MATH 310 ♦ 'TEST' Measure ♦ Nov 26, 2008 ♦ ANSWERS + SOLUTIONS + COMMENTS

() 1. Convert each of the following units, showing your work.

a. 0.052 km = __________ mm

$$.052 \text{ km} = .052 \text{ km} \cdot \frac{1000 \text{ m}}{1 \text{ km}} \cdot \frac{1000 \text{ mm}}{1 \text{ m}} = .052 \cdot 1000 \cdot 1000 \text{ mm} = 52000 \text{ mm}$$

b. 250 g = ___________ kg

$$250 \text{ g} = 250 \text{ g} \cdot \frac{1 \text{kg}}{1000 \text{ g}} = \frac{250}{1000} \text{kg} = .250 \text{ kg}$$

c. 0.53 m^3 = ____________ cm^3

$$0.53 \text{ m}^3 = 0.53 \text{ m}^3 \cdot \frac{100 \text{ cm}}{1 \text{ m}} \cdot \frac{100 \text{cm}}{1 \text{ m}} \cdot \frac{100 \text{cm}}{1 \text{ m}} = .53 \cdot 100 \cdot 100 \cdot 100 \text{ cm}^3 = 530{,}000 \text{ cm}^3$$

d. 2500 mL water (at 4°C) = ____________ kg.

$$2500 \text{ mL} = 2500 \text{ mL} \cdot \frac{1 \text{ g}^*}{1 \text{ mL}} \cdot \frac{1 \text{ kg}}{1000 \text{ g}} = \frac{2500}{1000} \text{kg} = 2.5 \text{ kg}$$

* (this equivalence good only for substances which have the same density as water at 4°C)

() 2. Place the following measures in increasing order: 0.5 in 1 m 2 ft 50 cm 2 dm 1.5 yd

.5 in < 2dm < 50 cm < 2 ft < 1 m < 1.5 yd

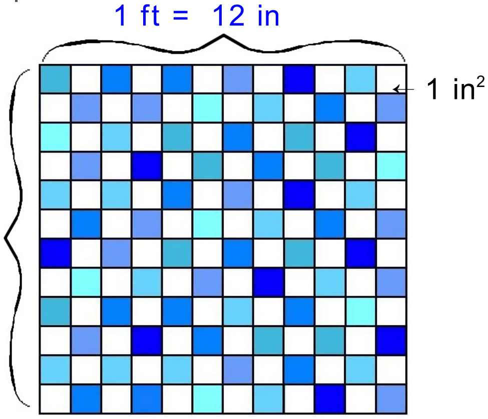
Smallest Largest

Use the fact that 1yd is close to 1 m
to "ballpark" these lengths!
So, for instance, 1m < 1.5 yd
and 50 cm = ½ m < 2 ft
(since 2ft = ⅔yd ≈ ⅔m)

() 3a. Draw a sketch which illustrates the relationship between square inches and square feet.

1 ft = 12 in

1 ft = 12 in



ONE SQUARE FOOT
= 144 square inches !

3b. Which gold leaf is less expensive?
X charges \$20/sq ft; Y charges \$1/sq in
Which should Jean buy? Show why.

To buy 1 square foot at \$20 per ft^2, cost is \$20

To buy one sq ft at \$1 per in^2, cost is:

$$1 \text{ ft}^2 = 1 \text{ ft}^2 \cdot \frac{\$1}{\text{in}^2} \cdot \frac{12\text{in}}{\text{ft}} \cdot \frac{12\text{in}}{\text{ft}} = \$144$$

Notice how dimensional analysis handles ALL units above.

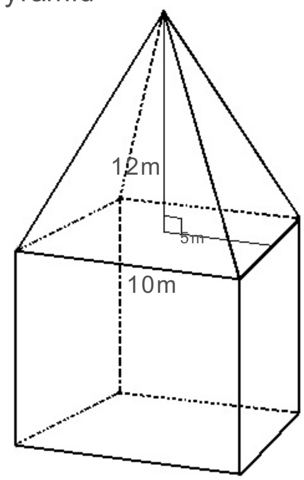
X, at \$20 per square foot, is far less expensive!

Another way to figure this– takes some realization of the relative sizes of in^2 and ft^2:
At \$1/in^2, \$20 buys 20 in^2, a lot less gold leaf than one ft^2.

People often do not *picture a square* when they talk about square feet (or square yards, or square meters, etc.) Unscrupulous retailers can take advantage of that.

Here's another example:
Look at the classroom floor tiles! Each tile covers 1 ft^2– area of a square 1 ft on each side.
As we previously discussed, a square yard is the **area of a square** 1 yard on each side.
It takes NINE of those 1-sq-ft tiles to cover a square yard! (Not THREE !)

() 4. A building has the form of a 10 m cube, topped by a 12 m high square-based pyramid (shown at right). Find the VOLUME of this building. Show your work in a clear, orderly manner.



