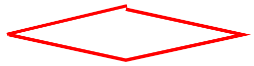
Math 310 ☆ Spring 2007 ☆ Test #2 ☆ 100 pts ☆ NAME▸ ______________________

(9) 1-3. True-False: Write in the blank T for each true statement, F for false.

1. a. __T__ Every square is a rectangle.

   b. __T__ Every rectangle is a trapezoid.

   c. __F__ A rhombus is a square. THIS  rhombus is NOT a square.

2. a. __F__ The least possible number of faces on a polyhedron is five. Tetrahedron has four faces!

   b. __T__ A prism may have eight faces. A hexagonal prism has 8 faces.

   c. __T__ A pyramid may have exactly one face that is a regular pentagon. (See problem 7 !)

3. a. __T__ A quadrilateral may have exactly two right angles. 

   i. __F__ There is only one* triangle with sides of lengths 15 cm, 7 cm, 6cm. 

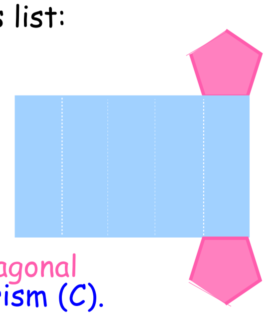
   k. __F__ There is only one* quadrilateral with sides 10cm, 7cm, 6cm, & 8cm long (in that order.)

   *"only one" in the sense that any two figures satisfying these conditions must be congruent.

(12) 4. Multiple choice.

For each statement, choose the BEST completion of the statement from this list:

A tetrahedron	B dodecahedron	C prism	D pyramid	E cube
F octahedron	G icosahedron	H square	J polygon	K cone
L quadrilateral	M parallelogram	N rectangle	O rhombus	P polygon



a. A quadrilateral with all sides congruent is a rhombus (O).

b. A quadrilateral with all angles congruent is a rectangle (N).

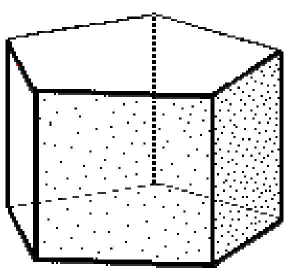
c. The net above right can be folded up into a polyhedron known as a ^ right pentagonal prism (C).

d. A simple closed curve consisting of line segments in a plane is a polygon (J).

e. A surface that encloses space, with a pentagonal base & triangular sides is a pentagonal pyramid (D).

f. The regular polyhedron in which three triangles meet at each vertex is a tetrahedron (A).

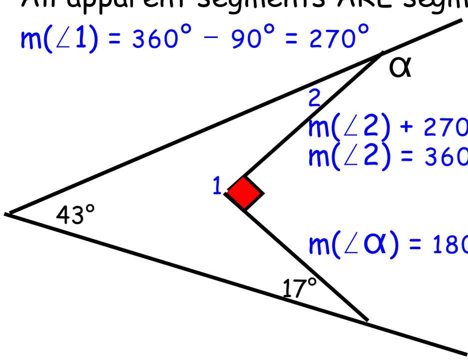
(4) 5. Sketch a prism with fifteen edges.



(6) 6. Find the measure of the angles marked α . (Figures are not drawn exactly to scale!)

All apparent segments ARE segments. Segments $\overline{AB}$ and $\overline{CD}$ are parallel.

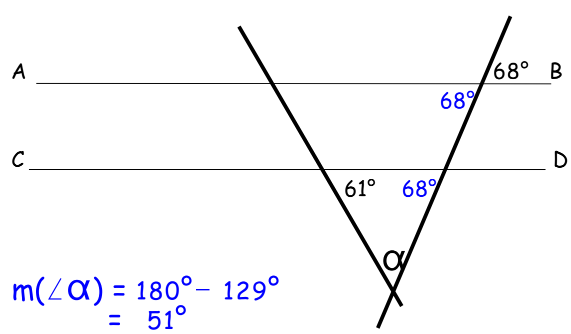
$m(\angle 1) = 360° - 90° = 270°$



$m(\angle 2) + 270° + 43° + 17° = 360°$

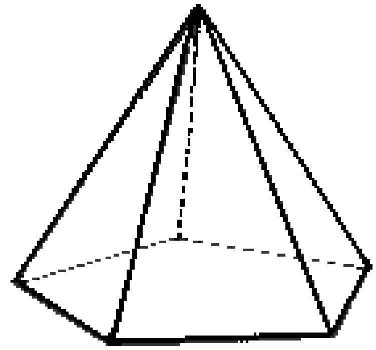
$m(\angle 2) = 360° - 330° = 30°$

$m(\angle \alpha) = 180° - 30° = 150°$



$m(\angle \alpha) = 180° - 129°$
$= 51°$

(8) 7a. Sketch (neatly) a **pyramid** with a pentagonal base.



