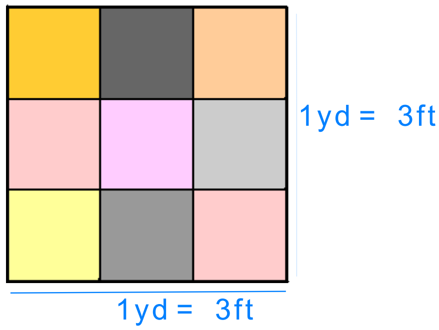
MATH 310 ♦ TEST Measure ♦ Dec 6, 2006 ♦ NAME: ______________________________

( )1a. Draw a sketch which illustrates the relationship between square yards and square feet.

1b. Use your illustration for part a to show why carpet that costs \$4 per square foot is not cheaper than carpet that costs \$33 per square yard.



The illustration demonstrates that 1 yd^2 is 9 ft^2.
This is confirmed by the dimensional analysis:

$$1\ yd^2 = 1yd^2 \frac{3\ ft}{1\ yd} \cdot \frac{3\ ft}{1\ yd} = 9\ ft^2$$

$$\frac{\$4}{ft^2} = \frac{\$4}{ft^2} \cdot \frac{9\ ft^2}{1\ yd^2} = \frac{\$36}{yd^2}$$

So carpet at \$4 per ft^2 costs more than \$33 per yd^2.

1yd = 3ft

IT IS IMPORTANT TO RECOGNIZE THAT
A SQUARE YARD IS NOT THE SAME THING AS A YARD...
A SQUARE FOOT IS A QUITE DIFFERENT THING FROM A FOOT....
A CUBIC YARD IS NOT THE SAME THING AS A YARD (Hey, a cubic yard contains 27 cubic feet !),
and so on....

( ) 2. Convert each of the following units, showing your work.

a. 0.52 km = ____________ cm

1 km = 1000 m
1 m = 100 cm

$$0.52\ km = \frac{.52km}{} \cdot \frac{1000\ m}{1\ km} \cdot \frac{100\ cm}{1\ m} = .52 \cdot 1000 \cdot 100\ cm = 52000\ cm$$

b. 0.5 mi = ____________ ft

$$.5\ mi = \frac{.5\ mi}{} \cdot \frac{5280\ ft}{1\ mi} = 2640\ ft$$

1 mi = 5280 ft

c. 5.2 m^2 = ____________ cm^2

$$5.2\ m^2 = \frac{5.2\ m^2}{} \cdot \frac{100\ cm}{1\ m} \cdot \frac{100\ cm}{1\ m} = 52000\ cm^2$$

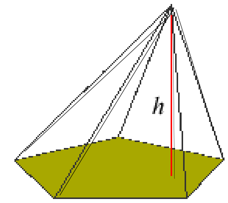
The number of cm^2 in a m^2 is NOT 100. Draw a sketch (in the manner of question #1) and see!

d. 325000 m^3 = ____________ km^3

$$325000\ m^3 = \frac{325000\ m^3}{} \cdot \frac{1\ km}{1000\ m} \cdot \frac{1\ km}{1000\ m} \cdot \frac{1\ km}{1000\ m} = .000325\ km^3$$

It takes m^3 to cancel m^3 ! (cubic meters cancel cubic meters.)

( ) 3. Find the volume of the pentagonal pyramid shown at right, given that the base of the pyramid has area 800 m^2 and the height (**h**) is 60m. Show your work.



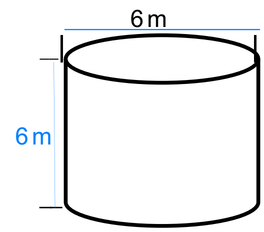
Volume of Cone or Pyramid is 1/3 the Volume of the corresponding cylinder.

Volume = (1/3) (Area of base) (height)

= (1/3) (800 m^2) (60 m)

= 16,000 m^3

( ) 4. Find the surface area of the right circular cylinder with measurements given:



Surface Area = Area of Base + Area of Top + Area of Side

(Base and Top are both circular regions with radius 3m...)