VOLUME of building = VOLUME OF CUBE + VOLUME OF PYRAMID

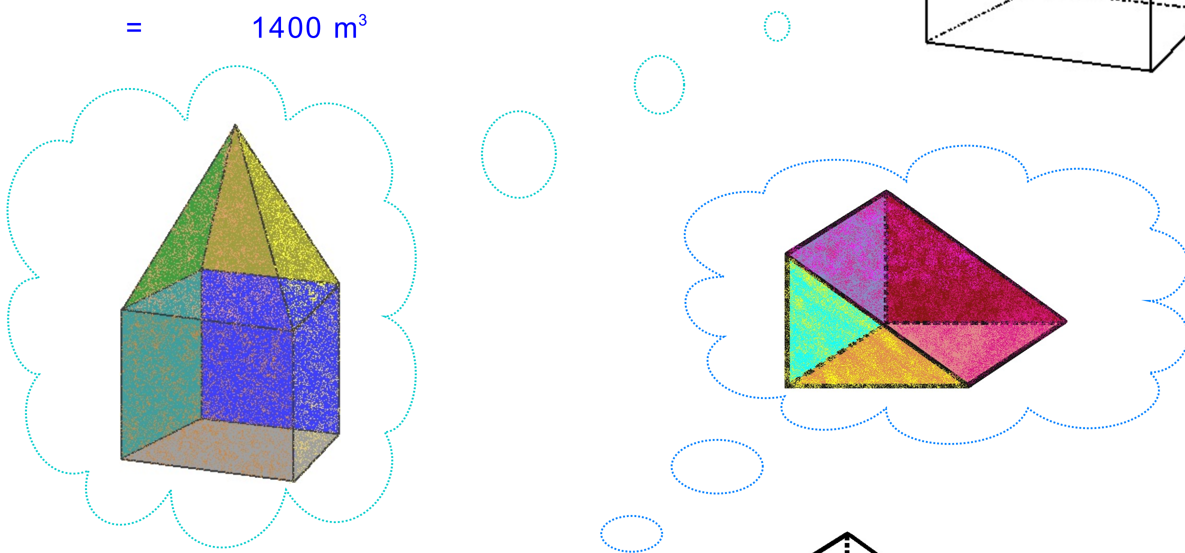
= (length of side)^3 + (⅓) (area of base) (height)

= $(10\text{ m})^3$ + (⅓) $(10\text{ m})^2$ 12m

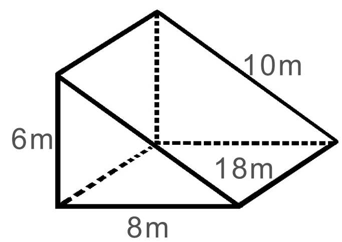
= 1000 m^3 + (⅓) 100m^2 12m

= 1000 m^3 + 400 m^3

= 1400 m^3



() 5. Find the SURFACE AREA of this right triangular prism.



The SURFACE AREA of this prism consists of the areas of the triangular bases, and its three rectangular faces.

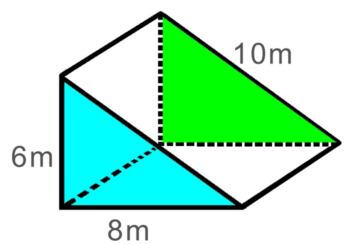
Note the dimensions (6-8-10) show base is a right triangle.

Areas of the two congruent bases add up to

2 · (½) base · height

2 · (½) · 8m · 6m

48 m^2



Areas of the rectangular faces add up to

8m · 18m + 6m · 18m + 10m · 18m

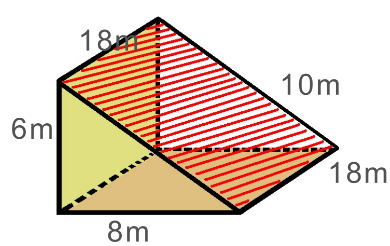
144 m^2 + 108 m^2 + 180 m^2

432 m^2

or (8m + 6m + 10m) · 18m (One long “strip”)

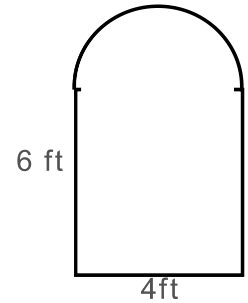
= 24 m · 18 m

= 432 m^2



The total surface area of this prism is 48 m^2 + 432 m^2 = 480 m^2

() 6. Find the PERIMETER and AREA of the demilune* window shown at right.
* (rectangular with a semicircular window on top)



Perimeter = rectangular parts length + length of semicircular arc on top

= 6 ft + 4 ft + 6 ft + (½) π(diameter)

= 16 ft + (½) π (4 ft)

= (16 + 2π) ft

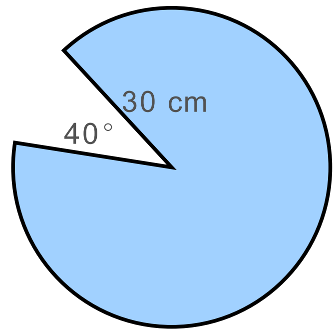
Area = area of rectangular part + area of semicircular top

= (6 ft) (4 ft) + (½) π (2 ft $)^2$

= 24 ft ^{2} + 2 π ft^2

= (24 + 2 π) ft^2

() 7. Find the AREA of the shaded sector of the circle given.



