A. PROBABILITY The solutions given are not the only method of solving each question.

1. A fair coin was flipped 5 times and landed heads five times. What is the probability of a head on the next toss?

   A fair coin by definition has equal chance of turning up ‘heads’ or ‘tails’ on each toss, therefore the probability of heads on the next toss is just what it always is: ½. (The coin has no memory of what it has been doing, and no mind to say, oops, better turn up tails for a change!)

2. One card is selected at random from an ordinary deck of 52 cards. Find the probability of each of the following events.

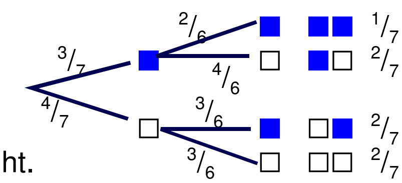
2a. P(♠) = 1/4 because there are four equally represented suits. Or: 13/52 because 13 cards are ♠.
2b. P(K♠) = 1/52
2c. P(♥ or Face) = P(♥) + P(Face) – P(♥& Face) = 13/52 + 12/52 – 3/52
2d. P(NOT 2♣) = 1 – P(2♣) = 1– 1/52 = 51/52

3. A box contains three blue cards and four white cards. Two cards are drawn one at a time.

3a. P(2blue) = P(■■) = P(1^{st} is ■) · P(2^{nd} is ■) = (3/7) (3/7) = 9/49

3b. P(■■) = P(1^{st} is ■) · P(2^{nd} is ■ | 1^{st} is ■) = (3/7) (2/6) = 1/7

3c. Diagram for experiment (b) [draw two cards without replacement] at right.



4. In firing a NASA three-stage rocket, the probability of the success of stage 1 is 95%, of stage 2, 97%, and of stage 3, 98%. Assuming that the stages are independent, what is the probability for success in all three stages during a single launch?

   P(successful launch) = P(1^{st} stage success)·P(2^{nd} stage success)·P(3^{rd} stage success)
   = .95 · .97 · .98 = .90307

5a. If the odds in favor of the Dodgers winning the game are 8 to 5, what is the probability that they will win?

   This is what that says to me:
   Out of every 13 possibilities, 8 are in the Dodgers’ favor, and 5 are against them.
   It’s the same as drawing a card at random from a box containing 8 ■ and 5 □.
   The odds favor blue, 8 to 5, as in this question. And the *probability* of getting a ■ is 8/13.

   So the probability of Dodgers winning is 8/13.

5b. If the probability that our team will win is 2/5, what are the odds against our team?

   Our team’s chance of winning is 2 out of 5.
   Out of 5 equally likely outcomes, 2 are winning and 3 are losing.
   The *odds are against* our team, 3 versus 2, or 3 against 2.

   So the odds against our team are 3 : 2 .

6. Two standard dice are rolled. What are the odds in favor of rolling a sum of 3?

   Comment: this goes against the grain, because the odds are definitely NOT in favor of 3. So, ask yourself, how much *against* a sum of 3 are they? Well, in this SS only two outcomes give a sum of 3, the other 34 do not. So the odds against a sum of 3 are 34 to 2. Thus we conclude:

   The odds in favor of the sum being 3 are 2 : 34 or 1 : 17 .

7. The English names of the months of the year are placed in a hat and one is drawn at random.
(a) List the sample space for this experiment.
(b) List the event consisting of the outcomes that the month drawn starts with the letter J.
(c) What is the probability of drawing the name of a month that starts with J?

7a. { January, February, March, April, May, June, July, August, September, October, November, December}
7b. E = {January, June, July}
7c. P (E) = 3/12 or 1/4

8a. If P(A) = 2/3, P(B) = 1/2, and P(B|A) = 1/3, find P(A|B).
This question exercises the probability fact that: P(A and B) = P(A) · P(B | A) (1)
... in which A & B can be interchanged to say: P(B and A) = P(B) · P(A | B) (2)
Solving the second statement for P(A | B): $P(A \mid B) = \dfrac{P(B \text{ and } A)}{P(B)}$ (3)

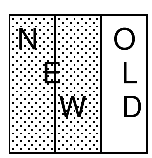
We use statement (1) to find P(A and B) = P(A) · P(B | A) = (2/3)(1/3) = 2/9

We use statement (3) to find $P(A \mid B) = \dfrac{2/9}{1/2} = \dfrac{4}{9}$

8b. If two-thirds of the books in the library are new, and half of the books in the library are non-fiction, but only one-third of the new books are non-fiction, find the probability that a book selected at random from the library is new, given that it is a non-fiction book.
"Formula" solution: This is identical to the above question. Let A be the event that a book is new, and let B be the event that a book is non-fiction. Two-thirds of books are new says P(A) = 2/3. 1/3 of the new books are non-fiction ("NF") says P(NF | new) = 1/3. And so on.... So this may be answered using the "formula" above.

