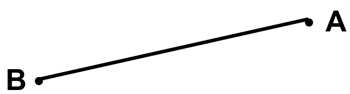
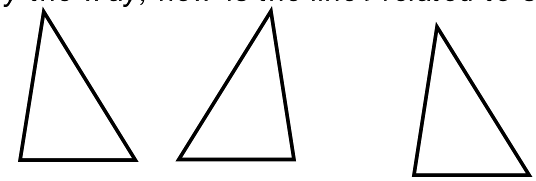
TG0-M **Mira Activities: Using the mira***

The mira is a *mirror* through which you can see (to the other side of the mirror).
This allows you to perceive the reflected image in the mira as actually being on the other side.

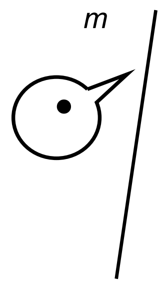
MA1. Place the mira between points A & B so that the image of AB is on itself, and B's image is on A.
The mira must be perpendicular to the paper, with its beveled edge at the bottom, facing you.
Trace the line at the beveled edge. This is the mirror line "ℓ".
The reflection of A through ℓ is B. (*by the way, how is the line ℓ related to segment AB?*)





MA2. Place the mira between the 1st & 2nd triangles above. *Experiment with the placement of the mira.*
Is one the reflection of the other?
What about the second and the third?

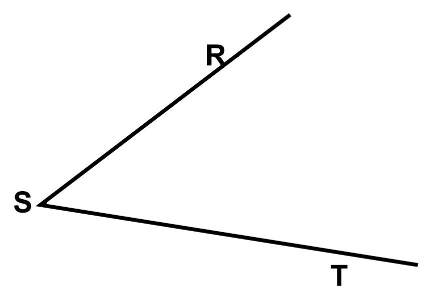
MA3. The mira can be used for drawing reflected images.
Place the mira line on the mirror line *m*, and trace
the image of the figure *F* reflected through *m*.

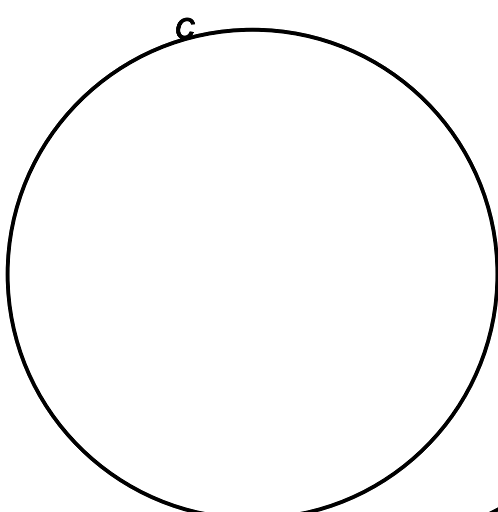


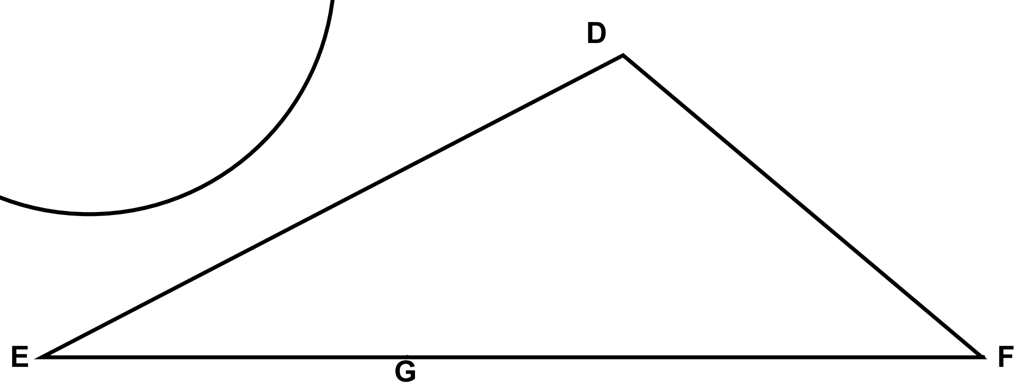
Many of the constructions we have done with compass &
straightedge involved bisectors and perpendicular lines.
These constructions can be very simple with the mira.

