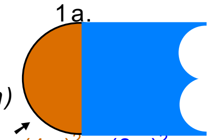
**MATH 310 ✔ MEASURE Self-Test ANSWERS ✔** (DETAILED answers start on NEXT PAGE.)

1. a. $(½)•π(4u)^2 + [(8u)^2 − π(2u)^2] = (4π + 64)u^2$ *(see diagram↘)*
Perimeter = π4u + 2•8u + 2• ½ •2π(2u) = (8 π + 16)u

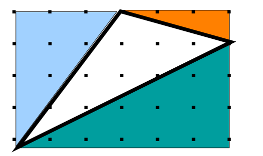
1a.




b. 7.5 $u^2$ [ 4•6 – 4•3•½ – 1•3•½ – 6•3•½ = 7.5 ] *(work from outside in)*
*Perimeter* = ( 5 + √10 + √45 ) units *(Use the Pythagorean theorem.)*

c. whole + ½ partial = $10u^2$ + (½ )($10u^2$) = $15u^2$
d. $2\ell•2w•2h = 8\ell wh$ = 8•(original volume)
(6+1)(8+1)(7+1) ≠ 6•8•7 + 1 ! It's 6•8•7 + 6•8 + 8•7 + 6•7 + 1

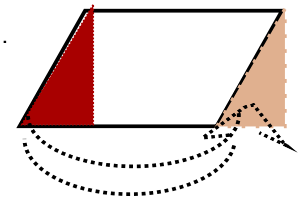
1b.



3. a. $h·(a + b)/2$ b. $πr^2$ c. $(x/360)•πr^2$
d. C = $2πr$ e. V = $\frac{4πr^3}{3}$ f. SA = $4πr^2$

4. a. $\sqrt{4^2 + 8^2}$ u = $4\sqrt{5}$ units.
b. D[(−1,4),(2,0)] = 5; D[(2,0),(−2,−3)] = 5; D[(−1,4),(−2,−3)] = $50^{1/2}$ .
Yes: $d_1^2 + d_2^2 = d_3^2$.

2.

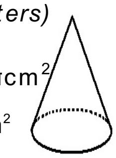


5. Area in square – Area in circle = $(12\ cm)^2 − π(6\ cm)^2 = (144 − 36π)cm^2$

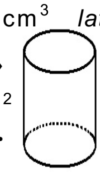
6. *V of prism or cylinder = (Area of base)•(Height)* = 8000 $m^3$

7. a. $(⅓)•(30\ cm)^2 20\ cm$ = 6000 $cm^3$ *Volume of pyramid or cone = (1/3) (area of base)(height)*
b. $(⅓)•(80\ cm^2)•20cm$ = 1600/3 cc. *("cc" is an alternate abbreviation for $cm^3$ – cubic centimeters)*

8. a. V = $(⅓)•π(5\ cm)^2•25cm$ = 625 π/3 $cm^3$ *lateral SA* = $(1/2)•π(10\ cm)(5\sqrt{26}\ cm) = 25\sqrt{26}\ πcm^2$

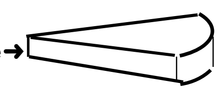


b. V = $π(5\ cm)^2•25cm$ = 625π cc ➜ BTW: Total SA = ( 25π + $25\sqrt{26}$ π ) $cm^2$



*lateral SA* = π(10cm)•25cm = 250π$cm^2$
BTW: Total SA = ( 2•25π + 250π ) $cm^2$ ←

radial face➜

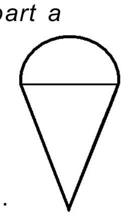


➜part of Circumference

9a. *SA = top+bottom + radial faces + strip of C* *V = one-sixth of the V of the original wheel*
SA = 2•(15cm•3cm) + $2•(1/6)•π(15cm)^2$ + $3•30πcm^2/6$ V = $(1/6)•π(15cm)^2•3cm$ = $225π/2cm^3$

9b. SA = 2•(15cm•3cm) + $2•(30/360)•π(15cm)^2$ + $(1/12)3•30πcm^2$
V = $(30/360)•π(15cm)^2•3cm$ = $225π/4\ cm^3$ *30° is 1/12 of the circle, so V is half V of part a*

10. SA = $(1/2)•4π(5cm)^2$ + (1/2)•π(10cm)•13cm = 115π$cm^2$ ←
V = $V_{hemisphere} + V_{cone}$ = $(1/2)•(4/3)π(5cm)^3 + (1/3)•π(5cm)^2•(12cm)$ = $(550π/3)cm^3$ ←