b. State the number of faces: F= 6

the number of edges: E= 10

and vertices: V= 6

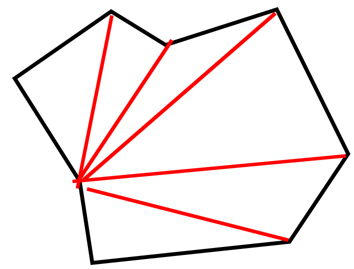
c. Show Euler's formula (section 9.4) holds.

Euler's formula says this must be true:

$$F + V = E + 2$$

The numbers above fit: $6 + 6 = 10 + 2$

(6) 8. Without using a protractor, showing your work, find the SUM of the measures of the interior angles in the polygon at right:



Triangulation of the interior shows the sum of the interior angles of this octagon must be 6(180°) = 1080°

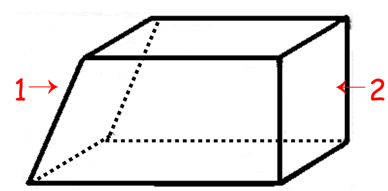
Showing your work, find the measure of one interior angle of a regular ICOSAGON.

An icosagon has 20 sides, so the sum of the measures of the interior angles is 18(180°) = 3240° .

In a regular icosagon, these degrees must be shared equally among all 20 interior angles.

So each must measure 3240°/20 = 162° !

(8) 9. The sketch at right is NOT that of a RIGHT RECTANGULAR PRISM. Explain why not (details), and state the most specific possible name for a polyhedron that could be illustrated with this sketch. (Assume angles that appear right are right, and so forth.)



A prism has two opposite faces as bases, which must be congruent, lie in parallel planes, and have their corresponding edges parallel.

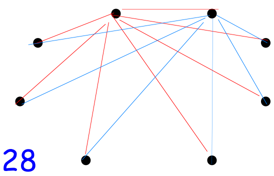
Faces 1 & 2 cannot be bases, as face 1 is longer than face 2, and they are not in parallel planes.

The top and bottom cannot be the bases, as they are not congruent.

The front and back are congruent trapezoids, thus the figure could be a trapezoidal prism.

The faces joining the two trapezoids appear to be rectangular, thus the figure could represent a RIGHT TRAPEZOIDAL PRISM.

(6) 10. Showing your work, find the number of segments connecting eight distinct points, no 3 of which are collinear. (Drawing them is not efficient.)



The first point must be connected to all seven other points (7).
The next point must be connected to only SIX other points (6).
...and so on.

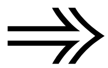
So the total number of segments joining these points is 7+6+5+4+3+2+1+0 = 28

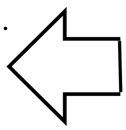
What is the number of diagonals in a convex octagon?

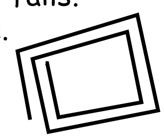
The SEGMENTS counted above that are NOT SIDES of the octagon are diagonals.

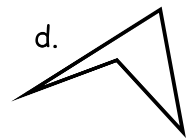
There are 28 - 8 = 20 diagonals in an octagon.

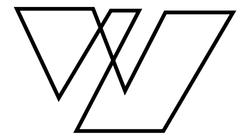
(7) 11. Of the following (curves in parts a through f), which are NOT simple, closed plane curves ? If a figure is not, EXPLAIN in what way it fails.

a. 

b. 

c. 

d. 

e. 

a. NOT

SCPC

c. NOT

SCPC

e. NOT

NOT a CURVE
because
this set of points is
NOT CONNECTED
(You must "lift your pencil"
to draw this set of points.)

Since it is not even a curve,
it cannot be a
simple closed plane curve.

NOT CLOSED

This curve does NOT
"end where it begins".

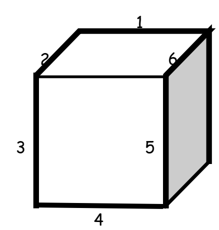
Since it is not closed
it cannot be a
simple closed plane curve.

