CONGRUENCE-101

Two figures are congruent if and only if they are the same size and shape.

Two line segments are congruent if and only if they are the same length.

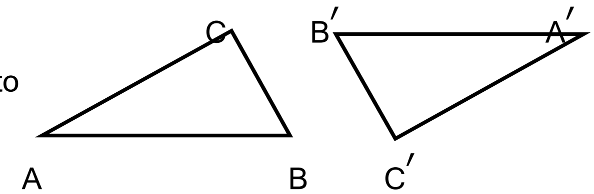
Two angles are congruent if and only if they have the same measure.

Why are triangles used in buildings?

Three theorems regarding triangle congruence inform us:

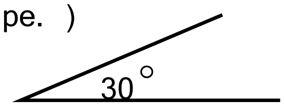
(1) “SSS” (“side-side-side”)

If the three sides of $\triangle ABC$ are congruent to the respective three sides* of $\triangle A'B'C'$ then the two triangles are congruent.



IF $AB \cong A'B'$ & $BC \cong B'C'$ & $AC \cong A'C'$ THEN $\triangle ABC \cong \triangle A'B'C'$

(A $\triangle$ with sides 9', 7' and 4' can be only one size and shape.)

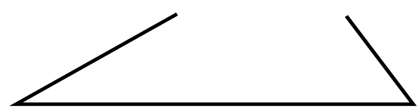


(2) “SAS” (“side-angle-side”)

If two sides and the included angle of $\triangle ABC$ are congruent to the two respective sides and included angle of $\triangle A'B'C'$, then the two triangles are congruent.

IF $AB \cong A'B'$ & $BC \cong B'C'$ & $\angle B \cong \angle B'$ THEN $\triangle ABC \cong \triangle A'B'C'$

(A $\triangle$ with sides 9', 7' and a 30° angle between, can be only one size & shape.)



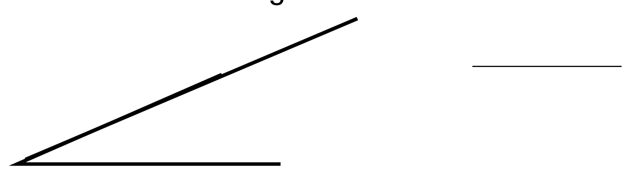
(3) “ASA” (“angle-side-angle”)

If two angles, and the included side, of $\triangle ABC$ are congruent to the two respective angles and included side of $\triangle A'B'C'$, then the two triangles are congruent.

IF $\angle A \cong \angle A'$ & $AB \cong A'B'$ & $\angle B \cong \angle B'$ THEN $\triangle ABC \cong \triangle A'B'C'$

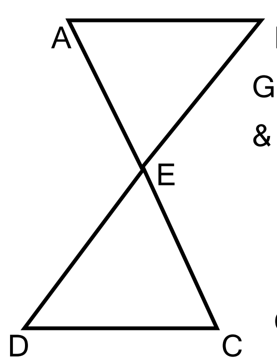
(There’s only one $\triangle$ with a 9' side, and 30° & 50° angles on either end of it.)

(4) “SSA” is NOT a theorem.



(5) What about “AAS”?

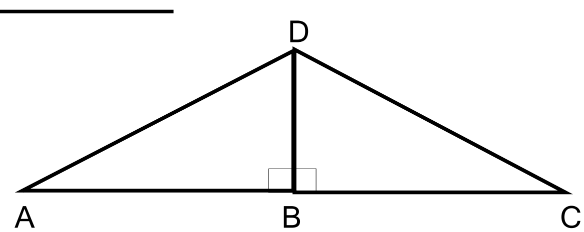
Ex:



Given that lines AC & BD intersect at E, and $AB \cong DC$

Can we be sure $\triangle ABE \cong \triangle CDE$?

...what if AB & DC are parallel ?

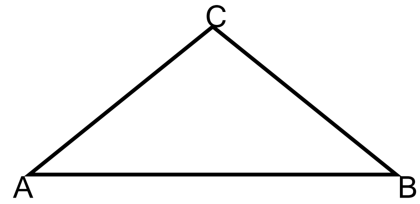


Given what’s shown & $\angle A \cong \angle C$, can we be sure $\triangle ABD \cong \triangle CBD$?