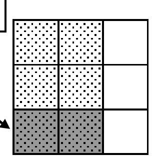
"Numeric" solution: Suppose there are 900 books in the library (this number was chosen to be divisible into thirds, halves, and thirds again).
Two-thirds are new, so 600 are new.
Half the books in the library are non-fiction, so 450 are non-fiction (and thus 450 are fiction).
One-third of the new books are non-fiction, so 1/3 of the 600 new books... i.e. 200 new books are NF.
Probability a book is new, given that it is non-fiction, must be 200/450, because it must be one of the 450 non-fiction books in the library, and the chance that such a book is new is only 200 (new non-fiction) out of 450 (non-fiction).

"Diagram" solution:
Two thirds of the books are NEW:



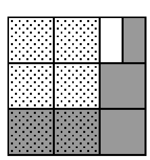
Half are non-fiction,
but we don't know how to allocate that yet.

But we do know:
One third of the NEW books are non-fiction:
NEW, NON-Fiction ↘

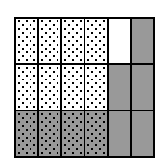


Since HALF of all the books are non-fiction, half of all 9 blocks (at left) must be shaded, so 2½ of the OLD blocks must be shaded

Here we see all 4½ blocks of non-fiction books shaded (■) – 2 NEW & 2½ OLD



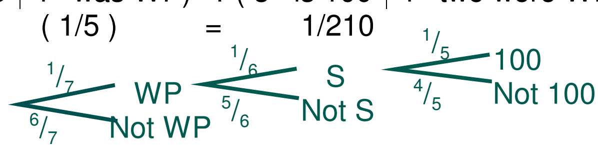
Dividing the total into equal parts shows 4 out of the 9 non-fiction parts are NEW.



9. Sara has a collection of 7 books: WAR AND PEACE, STEPPENWOLF, ONE HUNDRED YEARS OF SOLITUDE, HUCKLEBERRY FINN, LORD OF THE FLIES, THE OLD MAN AND THE SEA, THE GRAPES OF WRATH. If she randomly selects three of her books to read in sequence this month, what is the probability that she will read WAR AND PEACE, STEPPENWOLF, and ONE HUNDRED YEARS OF SOLITUDE in that order?

Abbreviating these titles to WP, S, 100, H, L, O, and G, the answer is:
P(WP, then S, then 100) = P(1^{st} is WP) · P(2^{nd} is S | 1^{st} was WP) · P(3^{rd} is 100 | 1^{st} two were WP & S)
= (1/7) · (1/6) · (1/5) = 1/210

An abbreviated tree diagram may
Be used to illustrate the solution.



B. STATISTICS

1. Mrs. Jurado's class of 20 students averaged 75 on a standardized reading test. Miss Johnson's class of 25 students averaged 66. What is the mean of the two classes combined?

   The total of all the points is: $(20\cdot75 + 25\cdot66) = 3150$
   Dividing equally among the 20+25 students: $3150/45 = 70$, the mean score.

2. Claude paid $38.80 for dinner for himself and two friends. If one friend's meal cost twice as much as Claude's and Claude's meal cost the same as his other friend, answer the following:
   (a) What is the mean cost of the meals? (b) What is the median cost of the meals?
   (c) What is the modal cost of the meals?

   Before trying to find the answers to those questions, we should know what the meals cost!
   Let $\boxdot$ be the cost of Claude's meal. One friend's meal cost $\boxdot\boxdot$ and the other friend's meal cost $\boxdot$.
   The total then was $4\,\boxdot$, which we are told was $38.80 (assuming no tax or tip included in the figure).
   Thus, the cost of Claude's meal was one-fourth of $38.80... Or $9.70. One friend's was also $9.70, the other's twice as much, or $19.40.

2a. Mean cost of the meals is $\$38.80/3 = \$12.9\overline{3}$
2b. Median cost of the meals was $9.70, the middle number.
2c. Modal cost of the meals was $9.70, the price of Claude's meal and one friend's meal.

3. Find the standard deviation of this set of numbers: 5, 14, 12, 6, 7, 3, 15, 10.
   Simplify your answer and express it in terms of a radical.

   First we compute the mean: $(5+14+12+6+7+3+15+10)/8 = 72/8 = 9$
   Then add up the squared distances to the mean:
   $4^2 + 5^2 + 3^2 + 3^2 + 2^2 + 6^2 + 6^2 + 1^2 = 136$ and divide by 8 (following the method in the text): $136/8 = 17$

   Then extract the square root: (well, we leave it at $\sqrt{17}$).

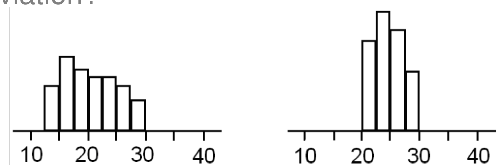
4. Which of the following data sets is likely to have the greatest standard deviation? Why?
   A: the ages of the 37 members of the Girl's Senior Varsity Soccer Team at SFVHS
   B: the ages of the 70 teachers at SFVHS
   C: the ages of the 94 members of the Retired Faculty Association of SFVHS

   Answer: B Standard Deviation is all about variation, the spread of the data relative to the mean. The ages of the girls on the SV Soccer Team are likely very close, as the girls are all seniors in high school. The ages of the Retired Faculty (C) are likewise relatively close (though probably not nearly as close together as the ages of the girls on the Soccer Team). But the ages of 70 teachers at SFVHS (B) are most likely spread out from beginning teachers in their 20s to about-to-retire teachers in their 60s. The data in set (B) would have the most variation.

