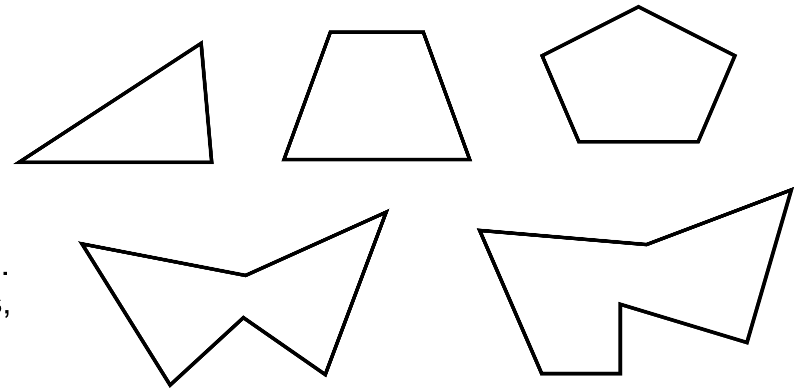
TWO INVESTIGATIONS OF POLYGONS—Angles and Diagonals **GB-2**

ANGLES:



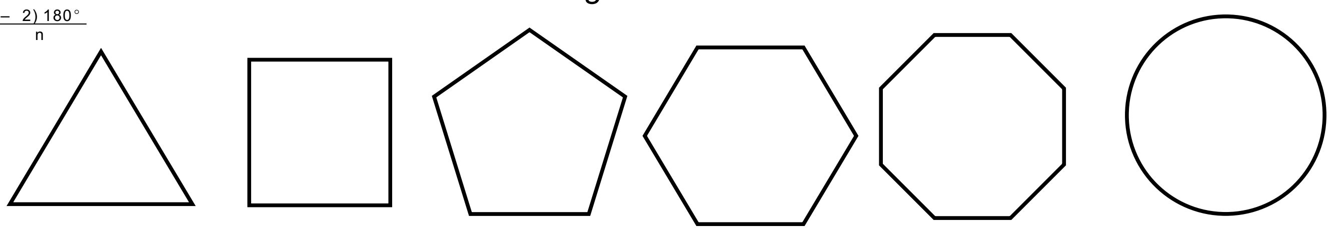
Consider any polygon…

Mark the interior angles.
What is the sum of their measures?

One approach— triangulate the polygon.
That is, break the polygon into triangles, whose vertices are ALL vertices of the original polygon. You will find …
a polygon with n sides can be divided into ________ triangles using original vertices. So…
$n-2$

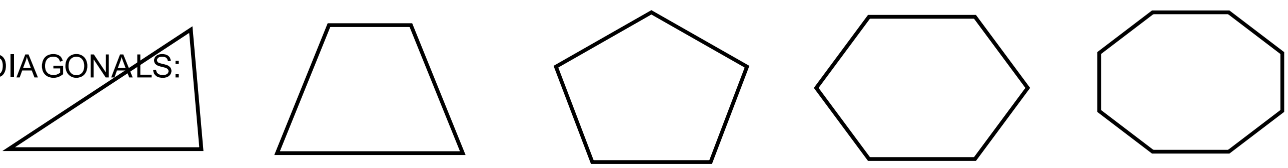
The sum of the measures of the interior angles of any *convex* polygon of **n** sides is ________
$(n-2)180°$

A **REGULAR polygon** has all sides congruent, and all interior angles congruent.
Since we know the sum of the measures of the n interior angles of a regular n-gon is _____,
we know the measure of each interior angle must be
$\frac{(n-2)\,180°}{n}$



Notice, also, that as the number of sides increases, these interior angles open up, getting closer and closer to ____.

DIAGONALS:



Our text asks the question: how many diagonals has a convex n-gon?

Far easier—and more important, too—is the question:
How many ways can we connect* n vertices (points in a plane), provided no 3 are collinear?
* (with line segments)

…and as each new point is added, that point may be connected to every one of the preceding points.
E.g. the 5th point connects to the previous 4 points….
What's the pattern?
# of diagonals for octagon?

ways = | #ways = 1+ | #ways = 1+ + | # ways = 1+ + +

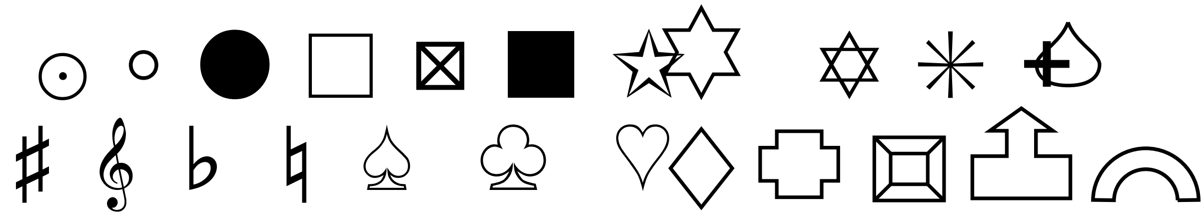
So the number of ways to connect* n points, no 3 collinear, is
It helps to know that this sum is $\frac{n(n-1)}{2}$.

So the number of diagonals must be $\frac{n(n-1)}{2} - n$. (Which will give the same silly formula as the book.)

Math 310 GB1 Fn

CURVE: a connected set of points that is 1-dimensional* which can be traced without retracing except for "(isolated) single points".

> * 1-dimensional: at almost every point on the curve, there are only two directions of movement possible, where movement in that direction, forward and backward, is possible, but no other direction....

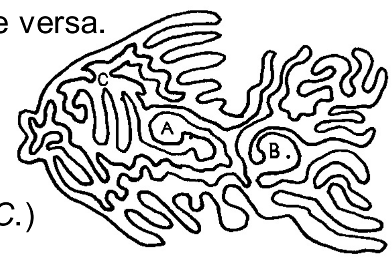


A B C D E L M P Q S X Y Z

PLANE **curve**: curve whose points lie in one plane.
CLOSED **curve**: curve which ends where it began.
SIMPLE **curve**: one that does not intersect (meet) itself
(except possibly to end where it began)

Jordan Curve Theorem A *simple closed plane curve* divides the plane into three disjoint regions: interior ("bounded"), exterior, and the curve itself, lying between the other two. When we cross the curve we pass from interior to exterior or vice versa.

Tests for interior/exterior of SCPC
Color the region.
Draw a line from "x" to any point clearly on the exterior;
count the number of breaches of the boundary. (*Fails if not SCPC.*)



SCPC & points A,B,C

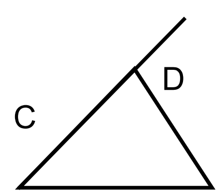
POLYGONS: simple closed plane curves composed of line segments.
(Some geometers define a polygon as a series of points in a plane where the last point = the first, and the line segments joining consecutive points [not guaranteed simple].)

POLYGONS are classed by number of sides, and INCLUDE:

TRIANGLES-
QUADRILATERALS-

PENTAGONS
HEXAGONS
HEPTAGONS (septagons)
OCTAGONS
NONAGONS
DECAGONS
DODECAGONS
ICOSAGONS
n-GONS

POLYGON terms:
DIAGONAL of a polygon: segment joining non-consecutive vertices.
EDGES (sides) of a polygon: the segments that make up the polygon.
VERTICES of a polygon: (corners) points where edges meet.
CONVEX polygon has NO "indentations"



REGULAR polygon: polygon with all sides ≅ AND angles all ≅
INTERIOR ANGLE: An angle determined by consecutive sides of a polygon A B
EXTERIOR ANGLE: An angle determined by the extension of one side, and the next side....