1. a. **T** Every square is a rectangle.
   b. **T** Every rectangle is a trapezoid.
   c. **F** A rhombus is a square. THIS rhombus is NOT a square.
   
2. a. **F** The least possible number of faces on a polyhedron is five. Tetrahedron has four faces!
   b. **T** A prism may have eight faces. A hexagonal prism has 8 faces.
   c. **T** A pyramid may have exactly one face that is a regular pentagon. (See problem 7!)

3. a. **T** A quadrilateral may have exactly two right angles.
   i. **F** There is only one* triangle with sides of lengths 15 cm, 7 cm, 6 cm.
   k. **F** There is only one* quadrilateral with sides 10 cm, 7 cm, 6 cm, & 8 cm long (in that order.)
   *"only one" in the sense that any two figures satisfying these conditions must be congruent.

4. Multiple choice.
   For each statement, choose the BEST completion of the statement from this list:
   A tetrahedron  B dodecahedron  C prism  D pyramid  E cube
   F octahedron  G icosahedron  H square  J polygon  K cone
   L quadrilateral  M parallelogram  N rectangle  O rhombus  P polygon
   
   a. A quadrilateral with all sides congruent is a rhombus (O).
   b. A quadrilateral with all angles congruent is a rectangle (N).
   c. The net above right can be folded up into a polyhedron known as a right pentagonal prism (C).
   d. A simple closed curve consisting of line segments in a plane is a polygon (J).
   e. A surface that encloses space, with a pentagonal base & triangular sides is a pyramid (D).
   f. The regular polyhedron in which three triangles meet at each vertex is a tetrahedron (A).

5. Sketch a prism with fifteen edges.

6. Find the measure of the angles marked $\alpha$. (Figures are not drawn exactly to scale!)
   All apparent segments ARE segments.
   
   - $m(\angle 1) = 360^\circ - 90^\circ = 270^\circ$
   - $m(\angle 2) + 270^\circ + 43^\circ + 17^\circ = 360^\circ$
   - $m(\angle 2) = 360^\circ - 330^\circ = 30^\circ$
   - $m(\angle \alpha) = 180^\circ - 30^\circ = 150^\circ$

   Segments $\overline{AB}$ and $\overline{CD}$ are parallel.
   
   - $m(\angle \alpha) = 180^\circ - 129^\circ = 51^\circ$
(8) 7a. Sketch (neatly) a pyramid with a pentagonal base.

b. State the number of faces:   F = 6

the number of edges:   E = 10

and vertices:   V = 6

c. Show Euler’s formula (section 9.4) holds.

Euler’s formula says this must be true:

\[ F + V = E + 2 \]

The numbers above fit:  \[ 6 + 6 = 10 + 2 \]

(6) 8. Without using a protractor, showing your work, find the sum of the measures of the interior angles in the polygon at right:

Triangulation of the interior shows the sum of the interior angles of this octagon must be \( 6(180°) = 1080° \)

Showing your work, find the measure of one interior angle of a regular Icosagon.

An icosagon has 20 sides, so the sum of the measures of the interior angles is \( 18(180°) = 3240° \).

In a regular icosagon, these degrees must be shared equally among all 20 interior angles.

So each must measure \( 3240°/20 = 162° \)!

(8) 9. The sketch at right is NOT that of a RIGHT RECTANGULAR PRISM. Explain why not (details), and state the most specific possible name for a polyhedron that could be illustrated with this sketch. (Assume angles that appear right are right, and so forth.)

A prism has two opposite faces as bases, which must be congruent, lie in parallel planes, and have their corresponding edges parallel.

Faces 1 & 2 cannot be bases, as face 1 is longer than face 2, and they are not in parallel planes.

The top and bottom cannot be the bases, as they are not congruent.

The front and back are congruent trapezoids, thus the figure could be a trapezoidal prism.

The faces joining the two trapezoids appear to be rectangular, thus the figure could represent a RIGHT TRAPEZOIDAL PRISM.
10. Showing your work, find the number of segments connecting eight distinct points, no 3 of which are collinear. (Drawing them is not efficient.)

The first point must be connected to all seven other points (7).
The next point must be connected to only SIX other points (6).
...and so on.

So the total number of segments joining these points is $7 + 6 + 5 + 4 + 3 + 2 + 1 + 0 = 28$

What is the number of diagonals in a convex octagon?
The segments counted above that are not sides of the octagon are diagonals.

There are $28 - 8 = 20$ diagonals in an octagon.

11. Of the following (curves in parts a through f), which are NOT simple, closed plane curves? If a figure is not, EXPLAIN in what way it fails.

a. NOT SCPC
b. NOT SCPC
c. NOT SCPC
d. NOT SCPC
e. NOT SCPC
f. NOT SCPC

NOT a CURVE because this set of points is NOT CONNECTED
(You must "lift your pencil" to draw this set of points.)

Since it is not even a curve, it cannot be a simple closed plane curve.

NOT CLOSED
This curve does NOT "end where it begins".

Since it is not closed, it cannot be a simple closed plane curve.

NOT SIMPLE
This curve crosses over itself... touches itself.

Since it is not simple, it cannot be a simple closed plane curve.