USE THE MIRA TO FIND:

MA4. the perpendicular bisector of the line segment **PQ**.
MA5. the bisector of the angle RST.
MA6. the center *O* of the circle *C*.
MA7. the altitude of triangle DEF passing through D.







A **translation** of the plane is one kind of isometry– in which the **vector** (directed line segment, or *arrow*) joining each point to its image is *constant* (*the same*).
In effect, all points *slide* per *one* vector (which our text calls the "slide arrow").

After translation, the image of a figure in the plane is congruent to the original. Notice since every point moves the same way, *the original & image face the same direction* in the plane.

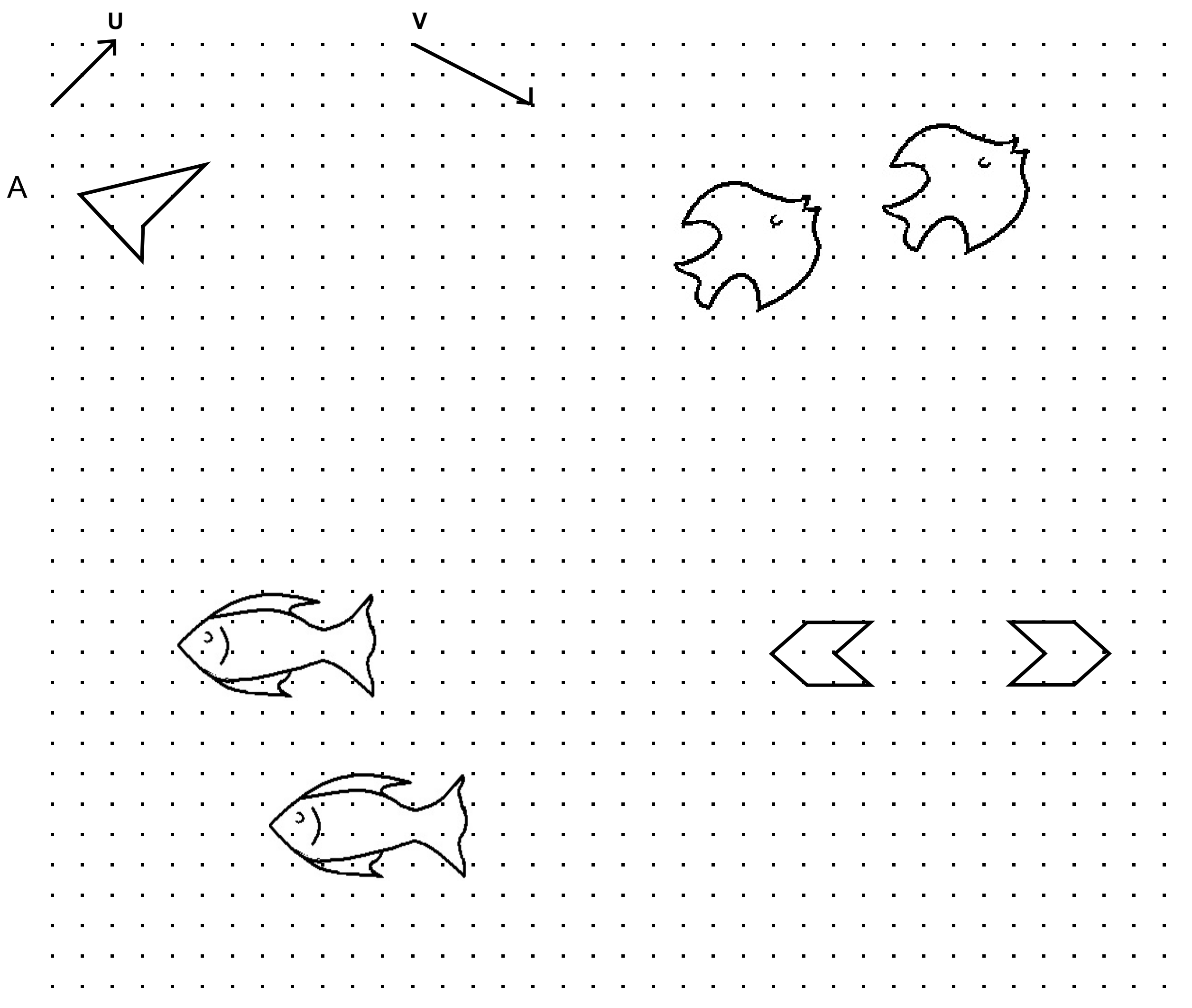
T1. Find the image of figure "**A**" under the translation indicated by vector "U".
Label the image **A'**.
Hint: Locate key points, and see where the translation vector moves each one.