NOT SIMPLE

This curve crosses
over itself... touches
itself.

Since it is not simple,
it cannot be a
simple closed plane
curve.

f. The six numbered edges of a cube illustrated at right
(Consider the edges numbered 1 - 6 ONLY).



f. NOT in a PLANE

The edges marked 1, 2, and 6 are edges of the top of the cube, while
the edges marked 3, 4, and 5 are on the front face of the cube (from our view).
These edges are in two planes that are perpendicular, not a single plane.

Since the curve is not a plane curve, it cannot be a simple closed plane curve.

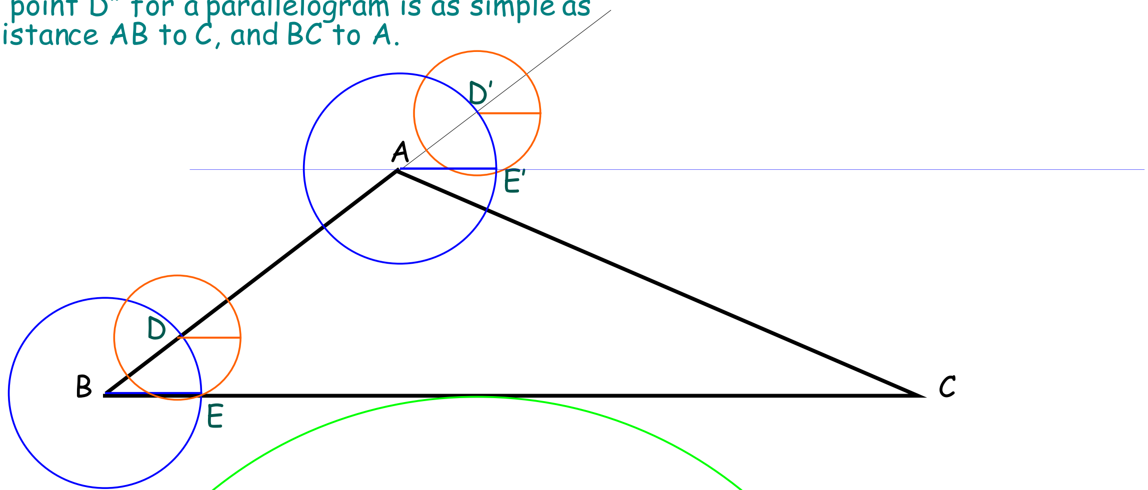
(3) 12. Which of the figures in #11 above are POLYGONS?

A polygon is a simple closed plane curve consisting of line segments.
Therefore, the only figures above that satisfy the first criterion (SCPC) are b and d.
Both b and d consist of line segments, so they meet the second criterion.