Area of entire circle would be π (radius $)^2$ ... and radius is 30 cm

π (30 cm $)^2$

Area of this sector = (appropriate fraction) of area of entire circle

$$= \frac{320°}{360°} \pi (30 \text{ cm})^2$$

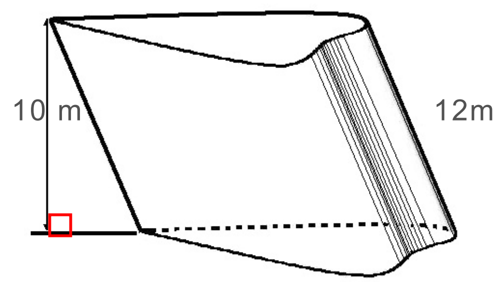
$$= \frac{8}{9} \pi \cdot 900 \text{ cm}^2$$

$$= 800 \pi \text{ cm}^2$$

360° – 40° = 320°

Or... 40/360 = 1/9, so the rest is 8/9 of the circle

() 8. The oblique cylinder shown at right has a base with area 77 m^2 . Find its volume.



Volume of any cylinder = (area of base) (height)

... and height is always perpendicular to the base !

Volume of this cylinder = (77 m^2) (10 m)

= 770 m^3

() 9. Write a formula which gives the volume of a right circular cylinder with base radius r and height h.

Volume = (area of base) (height) = $\pi \cdot r^2 \cdot h$

What happens to the volume of the cylinder if the base radius is doubled and the height is tripled?

F 12 times as great.

New V= $\pi \cdot (2r)^2 \cdot 3h = \pi \cdot 4r^2 \cdot 3h$
= $12 \pi \cdot r^2 \cdot h$
= 12 (original volume)

() 10. Write a formula for the surface area of a sphere in terms of its radius.

$4 \pi r^2$

It's area!

It's round!

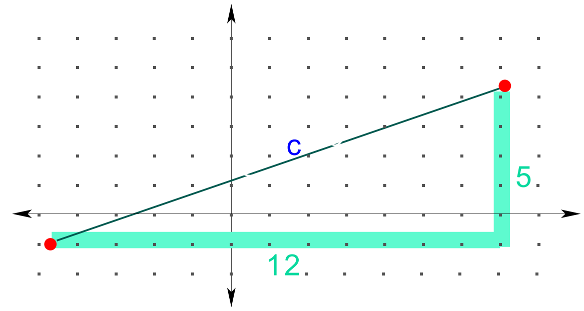
Write a formula for the volume of a sphere in terms of the radius of the sphere.

$\frac{4}{3} \pi r^3$

It's volume!

It's round!

() 11. Find the distance between the points (– 5,– 1) and (7,4).



Distance is length of line drawn; call it “c”.

$$c^2 = 12^2 + 5^2$$

$$c^2 = 169$$

$$c = 13$$

() 12. Find the area of an equilateral triangle whose sides are 2 cm long.

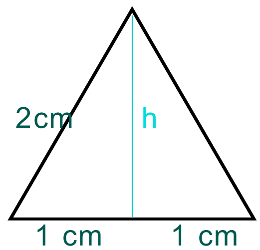
Height of triangle is h where

$$h^2 + (1\text{ cm})^2 = (2\text{ cm})^2$$

$$h^2 = 4\text{ cm}^2 - 1\text{ cm}^2$$

$$h^2 = 3\text{ cm}^2$$

$$h = \sqrt{3}\text{ cm}$$

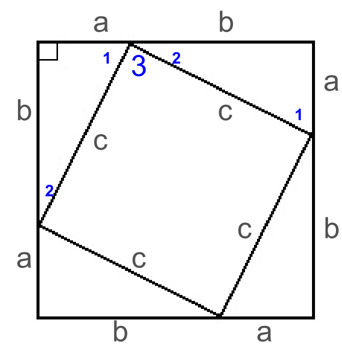


Area of triangle = (½) width · height

$$= (½)\ 2\text{ cm} \cdot \sqrt{3}\text{ cm}$$

$$= \sqrt{3}\text{ cm}^2$$

() 13. Use the sketch at right to prove the pythagorean theorem.
Given– the entire figure is a square, a+ b on each side....
(Hint: area of the entire figure; areas of parts; how do you know the c by c figure in the center is a square?)



Area of the entire (large) square is $(a + b)^2 = a^2 + 2ab + b^2$

Areas of the parts = Areas of the 4 △s + area of inner square*

$$= 4\,(½)\,a\,b + c^2$$

$$= 2\,a\,b + c^2$$

The area must be the same either way it is calculated,
(The whole is the sum of its parts)

so:

$$a^2 + 2ab + b^2 = 2\,a\,b + c^2$$ … subtracting 2ab from each side yields

$$a^2 + b^2 = c^2$$ QED

* As for the question as to whether the c by c by c by c rhombus in the center is really a square:

The four a-b-c triangles are congruent right triangles.
Because they are right triangles, their angles marked 1 and 2 are complementary, thus total 90°.
Therefore the angle marked 3 must be a right angle
(because, as shown at the top of the diagram, angles marked 1 and 2 and 3 together form a line, a 180° angle).