11. The relationship between °C and °F is a line going from (0,32) to (100,212);
that makes a rise of 100°C equal to a rise of 180°F: i.e. 100°C = 180°F, so ( 100°C/180°F ) = 1!.
100°C/180°F *reduces to* 5°C/9°F. *And, yes,* 5°C/9°F = 1 *also.* So °C = (5°C/9°F)•(°F − 32°F).
*(Notice °F must be adjusted down to 0 before multiplying, so that 32°F will end up being 0°C.).*

| |
|---|
| C = $(^5/_9)(F − 32)$ |
| F = $(^9/_5)C + 32$ |

a. 72°F = 22°C (18°Reaumur) c. 98.6°F = 37°C
b. 30°C = 86°F

12. a. cm b. cm c. m d. km e. kg f. g g. g *(actually- mg, but that's not in the list)*
h. t i. mL or cc j. L k. mL or cc l. kL

13. 1 kL = 1000 L = 1000•1000 mL = $10^6$mL = $10^6$cc = ☆ $10^6$g = 1000 kg = 1 metric ton. *OR:*

1 kL = 1 kL • $\frac{1000L}{1\ kL}$ • $\frac{1000\ mL}{1\ L}$ • $\frac{1\ cc}{1mL}$ • $\frac{1\ g^*}{cc}$ • $\frac{1\ kg}{1000g}$ • $\frac{1\ metric\ ton}{1000\ kg}$ = 1 metric ton

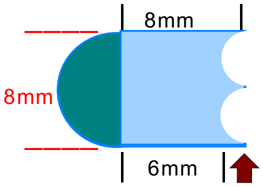
☆ *valid ONLY for water at 4°C*

14. a. *Carpet costs* \$32/yd²; *wood costs* \$4/ft² = \$4/ft²•3ft/yd•3ft/yd = \$36/yd²; *carpet is cheaper.*
b. 1 ft² = (12 in)² = 144 in²; 1 yd² = (3 ft)² = 9 ft² *(Use the method shown in #13.)*
c. (1 ft)³ = (12 in)³ = 1728 in³; (1 yd)³ = (3 ft)³ = 27 ft³; 1 yd³ = 27 ft³ = 27•1728 in³ = 46656 in³
d. 1 m³ = (100 cm)³ = 1000000 cm³; (see #13) = 1000000 mL
e. (see #13; the conversion at * is valid for water only!) f. .13 daL
g. 300 *dam* = 3000 *m* = 300000 *cm* g. 200 *cm*² = 200 cm² • $\frac{1m}{100cm.}$•$\frac{1\ m}{100cm}$ = .02*m*² h. 328dl = 328dl •$\frac{100ml}{dl}$•$\frac{1cc}{ml}$ = 32800 *cc*

15. *Just barely not. The longest dimension in a 24"x7"x7" box is the extreme diagonal, $(674)^{½}$ " ≐ 25.96"*

16. *New area* = *100cm² • 2.5 • 2.5 = 625 cm²*

1a. *We assume arcs are semi-circular.* Perimeters are shown on pag




We view this as a **half-circular region[I]** (with diameter 8mm) joined to a **rectangular region [II]** (8mm by 8mm) from which **two small semicircular regions [III]** have been removed

radius of the small circles must be the difference 8mm – 6mm= 2mm. (And must also be ½ of 8mm/2.)

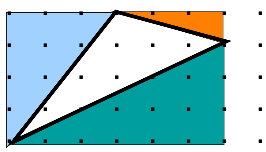
Area **I**: radius is ½ of 8mm, or 4mm. Area of half-circular disc is $(\frac{1}{2})\pi(4mm)^2$

Area **II**: area of 8m by 8mm square is $64mm^2$

Area **III**: area of two half-circular "cutouts" [diameter $^{8mm}/_2$ = 4mm]... is $2\cdot\frac{1}{2}\cdot\pi(2mm)^2$

Thus Total Area of figure is $(\frac{1}{2})\pi(4mm)^2 + 64mm^2 - 2\cdot\frac{1}{2}\cdot\pi(2mm)^2 = (4\pi + 64)mm^2$

1b.

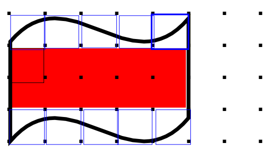


We start with the smallest vertical-horizontal bounded rectangle that contains the polygon given (which happens to be a triangle, but that's not important, this works for any polygon with vertices on lattice pts).