T2*. Next translate the figure **A'** (*NOT* A) by the vector "V", to figure **A''**.

T3*. You have just found the image of A under the **composition** of the two translations, U *followed by* V. [The notation for this composition is V◦U.]

✱ What transformation of the plane would move figure **A** directly to **A''**?

T4. Can you find a translation of the plane (ie find the *vector* of translation), that transforms figure **B** to **B'**? **C** to **C'**? **D** to **D'**?



A **reflection** of the plane through the line ℓ is an isometry in which ℓ is a perpendicular bisector of each segment connecting a point with its image. *A reflection "FLIPS" the plane about the line* ℓ. The points on ℓ *are fixed (do not move when the plane is reflected through* ℓ).

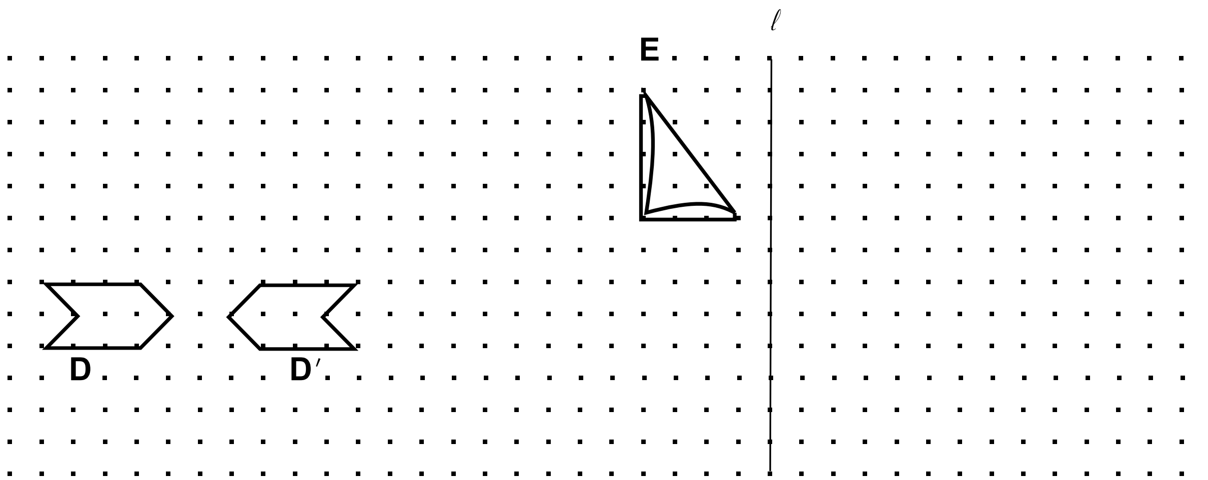
M1. Drawing the perpendicular bisector of one such segment suffices to determine the line of reflection, ℓ.
Find the line of reflection for the isometry taking D to D'.