*sides must be named in correct order

CONGRUENCE-102 Some examples:

1. It is well known that the angles opposite the equal sides of an isosceles triangle are congruent.



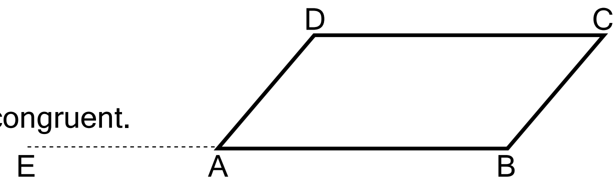
Proof:

Given that AC ≅ BC in △ABC ...

△BAC ≅ △ABC by ...

∠BAC ≅ ∠ABC by ...

2. We can "see" that (i) opposite angles & (ii) opposite sides of a parallelogram are congruent. But how can we know for sure?



(i) Proof: Given ABCD is a parallelogram.

Sides AB & DC are ...

Similarly, sides AD & BC are

Extend AB to EAB

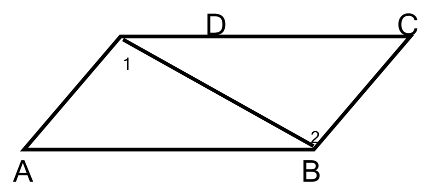
(ii) Since AD || BC, ∠ADB ≅ ∠CBD (∠1≅∠2) by

and since AB || CD, ∠CDB ≅ ∠ABD by

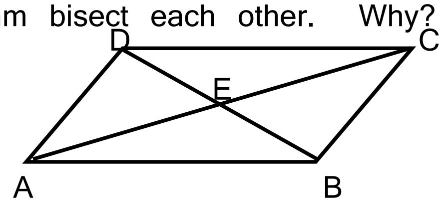
Noting that DB ≅ BD,

we know △ADB ≅ △CBD by

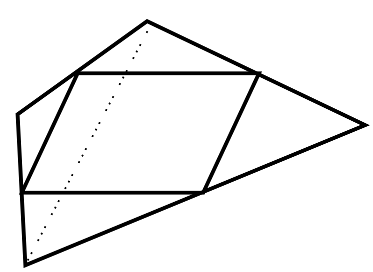
Conclusion:



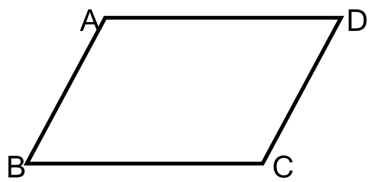
3. It is well known that the diagonals of a parallelogram bisect each other. Why? (Mark all the parts we can show congruent.)



4. Brain teaser: It is less well known that the midpoints of any quadrilateral form vertices of a parallelogram. How can we prove this? We can do so now. However, this will be much easier to prove after we learn about similarity. The dotted line is a hint.



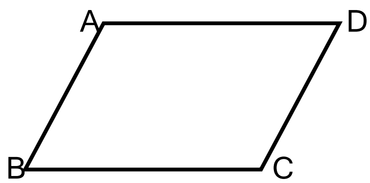
Any quadrilateral with a pair of opposite sides equal & parallel is a parallelogram.



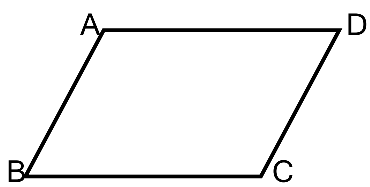
Assume ABCD has AB || CD and AB = CD.

To prove this is a parallelogram, we must show...

Any quadrilateral with two pairs of opposite sides congruent is a parallelogram.

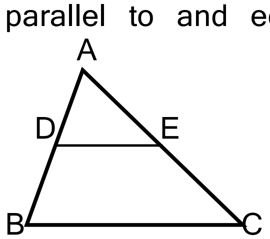


Any quadrilateral with two pairs of opposite angles congruent is a parallelogram.



Midpoint Theorem

The line seqment joining the midpoints of two sides of a triangle is parallel to and equal to half the third side.

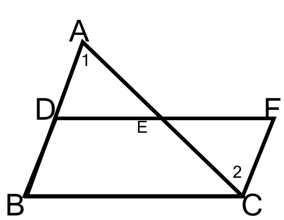


D & E are the midpoints of the sides AB & AC, of △ABC

There is a line through C, ||BA...

The line through DE meets said line at some point, call it F.

We will use this to prove that DE||BC & DE = (½)BC



PF: We first note that $\angle A \cong \angle ECF$ ($\angle 1 \cong \angle 2$) ...since