Area within polygon = area of rectangle – areas of "take-away" parts:

$(4u)(6u) - \frac{1}{2}(4u)(3u) - \frac{1}{2}(3u)(1u) - \frac{1}{2}(6u)(3) = 24 - 33/2 = 7.5\ u^2$

1c.



We cover the region with squares.

We count the squares which lie *entirely* *within the region outlined.*

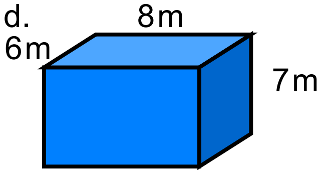
We count the squares which lie *partially* *within the region outlined.*

We then make the ESTIMATE:

Area within outline $\approx 10u^2 + (\frac{1}{2})(10)u^2 = 15u^2$ (approximate area)

*(This is based on the assumption that the squares which are partially enclosed average half inside, half outside)*

1d.




The volume of the box is (area of base)(height) = length•width•height = $(6m \cdot 8m)\cdot 7m = 336\ m^3$

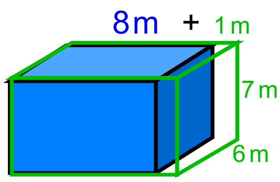
Doubling all the dimensions, we can compute:

NEW V= $(12m \cdot 16m)\cdot 14m = 2688\ m^3$ ... that is, $8\cdot 336\ m^3$

In fact, we could have anticipated this, since

$V = 2\ell \cdot 2w \cdot 2h = 8\ell w h = 8\cdot$(original volume)

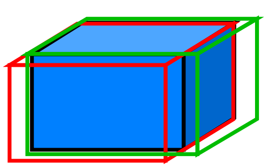
*If each dimension is increased by 1m, is the volume increased by* $1m^3$ *? Explain.*




NO! The increase in volume is much, much greater than that!

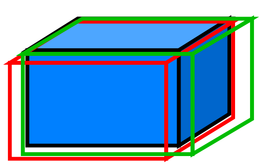
Just consider, first, the effect of increasing just ONE dimension– e.g. increase the **length** from 8m by 1m to 9m.

As illustrated at left, this would add a 1m by 6m by 7m "slab" to one end of the box. That alone, then, adds $42m^3$ of volume to the box.



Here we illustrate the addition of a 1m extension to the **depth** of the box. Just for the old box, this extension results in an additional 1m •8m•7m or $56m^3$, and that does not even take into account the extension of the length from 8m to 9m!

To account for that, we need another 1m by 1m by 7m piece in the corner!



Extending the **height** an additional 1m, from 7m to 8m results in a new slab added to the top of the box, 1m by 6m by 8m to cover the original box, but 1m x 7m x 9m to cover the box with its new extended length and depth. ←Picture it (draw the top extension in) yourself!

Here's the entire difference, found algebraically: Original volume, $V = \ell w h$,

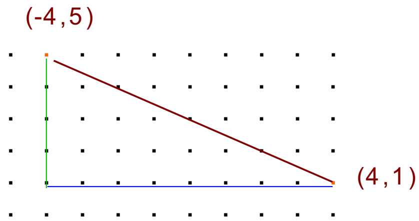
After each dimension is increased by amount a:

NEW V = $(\ell+a)(w+a)(h+a)$

$= \ell w h + \ell w a + \ell a h + a w h + \ell a a + a w a + h a a + a a a$

**Difference:** $\ell w \mathbf{a} + \ell \mathbf{a} h + \mathbf{a} w h + \ell \mathbf{a}\mathbf{a} + \mathbf{a} w \mathbf{a} + h \mathbf{a}\mathbf{a} + \mathbf{a}\mathbf{a}\mathbf{a}$

4. a. Sketch the points (-4,5) and (4,1) in the plane. Find the distance between the points.




The change in x = 5 - 1 = 4
The change in y = 4 -(-4) = 8

$4^2 + 8^2 = c^2$
$c^2 = 80$
$c = \sqrt{80} = \sqrt{16 \cdot 5} = 4\sqrt{5}$ (Units)

b. Do points (-1,4), (2,0), (-2,-3) lie at the vertices of a right triangle? How do you know?

The solution is based on the Pythagorean Theorem: the sum of the squares of the two short legs is equal to the square of the hypotenuse if, and *ONLY if, the triangle is a right triangle*. So find the distances between the three pts. They are 5, 5, and sqrt(50). We find the pythagorean relationship holds true ($5^2 + 5^2 = \sqrt{50}^2$). So the triangle must be a right triangle.