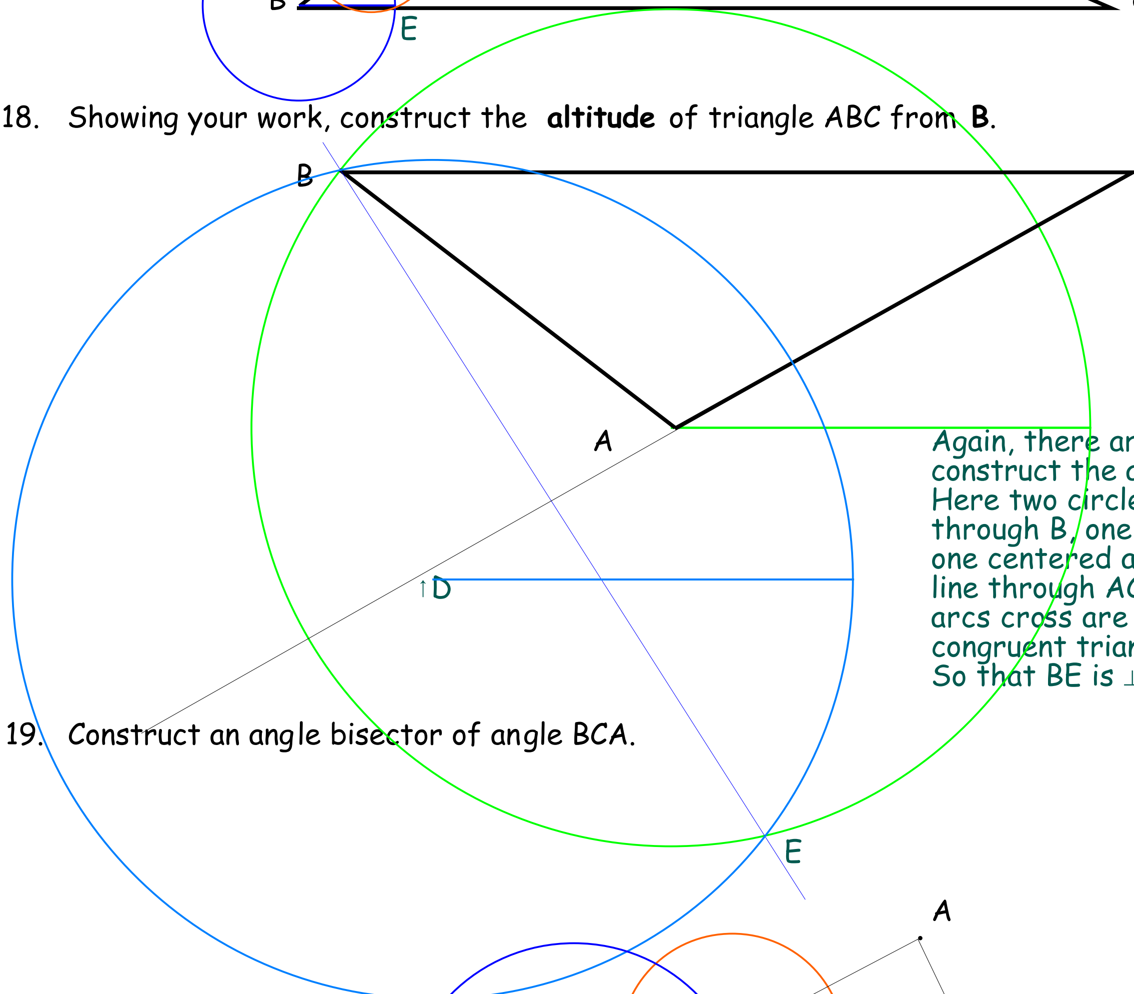
Therefore, the polygons in #11 are b and d.

(3) 17. a. Showing your work, construct a **line** parallel to BC through A.

There are several ways to construct the required line.
One of the most obvious is to copy the angle at B at point A. (Copying angle is basic.)
Another is to create a parallelogram.
Locating “point D” for a parallelogram is as simple as copying distance AB to C, and BC to A.

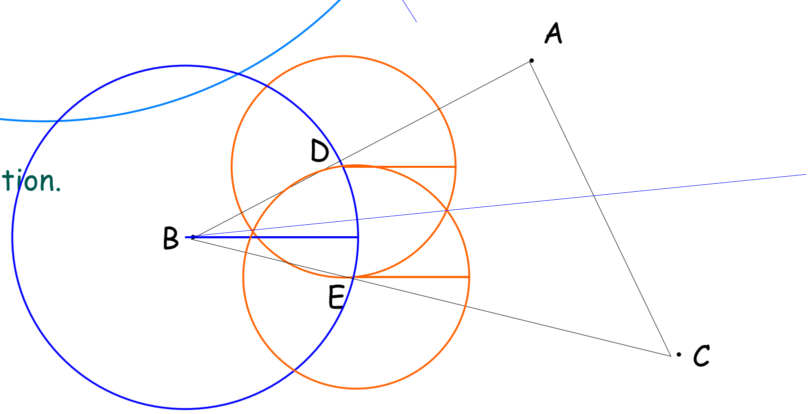


(3) 18. Showing your work, construct the **altitude** of triangle ABC from **B**.



Again, there are many ways to construct the altitude.
Here two circles were constructed through B, one centered at A, and one centered at D, a point on the line through AC. Where those arcs cross are vertices of two congruent triangles, ABE & DBE, So that BE is ⊥ the line AC.

(3) 19. Construct an angle bisector of angle BCA.



Another basic construction.

Note: full-circle arcs were used here, as that is far easier than illustrating partial-circle arcs. but there is no need to draw the full circles in your constructions.
Each circle is illustrated with a radius which shows the location of the center of the arc.
Also note methods shown are not the only methods, as mentioned above!

