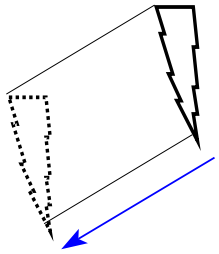
A **TRANSFORMATION** is a one-to-one mapping of the points of the plane to new points of the same plane.

An **ISOMETRY**, also called a "**rigid motion**", is a transformation which preserves distances. Preserving all distances preserves figures (think of triangles).

There are *only* four types of isometries of the plane:

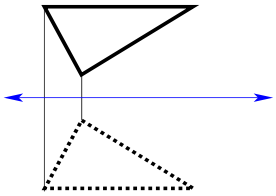
Translation	("Slide")
Reflection	("Flip")
Rotation	("Turn")
Glide reflection	("Flip'nSlide")

TRANSLATION



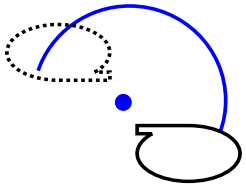
- Determined by a vector (an arrow with specific length and direction)
- Moves all points of the plane in one direction, the same distance... determined by the "slide arrow" or vector of the translation.
- Since all points move the same direction, points move on parallel straight paths.

REFLECTION



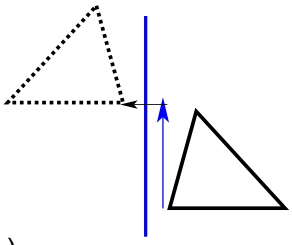
- All points of the plane, except those on the line of reflection, move across the line of reflection; points equally distant from the line of reflection, but on opposite sides, essentially swap places.
- The reflection line is the $\perp$ bisector of the segment joining a point and its image.
- Clockwise vs counter-clockwise sense/orientation reverses (ie figures "flip").

ROTATION



- Determined by a center and directed angle of rotation
- Every point in the plane, except the center of rotation, moves on a circular path around the center of rotation, through the same angle.
- The center of rotation stays fixed.

GLIDE-REFLECTION



- Determined by a line of reflection and vector parallel to the line.
- All points of the plane flip across the line of reflection, then "glide".
- No point stays fixed.
- The reflection line *contains the midpoints* between points and their images.
- Clockwise vs counter-clockwise sense (orientation) reverses. (i.e. figures "flip".)

Details:
✔To determine what isometry moved Figure 1 to Figure 2, find at least three pairs of matching points, and name them, e.g. ABC and A'B'C'.
✔Check the orientation of the figure & image. ✔ If path ABC is clockwise & A'B'C' is counter-clockwise, then image reversed, and the isometry must be a Reflection or Glide-Reflection. In that case, draw arrows from A to A', B to B'. If they are parallel, the isometry is a reflection. ✔If the image is not reversed, then the isometry is a translation or rotation. In that case, draw arrows from A to A', B to B'. If they are the same length & direction, the isometry is a translation.

To find the:
- Vector of a Translation, draw an arrow from a point A to its image, A'.
- Line of a Reflection, construct/find the perpendicular bisector of segment A to its image, A'.
- Center of a Rotation, use perpendicular bisectors of AA' and BB' to locate the center !
- Angle of Rotation, draw AOA' after locating the center, "O".
- Line of Reflection of a Glide-Reflection: passes through the midpoints of two segments, AA' and BB'.

- Composition...
- of two translations is another translation
- of two rotations is another rotation...
- of two reflections may be a rotation (if lines intersect) or a translation (if lines are ||).
- of two glide-reflections may be a rotation... or translation (if lines are ||).

PS: Another type of transformation preserves shapes, **not** distances: Dilation, creates similar figures. (See "Extended Notes– on Dilations" online on Notes page, for information about dilations.)