5. Area inside the square is $(12cm)^2$ . D of the circle = 12cm, so A = $\pi\ (6cm)^2$ . Now subtract!

6. No matter what shape the base of a prism is, the
**Volume of the prism** is just the product: **(Area of base) (Height)**

7. No matter what shape the base of a pyramid is, the
**Volume of the pyramid** is just **(one third)• the volume of the corresponding prism**
(See #6 note above!)

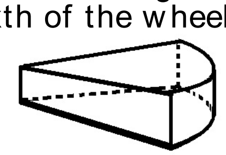
**These ideas apply to cylinders and cones as well.**

8a. Volume of cone = ( ⅓) (Area of Base)(Height) [see note above!]

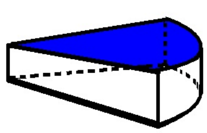
Finding the surface area of cone is similar to surface of pyramid.
Pyramids have lateral faces that are triangular.
The area of each triangle is (½ )(base length)(height)... When these are added up around the pyramid, the **total (for lateral surface area) is (½ ) (Perimeter of base) (slant height).**
[See more details on the answers to the Surface Area Quiz).]
Similarly,
the **lateral surface area of a cone is (½)(Perimeter of Base of cone)(Slant height of cone)**

9. The entire wheel of brie has the shape of a right circular cylinder, with height of only 3 cm. A wedge which is one-sixth of the wheel has volume = one-sixth of the wheel.

A wedge with a central angle of 30° contains only one-twelfth of the wheel, since $\frac{30°}{360°} = \frac{1}{12}$.



The Surface Area consists of three parts:



The top and bottom of the wedge are surfaces in the shape of a sector of a circle (radius 15cm). Thus the area is (PART) $\pi\ (15cm)^2$
In part a, this is $(1/6)\pi(15cm)^2$. The bottom has the same area.


The remaining surface of the wedge might be covered with a strip of paper 3 cm high. The length would be (2(radius) + (1/6) (Circumference)
(2(15cm) + (1/6) ( 2π(15cm) )
So the total lateral area is (3cm) (2(15cm) + (1/6) ( 2π(15cm))

The total surface area of the 1/6-wheel wedge of brie is:
$2\cdot(1/6)\pi(15cm)^2$ + (3cm) (2(15cm) + (1/6) ( 2π(15cm))

(For the 30° wedge of brie, replace (1/6) with (1/12) in all parts!)

10. Finding the total surface area and volume of an ice cream cone, topped with a hemisphere of ice cream, given the diameter of the top of the cone is 10 cm. and the height of the cone is 12 cm.

$$\text{Volume} = V_{hemisphere} + V_{cone}$$
$$= (\tfrac{1}{2})\bullet(\tfrac{4}{3})\pi(r)^3 + (\tfrac{1}{3})\bullet\pi(r)^2\bullet(h)$$
$$= (\tfrac{1}{2})\bullet(\tfrac{4}{3})\pi(5cm)^3 + (\tfrac{1}{3})\bullet\pi(5cm)^2\bullet(12cm) = (550\pi/3)cm^3$$

$$\text{Surface Area} = SA_{hemisphere} + \text{Lateral } SA_{cone}$$
$$= (\tfrac{1}{2})\bullet 4\pi r^2 + (\tfrac{1}{2})\, 2\pi r\bullet(h)$$
$$= (\tfrac{1}{2})\bullet 4\pi(5cm)^2 + (\tfrac{1}{2})\, 2\pi(5cm)\bullet(12cm)$$

11. Was/will be discussed in class.

13. Don't be intimidated by the number of steps here.
See how each multiplication just gives you new units.

$$1\text{ kL} = 1\text{ kL}\bullet\frac{1000\text{L}}{1\text{ kL}}\bullet\frac{1000\text{ mL}}{1\text{ L}}\bullet\frac{1\text{ cc}}{1\text{mL}}\bullet\frac{1\text{g}^*}{\text{cc}}\bullet\frac{1\text{ kg}}{1000\text{g}}\bullet\frac{1\text{ metric ton}}{1000\text{ kg}} = 1\text{ metric ton}$$

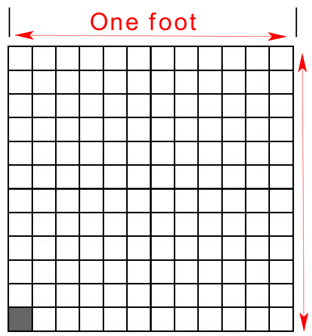
For instance, if we pause right here, we see that we now have L (liters) and since we know a conversion from milliliters (mL) to cc to grams*, we continue by converting the liters to milliliters. Then we need, at THIS POINT, to get from grams to kilograms, then to tons.