(3) 13. If $m(\angle\alpha) = 108^\circ\ 38'\ 5''$, what is the measure of α's supplement?

$$\begin{array}{r} 180^\circ \\ -\ 108^\circ\ 38'\ 5'' \end{array} \quad = \quad \begin{array}{r} 179^\circ\ 59'\ 60'' \\ -\ 108^\circ\ 38'\ \ 5'' \end{array} \quad = \quad 71^\circ\ 21'\ 55''$$

(5) 14. If $3x$ and $4x + 6^\circ$ are the measures of a pair of complementary angles, what are their measures, in degrees?

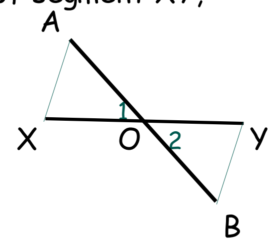
$$\begin{aligned} 3x + 4x + 6^\circ &= 90^\circ \\ 7x + 6^\circ &= 90^\circ \\ 7x &= 84^\circ \\ x &= 12^\circ \end{aligned}$$

So one angle measures $3x = 3 \cdot 12^\circ = 36^\circ$

and the other measures $4x + 6^\circ = 54^\circ$

(Sum = 90°)

(7) 15. Given that point O is the midpoint of segment AB, and also is the midpoint of segment XY, prove that the lines through A& X and through B & Y must be parallel.

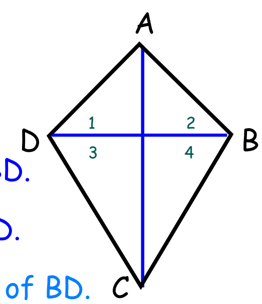


For convenience, $\angle AOX$ is called $\angle 1$ and $\angle BOY$ is $\angle 2$.
We will first show that triangles AOX and BOY are congruent.
$AO \cong BO$ because O is the midpoint of AB.
$XO \cong YO$ because O is the midpoint of XY.
$\angle AOX \cong \angle BOY$ ($\angle 1 \cong \angle 2$) as they are a pair of vertical angles.
$\triangle AOX \cong \triangle BOY$ since two pairs of corresponding sides and the included angle are congruent ("SAS theorem").

Since the triangles are congruent, their corresponding parts are congruent. Therefore: $\angle AXO \cong \angle BYO$.

Since the alternate interior angles $\angle AXO$ and $\angle BYO$ are congruent, the lines through AX and BY must be parallel.

(7) 16. Prove that the diagonals, AC & BD, of kite ABCD are perpendicular.



ABCD is a kite, so $AD \cong AB$ and $CD \cong CB$.

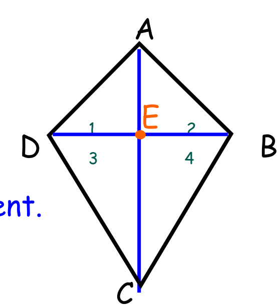
We note again, $AD \cong AB$.
Since A is equidistant from D and B, A lies on the perpendicular bisector of BD.
Similarly, $CD \cong CB$.
Since C is equidistant from D and B, C lies on the perpendicular bisector of BD.

As two points determine a line, A and C determine the perpendicular bisector of BD.

So AC is perpendicular to BD.

Note: Pivotal parts of argument are deep blue,
non-critical statements light blue.

A much longer, but valid, proof:



As above, we note that since ABCD is a kite, $DA \cong BA$ and $DA \cong BA$.

We assume that AC and BD meet at point E.

Triangle DAB is isosceles since $DA \cong BA$. Therefore the opposite angles, $\angle 1$ & $\angle 2$ are congruent.
Similarly, triangle DCB is isosceles, with $DC \cong BC$, and so $\angle 3$ & $\angle 4$ are congruent.
Since $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$, angles D and B are congruent– ie $\angle ADC \cong \angle ABC$.
Thus AD and angle D and side DC are congruent to
AB and angle B and side BC, respectively.
So triangles ADC and ABC are congruent (by side-angle-side theorem).
Corresponding parts of congruent triangles are congruent, so $\angle BAC \cong \angle DAC$.
That makes AE the vertex bisector of isosceles triangle DAB,
which by theorem makes it also the perpendicular bisector of the base DB.

(Alternatively, without using that theorem, we can quickly determine the same result, since triangles ADE and ABE are congruent by SAS.)