{ m} \text{ (starting with the base)} \\
P = 36 \text{ m}
\]

6. Which of the following is the volume in a sphere? A B C D E F G H (Circle the letter of your choice from the list below.)

\[
A \ 2\pi r^3 \quad B \ 2\pi r^2 \quad C \ 2\pi r \quad D \ \frac{4\pi r^3}{3} \quad E \ \pi r^2 \quad F \ \frac{2\pi r^3}{3} \quad G \ 4\pi r^2 \quad H \ \text{none}
\]
7. Estimate the area inside the curve shown in #7. If that figure were expanded to one of the same shape, but twice as high, and twice as wide, what would the area inside the new curve be?

\[ \text{# of whole square units within region} + \left( \frac{1}{2} \right) \text{# of partial units within region} \]

\[ 7 \text{ unit}^2 + \left( \frac{1}{2} \right) (17 \text{ unit}^2) = 15 \frac{1}{2} \text{ unit}^2 \]

8. Find the area enclosed by the figure in #8. (Curve turns only at points: (1,1) & (6,7) & (1,7) & (2,5))

\[ \text{Area of enclosing rectangle} - \text{(areas of rectangles and triangles not inside the curve)} \]

\[ 5 \cdot 6 \text{ unit}^2 - \left( \frac{1}{2} \right) 6 \cdot 1 \text{ unit}^2 - \left( \frac{1}{2} \right) 5 \cdot 6 \text{ unit}^2 = 12 \text{ unit}^2 \]

9. Find the perimeter of the triangle in figure #9 above.

\[ \text{Perimeter} = \text{Distance from } (-5,-2) \text{ to } (3,-2) + \text{Distance to } (3,4) + \text{Length of Hypotenuse} \]

\[ = 4 - (-4) + 3 - (-3) + \sqrt{8^2 + 6^2} \]

\[ = 8 + 3 + 6 + \sqrt{64 + 36} = 10 + 10 = 24 \text{ (units)} \]

10. Find the area inside the curve at right, given all arcs are semicircular.

This is a semicircular region with a semicircular “hole” taken out!

\[ \text{Area of the larger semicircular region} - \text{area of the smaller semicircular region} \]

\[ = \left( \frac{1}{2} \right) \pi R^2 - \left( \frac{1}{2} \right) \pi r^2 \]

\[ = \left( \frac{1}{2} \right) \pi (13 \text{ mm})^2 - \left( \frac{1}{2} \right) \pi (5 \text{ mm})^2 \]

\[ = \left( \frac{1}{2} \right) 169 \pi \text{ mm}^2 - \left( \frac{1}{2} \right) 25 \pi \text{ mm}^2 \]

\[ = 72 \pi \text{ mm}^2 \]
11. 52500 mL water (at 4°C) = _________ kg.

\[ \text{52500 mL} = \frac{52500 \text{ mL} \times \frac{1 \text{ g}}{1 \text{ mL}} \times \frac{1 \text{ kg}}{1000 \text{ g}}}{\text{52.5 kg}} \]

* At 4°C, water is at its most dense state and each 1 cc or 1 mL has 1 gram of mass.

12. Find the area inside a 60° sector of a circle with radius 2 cm.

The area inside a sector is just the appropriate fraction of the area inside the entire circle. The area of a 60° sector of a circle is just \( \frac{60}{360} \), or \( \frac{1}{6} \), of the area of the entire circle.

\[ \frac{60}{360} \pi R^2 = \frac{1}{6} \pi R^2 \]

\[ = \frac{1}{6} \pi (2\text{cm})^2 \]

\[ = \pi (2\text{cm})^2 /6 \]

\[ = \pi (4) \text{ cm}^2 /6 \]

\[ = \frac{2 \pi \text{ cm}^2}{3} \]

13. Find the surface area of the square-based right pyramid shown. Find the volume contained by this pyramid.

Surface area (SA) of this square-based pyramid is

Area of square base + 4(Area of triangular face)

\[ \text{SA} = (12 \text{ m})^2 + 4 \left( \frac{1}{2} \right) 12\text{m} (8\text{m}) \]

\[ = 144 \text{ m}^2 + 4 \cdot 48 \text{m}^2 \]

\[ = 336 \text{m}^2 \]

The Volume of the Pyramid is \( \left(\frac{1}{3}\right) \) (Area of base)(height)

The height is \( H \). The vertical arm of a right triangle whose hypotenuse is 8, the value we figured out above, the height of the triangular surface.

To find \( H \):

\[ \frac{H^2 + 6^2}{2} = 8^2 \]

\[ \frac{H^2 + 36}{2} = 64 \]

\[ H^2 = 28 \]

\[ H = \sqrt{28} = 2 \sqrt{7} \text{ m} \]

Volume = \( \left(\frac{1}{3}\right) \) (Area of base)(height)

\[ = \left(\frac{1}{3}\right) (12 \text{ m})^2 (2 \sqrt{7} \text{ m}) \]

\[ = 96 \sqrt{7} \text{ m}^3 \]

14. A right circular cylinder holds a volume of 4000 cm\(^3\). What is the capacity (volume) of a cylinder that is half as tall, but with a diameter twice that of the original cylinder?

Since volume = (area of base)(height)...
Cutting the height in half would halve the volume.
Doubling the diameter doubles the radius, which in turn quadruples the area... thus quadrupling \( V \).

OR merely compare \( V = \pi r^2 h \) to new \( V = \pi (2r)^2 \left(\frac{h}{2}\right) = 2 \pi r^2 \frac{h}{2} \) .... New \( V = 8000 \text{ cm}^3 \)