*☆ 1g = 1cc ...valid ONLY for water at 4°C*

14. b. $1\text{ ft}^2 = (12\text{ in})^2 = 144\text{ in}^2$

Using dimensional analysis: $1\text{ ft}^2 = 1\text{ ft}^2\bullet\frac{12\text{ in}}{1\text{ ft}}\bullet\frac{12\text{ in}}{1\text{ ft}} = 144\text{in}^2$

Using "common sense":



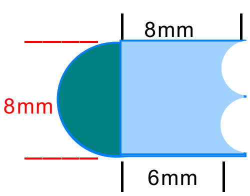

This 1-foot by 1-foot square is a "square foot".

12•12 (ie 144) square inches are required to cover this square foot!

ONE square inch

1a. PERIMETER:

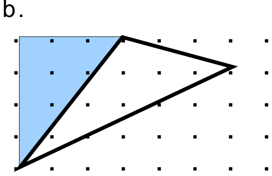
1a. We have a semi-circular arc of a circle with D= 8mm, and a full circle with diameter 4mm, In addition to two straight sides of length 8mm each.




So the full perimeter is

$$P = (\tfrac{1}{2})\pi\cdot 8mm + \pi(4mm) + 8mm + 8mm = (8\pi + 16)\ mm$$

1b.



If the sides were horizontal and vertical, we could just count the units of length on each side. But none of the sides are horizontal or vertical, So we must *compute* the length of each side, using the Pythagorean thm.

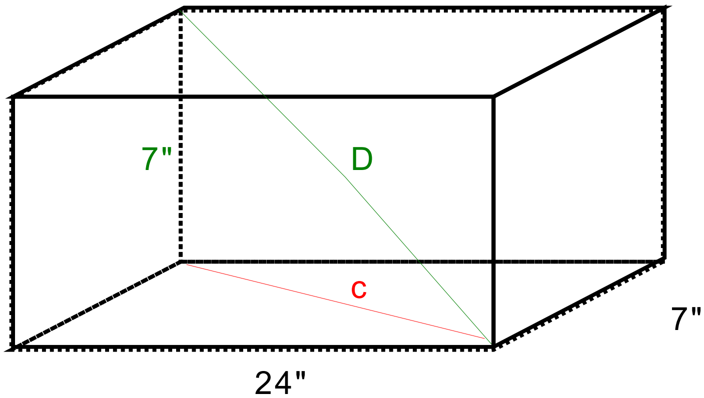
Each side of the given triangle is the hypotenuse of a right triangle with horizontal & vertical sides.

For instance, the upper left side of the given triangle is the hypotenuse of the shaded triangle. The shaded triangle is 4 units high & 3 units wide, so its hypotenuse must be ... 5units long.

For the upper right side: $3^2 + 1^2 = c^2$, so $c = \sqrt{10}$     $P = 5 + \sqrt{10} + \sqrt{45}$ units.

15. Find the longest diagonal inside a 24" by 7" by 7" right rectangular prism.

(BOX !)




The diagonal on the base of the box:

$(24")^2 + (7")^2 = c^2$

$576\text{ in}^2 + 49\text{in}^2$

$625\text{ in}^2 = c^2$

$25\text{ in} = c$

$c^2 + (7\text{in})^2 = D^2$

$(25\text{in})^2 + 49\text{in}^2 = D^2$

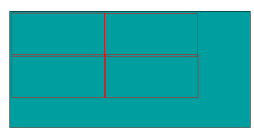
$674\text{in}^2 = D^2$

$25.9615\text{ in} \doteq D$

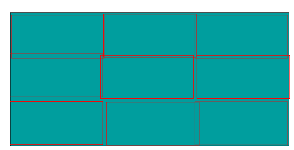
16. Explain why the answer to #16 is NOT just 2.5 times the old area!

The scale factor applies to both dimensions that contribute to area: height and width.
The scale factor is the ratio of new lengths to old, so the ratio of (New height)/(Old height)= 2.5 and the ratio of (New width)/(Old width) is also 2.5.
New Area = (Old Area) (Ratio of New height to Old height) (Ratio of New width to Old width)
= (Old area) (scale factor) (scale factor)

If you think in terms of the simplest area (think rectangle), this is obvious.

x by y  2.5x by 2.5y 

It becomes even more obvious when scaled up by a factor which is a whole number, say 3:

x by y  3x by 3y:  Area is 9 times as great.