5. Without calculating, decide which set of Data has the largest standard deviation. Why?
   A. 10, 20, 30, 40, 50, 60, 70, 80
   B. 110, 120, 130, 140, 150, 160, 170, 180
   C. 20, 40, 60, 80, 100, 120, 140, 160

   Answer: C Standard Deviation measures spread– variability– relative to the mean. Notice the values in B are exactly 100 more than the values in A. The data in A and B are identically "spread out". Further, since each set of data is symmetric, the mean for A is obviously 45, that for B, 145; the distances between the data and mean in set A are *identical* to the distances between the data and mean for set B. Thus the standard deviations for A and B must be the same. Finally, notice the data in set C are twice the values in set A. The distances in set C are all, therefore, double the distances in set A. For instance, the distance between 10 & mean 45 in set A is 35; the distance between 20 and the mean, 90, in set C is 70. Since all distances are doubled, the standard deviation must also be doubled.

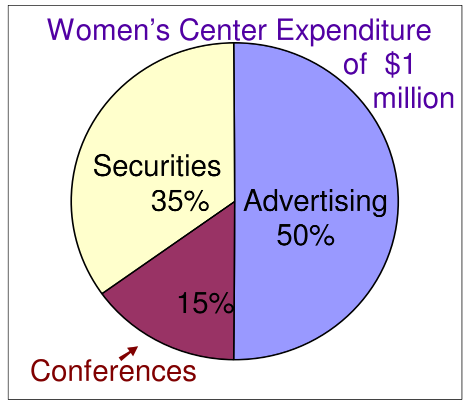
6. Which data distribution appears to have the greater standard deviation?



The data in the second distribution appears to be quite "clustered about" (close to) the mean, whereas the data in the first is "more spread out". Thus the first one has the greater std. deviation.

In more detail: The first, in which the range appears to be about 17, as opposed to the second, in which the range appears to be about 10. Standard deviation is somewhat akin to the notion of average distance to the mean, but gives great emphasis to any very large distances (by virtue of the squaring that is done in the computation). But when the average distance to the mean appears greater (as it does in the first distribution), the standard deviation is likely to follow.

7. The budget for the Women's Center is $1,000,000. If $500,000 is spent on advertising, $150,000 is spent on conferences, and the remainder is spent on long-term securities, construct a circle graph to indicate how the money is spent. Give the central angle for each sector.



Advertising comprises 500,000/1,000,00 = ½ the budget, so should be represented by a 50% (180°) sector.
Conferences eat up 150K of 1000K = 15%, thus should be represented by a .15(360°) = 54° sector.
The remainder (350K)is 35%, so the central angle for its sector must be 35% of 360° = 126°

8. Construct a box-and-whisker plot for the data:
6, 6, 8, 8, 1, 2, 3, 9, 10, 30, 11, 15

We order the data: 1 2 3 6 6 8 8 9 10 11 15 30
The median is 8. The first and third quartile marks are at (3+6)/2 = 4.5 and at 10.5
The Interquartile Range is 10.5 – 4.5 = 6.
The "test distance" for outliers is thus 9. Any value less than 4.5 – 9, or greater than 10.5 + 9, would be considered an outlier. That makes "30" an outlier. NOTE: There will be no outliers on the final.
The Box plot follows:

[No title since data is unidentified. All graphs should have informative titles and appropriate labels.]

9. Write a set of data consistent with the box-and-whisker plot [not shown here]
[Data is shown with a low value of 22, high value 38, and quartile marks 25, 29, and 34.]

There are, of course, many sets of data that would fit these requirements.
Here is one of the simplest and most obvious:

DATA (eight data):	22	24		26	28		30	33		35	38
Quartile marks			25 Q_1			29 Q_2			34 Q_3		

2 | 2 4
2 | 6 8
3 | 0 3
3 | 5 8 Legend: 3 | 0 3 represents values 30 & 33

Note: a title explaining the data is usually required.

Note: It is generally considered inappropriate to present data grouped in only 4 classes, but given the constraints on this problem, anything else would have been a perversion of the intent of graphics, which is to illuminate.

10. The mean age of 5 persons in a room is 30 years. A 36-year-old person walks in. What is the mean age of the persons in the room now?

New mean = sum of all the ages / 6 = (5·30 + 36) / 6 = 186/6 = 31. The new mean age is 31 years.

11. [There are seven test scores. The average of the first four tests was 72. The average of test 4 through 7 was 82. The 4th test score was 84.] What is the average of the seven tests?

The mean is the sum of all the scores divided by 7.

Mean score on the seven tests is (4·72 + 4·82 – 84) / 7 = 76

(You know the total points on the first 4 tests is 4·72 =288, and the last four total 4·82 = 328, but test #4 was included in both groups, so that score must be subtracted out once. Yes, the problem requires careful reading, but that's why it is multiple choice– to make you think again, when your answer doesn't show up!)