1. A jar contains four marbles: three red, one white. Two marbles are drawn with replacement.
   (i.e. A marble is randomly selected, the color noted, the marble replaced in the jar, then a second
   marble is drawn.)
   a. List a sample space containing four outcomes.
   b. List a sample space with sixteen outcomes.
   c. Write the probability of each of the four outcomes in (a).
   d. What are the probabilities of the outcomes in (b)?
   e. What is the probability the colors of the two marbles match?
   f. What is the probability the same marble is drawn twice?

2. We are playing with a short deck, as shown at right.
   Let "H" be the event the card drawn is a heart.
   Let "D" be the event the card drawn is a diamond.
   Let "A" be the event the card is an ace.
   a. \( P(H) = \) P(D) = \( P(A) = \)
   b. \( P(H \text{ or } D) = \)
   c. \( P(H \text{ or } A) = \)
   d. \( P(H \text{ and } D) = \)
   e. \( P(H \text{ and } A) = \)
   f. Are H and D independent events?
   g. Are H and A independent events?

3. If three cards are drawn from the deck in #2, one at a time, what is the probability that
   a. the 1\textsuperscript{st} card is the ace of hearts, the 2\textsuperscript{nd} is the 2 of diamonds, and the 3\textsuperscript{rd} is the 3 of clubs?
   b. all three cards are aces?

4. An airplane is built to be able to fly on one engine. If the plane's two engines operate
   independently, and each has a 1% chance of failing in any given four-hour flight, what is the
   chance the plane will fail to complete a four-hour flight to Oklahoma due to engine failure?

5. A pair of fair, standard dice are rolled. What is the probability the sum of the dice is 5?

6. Fifty marbles are to be drawn from the jar in problem #1 with replacement. If the first four
   marbles drawn are red, what is the probability the next marble drawn will not be red?

7. A probability experiment has four possible outcomes: \( e_1, e_2, e_3, e_4 \). The outcome \( e_i \) is four
   times as likely as each of the three remaining outcomes. Find the probability of \( e_i \).

8. What are the odds in favor of rolling a sum of seven in one roll of a pair of fair standard dice?

9. If \( P(A) = \frac{2}{3} \) and \( P(B) = \frac{1}{2} \) and \( P(B|A) = \frac{1}{3} \), find:
   a. \( P(A \text{ and } B) \)
   b. \( P(A \text{ or } B) \)
   c. \( P(A|B) \)

* 10. The deck of sixteen cards shown in #2 is thoroughly shuffled. Three cards are drawn from the
    top of the deck, one at a time. What is the probability the third card is an ace?

* 11. (The Birthday Problem) In a roomful of 30 people, what is the probability that at least two
    people have the same birthday? Assume birthdays are uniformly distributed and there is no
    leap year complication. (Hint: what is the probability that they all have different birthdays?)

* 12. A 1-inch-diameter coin is thrown on a table covered with a grid of lines two inches apart.
    What is the probability the coin lands in a square without touching any of the lines of the grid?
    (Hint: in order to not touch any of the grid lines, where must the center of the coin be?)

Try the Probability Problems from the Final Exam Practice.
Self-Test Probability Answers:

1a. \{RR, RW, WR, WW\}  
1b. \{R_1R_1, R_1R_2, R_1R_3, R_1W_1, R_2R_1, R_2R_2, R_2R_3, R_2W_1, R_3R_1, R_3R_2, R_3R_3, R_3W_1, W_1R_1, W_1R_2, W_1R_3, W_1W_1\} 
1d. The outcomes detailed in the sample space in 1b are equally likely; each has \( P = \frac{1}{16} \).

1e. \( P(\text{colors match}) = P(\text{RR}) + P(\text{WW}) = \frac{9}{16} + \frac{1}{16} = \frac{10}{16} \) or \( \frac{5}{8} \)

1f. \( P(\text{same marble twice}) = P(R_1R_1, R_2R_2, R_3R_3, W_1W_1} = \frac{4}{16} \) (using 1b; SS in 1a is no help at all)

...or, you can reason thus: \( P(\text{same marble twice}) = P(\text{second marble is same as the first}) = \frac{1}{4} \) because there are 4 marbles in the jar on the second draw, and only one is the same marble as the 1st.

2a. \( P(H) = P(\text{RR}) = P(A_1) = \frac{4}{16} \) 
2b. \( P(D) = P(\text{RR}) + P(\text{WW}) = \frac{4}{16} + \frac{1}{16} = \frac{5}{8} \) 
2c. \( P(H \text{ or } A) = P(H) + P(A) - P(H \text{ and } A) = \frac{1}{4} + \frac{1}{4} - \frac{1}{16} = \frac{7}{16} \)

2d. \( P(H \text{ and } D) = 0 \) (see 2b)
2e. \( P(A \text{ or } D) = P(A) \) 
2f. H & D are not independent, they are mutually exclusive. If one occurs, the other cannot!

3. a. \( P(\text{4\spadesuit | A\spadesuit gone}) \) \( P(\text{3\spadesuit | 2\spadesuit & A\spadesuit gone} = (1/16)(1/15)(1/14) \) 

b. \( P(\text{AAA}) = (4/16)(3/15)(2/14) \) ...by reasoning similar to part a.

4. \( E_1 \text{ fails} \) \( E_2 \text{ fails} \) \( E_1 \text{ OK} \) \( E_2 \text{ fails} \) \( E_1 \text{ OK} \) \( E_2 \text{ OK} \) 

The plane will fail to make the flight due to engine failure only if BOTH engines fail (because the plane can fly on one engine.) 
\( P(\text{flight fails}) = P(\text{both engines fail}) = P(1\text{st fails})P(2\text{nd fails}) = .0001 \)

5. \( P(\text{sum = 5}) = P(\text{rolling 14 or 23 or 32 or 41}) = \frac{4}{36} = \frac{1}{9} \)

6. Every time a marble is taken from this jar (assuming previously drawn marbles are replaced), the probability of obtaining a red marble is \( \frac{3}{4} \). Therefore, \( P(\text{not red}) = \frac{1}{4} \).

7. \( 4p + p + p + p = 1 \Rightarrow 7p = 1 \Rightarrow p = \frac{1}{7} \Rightarrow P(e_i) = 4p = 4(1/7) = \frac{4}{7} \)

8. There are six ways to roll a sum of 7: 16, 25, 34, 43, 52, 61. \( P(\text{sum = 7}) = 6/36 \) or 1/6 (not the question!)

9. There are six favorable outcomes in this SS with 36 equally likely outcomes, so 29 are unfavorable. The odds in favor of a sum of 7 are 6:29 (Because they are 29:6 against....)

9. a. \( P(A \text{ and } B) = P(A) P(B|A) = (\frac{2}{9})(\frac{1}{9}) = \frac{2}{9} \)

b. \( P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = (\frac{2}{9}) + (\frac{2}{9}) - \frac{2}{9} = \frac{5}{9} \)

c. \( P(A|B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{\frac{2}{9}}{\frac{5}{9}} = \frac{2}{5} \)

10. \( P(\text{all different}) = \frac{365 \cdot 364 \cdot 363 \cdot \ldots \cdot 336}{365 \cdot 365 \cdot 365} \) This turns out to be under 30%.

And if they are NOT all different, then at least two must be the same. Therefore, over 70%!

12. Where does the coin have to land in order to win? What determines the location of the coin? Where must the center of the coin be? Draw a picture of where it can be. The answer is one-fourth.