The six numbered edges of a cube illustrated at right (Consider the edges numbered 1-6 ONLY).

f. NOT in a PLANE

The edges marked 1, 2, and 6 are edges of the top of the cube, while the edges marked 3, 4, and 5 are on the front face of the cube (from our view). These edges are in two planes that are perpendicular, not a single plane.

Since the curve is not a plane curve, it cannot be a simple closed plane curve.

12. Which of the figures in #11 above are POLYGONS?

A polygon is a simple closed plane curve consisting of line segments.
Therefore, the only figures above that satisfy the first criterion (SCPC) are b and d.
Both b and d consist of line segments, so they meet the second criterion.

Therefore, the polygons in #11 are b and d.
17. a. Showing your work, construct a line parallel to BC through A.

There are several ways to construct the required line. One of the most obvious is to copy the angle at B at point A. (Copying angle is basic.) Another is to create a parallelogram. Locating "point D" for a parallelogram is as simple as copying distance AB to C, and BC to A.

18. Showing your work, construct the altitude of triangle ABC from B.

Again, there are many ways to construct the altitude. Here two circles were constructed through B, one centered at A, and one centered at D, a point on the line through AC. Where those arcs cross are vertices of two congruent triangles, ABE & DBE, so that BE is \( \perp \) the line AC.

19. Construct an angle bisector of angle BCA.

Another basic construction.

Note: full-circle arcs were used here, as that is far easier than illustrating partial-circle arcs. But there is no need to draw the full circles in your constructions. Each circle is illustrated with a radius which shows the location of the center of the arc. Also note methods shown are not the only methods, as mentioned above!
13. If $m(\alpha) = 108^\circ 38' 5''$, what is the measure of $\alpha$’s supplement?

$$180^\circ = 179^\circ 59' 60'' = 71^\circ 21' 55''$$

14. If $3x$ and $4x + 6^\circ$ are the measures of a pair of complementary angles, what are their measures, in degrees?

$$3x + 4x + 6^\circ = 90^\circ \quad \text{so one angle measures } 3x = 3*12^\circ = 36^\circ$$

$$7x + 6^\circ = 90^\circ \quad \text{and the other measures } 4x + 6^\circ = 54^\circ$$

$$x = 12^\circ \quad (\text{Sum} = 90^\circ)$$

15. Given that point $O$ is the midpoint of segment $AB$, and also is the midpoint of segment $XY$, prove that the lines through $A$ & $X$ and through $B$ & $Y$ must be parallel.

For convenience, $\angle AOX$ is called $\angle 1$ and $\angle BOY$ is $\angle 2$.

We will first show that triangles $AOX$ and $BOY$ are congruent.

$AO = BO$ because $O$ is the midpoint of $AB$.

$XO = YO$ because $O$ is the midpoint of $XY$.

$\angle AOX = \angle BOY$ ($\angle 1 = \angle 2$) as they are a pair of vertical angles.

$\triangle AOX \cong \triangle BOY$ since two pairs of corresponding sides and the included angle are congruent ("SAS theorem").

Since the triangles are congruent, their corresponding parts are congruent. Therefore:

$\angle AXO \cong \angle BYO$.

Since the alternate interior angles $\angle AXO$ and $\angle BYO$ are congruent, the lines through $AX$ and $BY$ must be parallel.

16. Prove that the diagonals, $AC$ & $BD$, of kite $ABCD$ are perpendicular.

$ABCD$ is a kite, so $AD \parallel AB$ and $CD \parallel CB$.

We note again, $AD \parallel AB$.

Since $A$ is equidistant from $D$ and $B$, $A$ lies on the perpendicular bisector of $BD$.

Similarly, $CD \parallel CB$.

Since $C$ is equidistant from $D$ and $B$, $C$ lies on the perpendicular bisector of $BD$.

As two points determine a line, $A$ and $C$ determine the perpendicular bisector of $BD$.

So $AC$ is perpendicular to $BD$.

A much longer, but valid, proof:

As above, we note that since $ABCD$ is a kite, $DA \parallel BA$ and $DA \parallel BA$.

We assume that $AC$ and $BD$ meet at point $E$.

Triangle $DAB$ is isosceles since $DA \parallel BA$. Therefore the opposite angles, $\angle 1$ & $\angle 2$ are congruent.

Similarly, triangle $DCB$ is isosceles, with $DC \parallel BC$, and so $\angle 3$ & $\angle 4$ are congruent.

Since $\angle 1 \equiv \angle 2$ and $\angle 3 \equiv \angle 4$, angles $D$ and $B$ are congruent--ie $\angle ADC \equiv \angle ABC$.

Thus $AD$ and angle $D$ and side $DC$ are congruent to $AB$ and angle $B$ and side $BC$, respectively.

So triangles $ADC$ and $ABC$ are congruent (by side-angle-side theorem).

Corresponding parts of congruent triangles are congruent, so $\angle DAC \equiv \angle DAC$.

That makes $AE$ the vertex bisector of isosceles triangle $DAB$.

Then by theorem makes it also the perpendicular bisector of the base $DB$.

(Alternatively, without using that theorem, we can quickly determine the same result, since triangles $ADE$ and $ABE$ are congruent by $SAS$.)