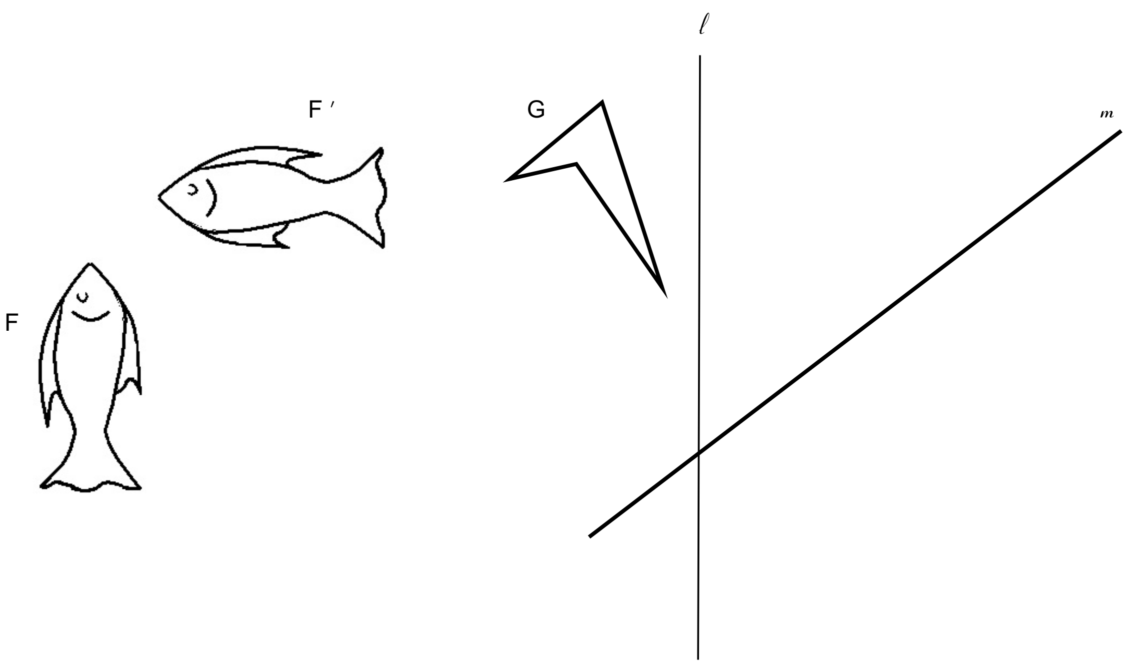
M2. Draw the reflection of figure E through the line ℓ. (*Note: dots are not aligned in squares.*)

M3*. Draw a figure on a piece of clean translucent paper. a. Draw a *separate* line. Use paper folding to draw the reflection of the figure through ℓ. b. Draw a line *through* the figure and find the reflection.

M4*. How can you use paper folding to find the line of reflection between figures D and D'? Between F and F'?

M5*. Reflect figure G through line *l* (call the image G'). Reflect G' through line *m* to G''.
Is G'' congruent to the original, G?
What isometry of the plane would move G to G'' directly? (Can this be generalized?)





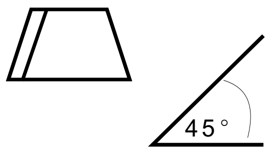
Select any three points on fig F, mark A,B,C.
Locate corresponding points on fig F′, label A′, B′, C′.
Circling from A to B to C goes ______________-wise
Circling from A′ to B′ to C′ runs ________________-wise.

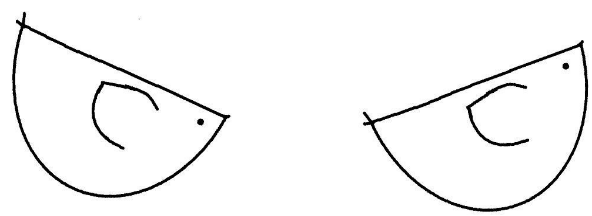
A **rotation** of the plane is an isometry which fixes one point, *O*, (the center of rotation); all other points move so that the point and its image are equidistant from *O*, and segments from the point to *O* and from *O* to the image, form constant (congruent) angles.
(A rotation "turns" or "pivots" the plane, through some angle, around some fixed point.)

R1. Use tracing paper to rotate the given trapezoid 45° about the point *O*.

R2*. Roughly estimate the center of rotation between the "birds". Connect two corresponding points on the birds. Notice the triangle formed with *O*. What kind of triangle? Where is *O* with respect to the line segment connecting the two points? connect another pair of points. How can we pinpoint *O*?

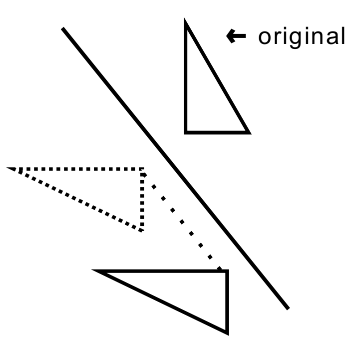
R3*. Find (construct) the center and angle of rotation for the boomerang figures in M5.