(Rectangle 6m high by C= 2π(3m) long)

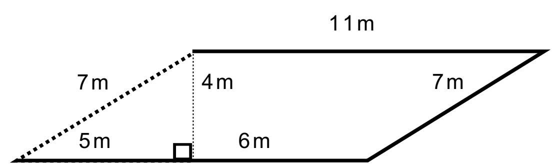
SA = $\pi(3m)^2 + \pi(3m)^2 + (6m)(2\pi\ 3m)$

= $18\pi\ m^2 + 36\pi\ m^2$

SA = $54\pi\ m^2$

Surface Area is not really different from "AREA" –
the key is to make sure to include the area of each face of the object.

( ) 5. Find the area contained within this polygon.
Find the perimeter.



Area = width • height

= 11 m • 4 m

= 44 m^2

Perimeter = total length of boundary

P = 11m + 7m + 11m + 7m (starting with the base)

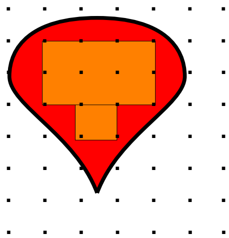
P = 36 m

( ) 6. Which of the following is the volume in a sphere ? A B C D E F G H
Which of the following is the area within a circle ? A B C D E F G H
Which of these is the circumference of a circle ? A B C D E F G H
(assume all have radius "r")

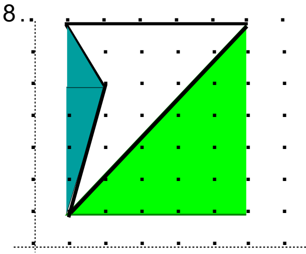
(Circle the letter of your choice from the list below.)

A $2\pi r^3$ B $2\pi r^2$ C $2\pi r$ D $\frac{4\pi r^3}{3}$ E πr^2 F $\frac{2\pi r^3}{3}$ G $4\pi r^2$ H none

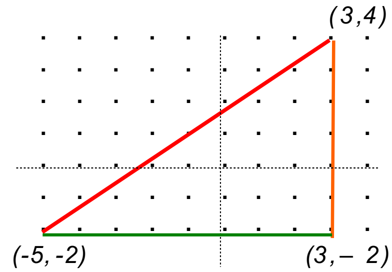
7.



8.



9.



In all the above, the area shown here →  *is one square unit.*

( ) 7. Estimate the area inside the curve shown in #7. If that figure were expanded to one of the same shape, but twice as high, and twice as wide, what would the area inside the new curve be?

\# of whole square units within region + (½) #of partial units within region

7 unit^2 + (½) (17 unit^2)

15 ½ unit^2

(3) 8. Find the area enclosed by the figure in #8. (Curve turns only at points: (1,1) & (6,7) & (1,7) & (2,5))

(Area of enclosing rectangle) – (areas of rectangles and triangles not inside the curve)

5•6 unit^2 – (½) 6 •1 unit^2 – (½) 5•6 unit^2

30 unit^2 – 3 unit^2 – 15 unit^2

12 unit^2

(5) 9. Find the PERIMETER of the triangle in figure #9 above.

Perimeter = Distance from (– 5,– 2) to (3,-2) + Distance to (3,4) + Length of Hypotenuse

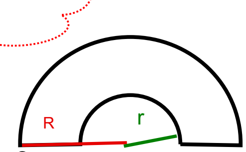
= 4– (– 4) + 3 – (– 3) + square root $(8^2 + 6^2)$

= 8 + 6 + 10

= 24 (units)

$8^2 + 6^2 = c^2$
$64 + 36 = c^2$
$100 = c^2$

(5) 10. Find the AREA inside the curve at right, given all arcs are semicircular.



This is a semicircular region with a semicircular "hole" taken out!

Area of the larger semicircular region – area of the smaller semicircular region

= ½ of πR^2 – ½ of πr^2

= (½) π (13mm)^2 – (½) π (5mm)^2

= (½) 169 π mm^2 – (½) 25 π mm^2

= 72 π mm^2

...where R is the radius of the outer semicircle, and r is the radius of the inner semicircle.

( ) 11. 52500 mL water (at 4°C) = ____________ kg.

* At 4° C, water is at its most dense state and each 1cc or 1 mL has 1 gram of mass.

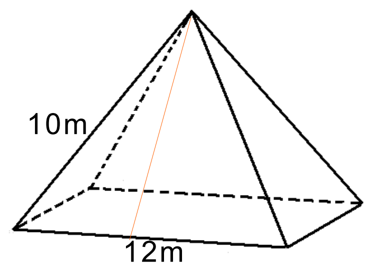
$$52500 \text{ mL} = 52500 \text{ mL} \cdot \frac{1 \text{ g}^{*}}{1 \text{ mL}} \cdot \frac{1 \text{ kg}}{1000 \text{ g}} = 52.5 \text{ kg}$$

( ) 12. Find the area inside a 60° sector of a circle with radius 2 cm.

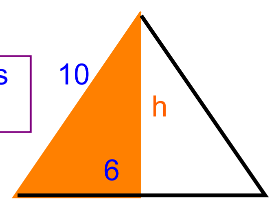
The area inside a sector is just the appropriate fraction of the area inside the entire circle.
The area of a 60° sector of a circle is just 60/360, or 1/6, of the area of the entire circle.

$$\frac{60}{360}\pi R^2 = \frac{1}{6}\pi R^2$$
$$= \frac{1}{6}\pi (2\text{cm})^2$$
$$= \pi (2\text{cm})^2 / 6$$
$$= \pi (4) \text{ cm}^2 / 6$$
$$= \frac{2\pi \text{ cm}^2}{3}$$

( ) 13. Find the surface area of the square-based right pyramid shown.
Find the volume contained by this pyramid.



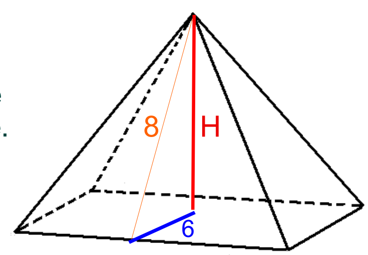
Surface area (SA) of this square-based pyramid is
Area of square base + 4(Area of triangular face)



$$SA = (12 \text{ m})^2 + 4(½)\, 12\text{m}\,(8\text{m})$$
$$= 144 \text{ m}^2 + 4 \cdot 48\text{m}^2$$
$$= 336 \text{ m}^2$$

$$h^2 + 6^2 = 10^2$$
$$h^2 + 36 = 100$$
$$h^2 = 64$$
$$h = 8 \text{ (m)}$$

The Volume of the Pyramid is (⅓) (Area of base)(height)
The height is H... The vertical arm of a right triangle whose hypotenuse is 8, the value we figured out above, the height of the triangular surface.



To find H:

$$H^2 + 6^2 = 8^2$$
$$H^2 + 36 = 64$$
$$H^2 = 28$$
$$H = \sqrt{28} = 2\sqrt{7} \text{ (m)}$$

Volume = (⅓) (Area of base)(height)

$$= (⅓)\,(12 \text{ m})^2\,(2\sqrt{7} \text{ m})$$
$$= 96\sqrt{7} \text{ m}^3$$

( ) 14. A right circular cylinder holds a volume of 4000 cm^3. What is the capacity (volume) of a cylinder that is half as tall, but with a diameter twice that of the original cylinder?

Since volume = (area of base) (height)...
Cutting the height in half would halve the volume.
Doubling the diameter doubles the radius, which in turn quadruples the area... thus quadrupling V.

OR merely compare $V = \pi r^2 h$ to new $V = \pi (2r)^2 (h/2) = 2\pi r^2 h$ .... New V = 8000 cm^3