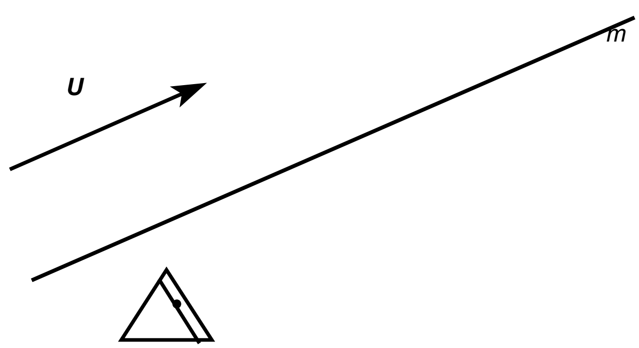


The fourth, and last, type of isometry of the plane is called a **glide reflection**– which reflects the plane through a line, then moves the plane in a direction parallel to the line of reflection.
(This isometry is a unique transformation in its own right, but may *be thought of as the composition of a translation and a reflection through a line parallel to the translation vector.)*

G1*. Consider the example below.
How does the transformation differ from translation? from rotation? from simple reflection?

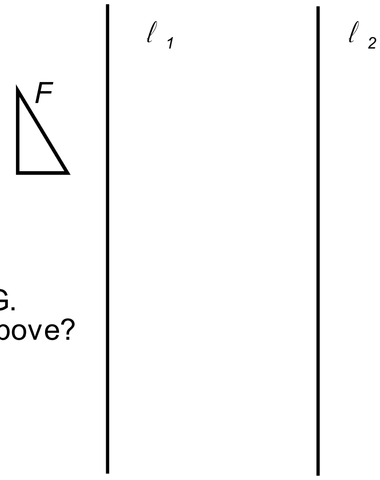


G2*. Given glide vector *U*, & line of reflection *m*, find the image of the figure under the corresponding glide reflection.

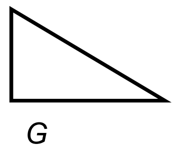


NOTE: We are not covering compositions

C1. ℓ_1 and ℓ_2 are parallel.
Reflect figure F through ℓ_1 to F'.
Reflect the image F' through ℓ_2 to F''.
What transformation moves F <u>directly</u> to F''?



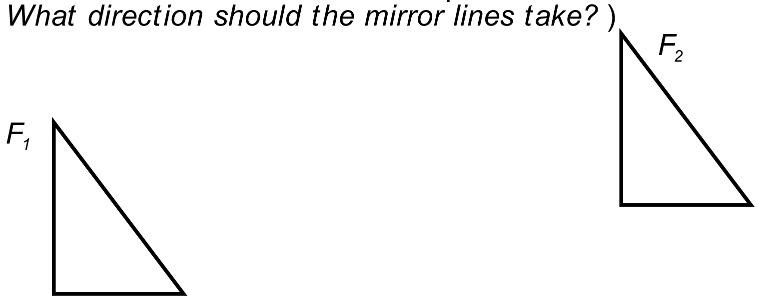
C2. Repeat the activities of exercise 1 for figure G.
Do your observations match the conclusion above?



C3. In M5, two reflections caused a rotation. What's different here?

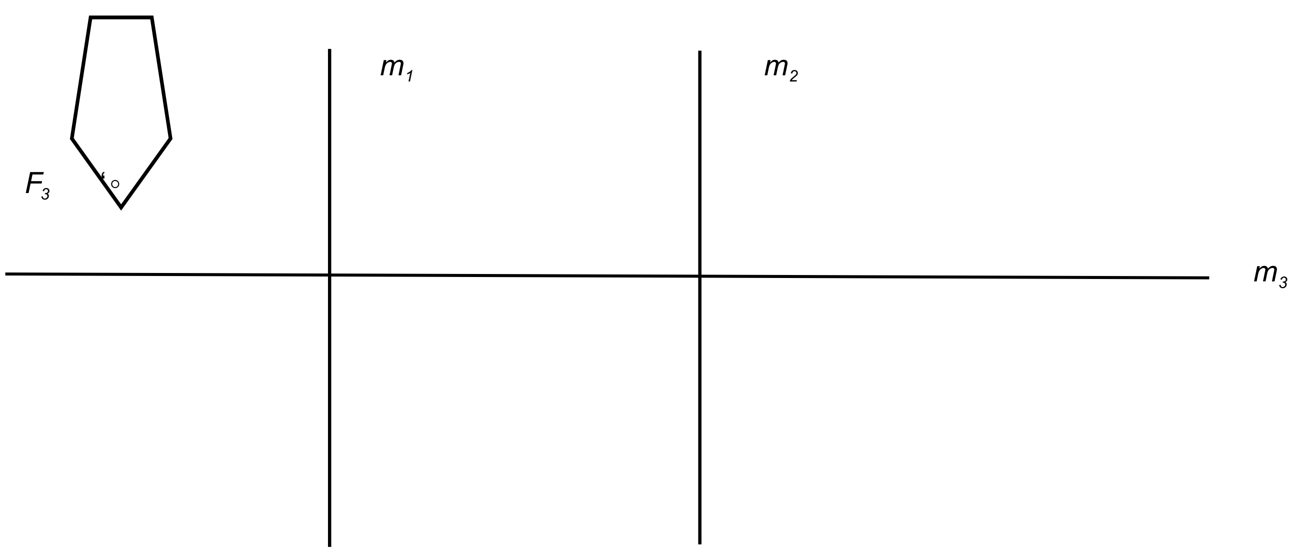
[*Two reflections through non-parallel lines cause a rotation, with center at the intersection, angle measure twice that of the angle between. Two reflections through parallel lines result in a translation, perpendicular to the two lines, twice the distance between.*)

C4. Find two reflections which accomplish the translation that moves F_1 to F_2:
(*What direction should the mirror lines take?*)



C5. Note that m_1 and m_2 are parallel, and m_3 is perpendicular to m_1 and m_2.
Reflect F_3 through m_1 to F_3', then reflect F_3' through m_2 to F_3'', then reflect F_3'' through m_3 to F_3'''.

What transformation moves F_3 to F_3''' directly?
Why is this not surprising?



A SERIES OF REFLECTIONS CAN BE USED TO DUPLICATE A TRANSLATION, A ROTATION, OR A GLIDE REFLECTION.

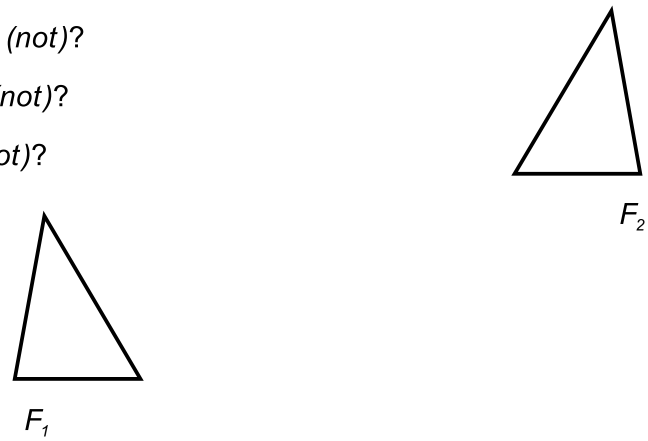
C6. Given $F_1 \cong F_2$… what transformation moves F_1 to F_2 ?

Could this be a translation? *Why (not)*?

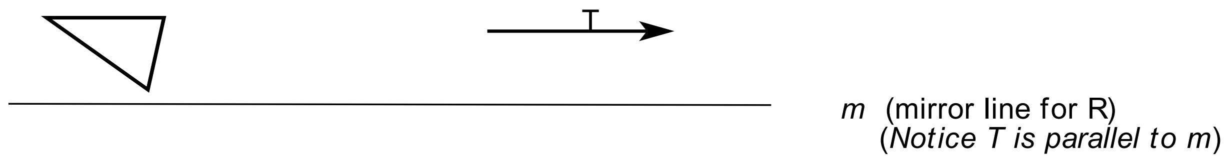
Could this be a reflection? *Why (not)*?

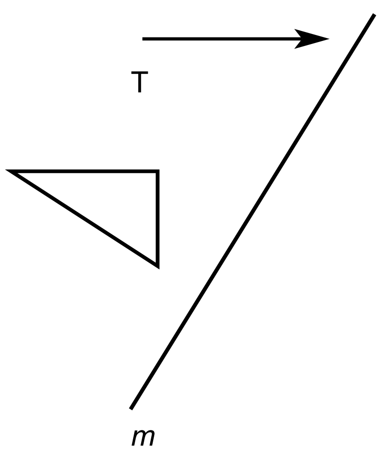
Could this be a rotation? *Why (not)*?

This leaves only one possibility:
(verify it)



C7. If T is a translation, and R a reflection, how does T∘R compare with R∘T? (Let *V* be the translation vector, and l the line of reflection; investigate.)





In general, are compositions of transformations commutative?

C8. a. Can every type of transformation be accomplished by a series of translations? Why? (*Does a translation ever change the orientation of an object?*)

b. Same question, for rotations.

c. Same question, for reflections.