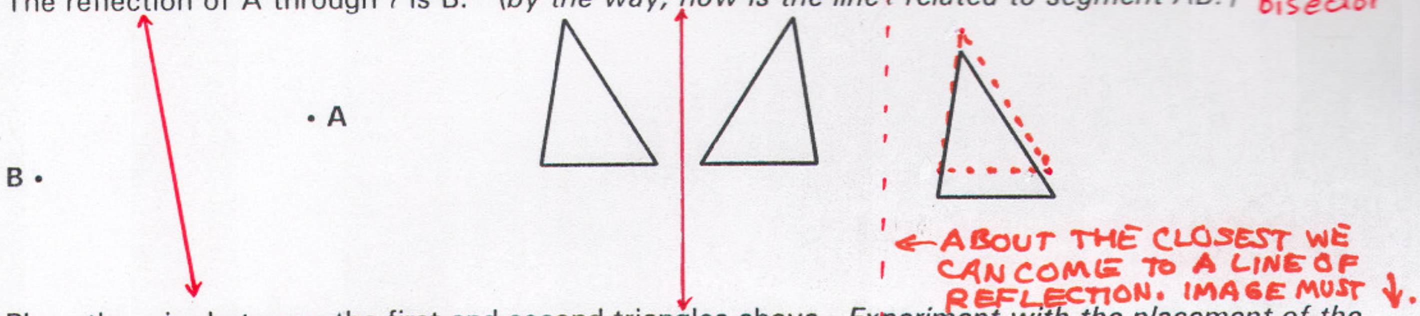
TG-0 **Mira Activities: Using the mira***

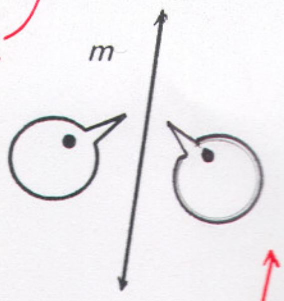
The mira is a *mirror* through which you can see (to the other side of the mirror).
This allows you to perceive the reflected image in the mira as actually being on the other side.

MA1. Place the mira between points A & B so that the image of A overlaps B.
The mira must be perpendicular to the page with bevelled side at the bottom, towards you.
Trace the line at the bevelled edge. This is the mirror line "ℓ".
The reflection of A through ℓ is B. *(by the way, how is the line ℓ related to segment AB?)* perpendicular bisector

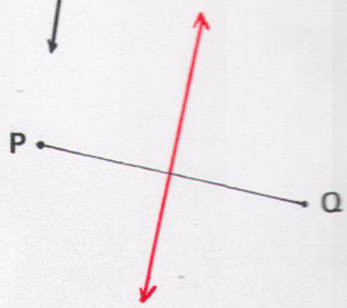


MA2. Place the mira between the first and second triangles above. *Experiment with the placement of the mira.*
Is one the reflection of the other? YES.
What about the second and the third? THIS CAN'T BE DONE.

MA3. The mira can be used for drawing reflected images.
Place the mira line on the mirror line *m*, and trace the image of the figure *F* reflected through *m*.

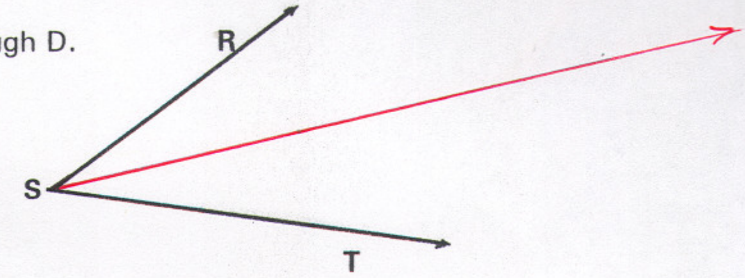


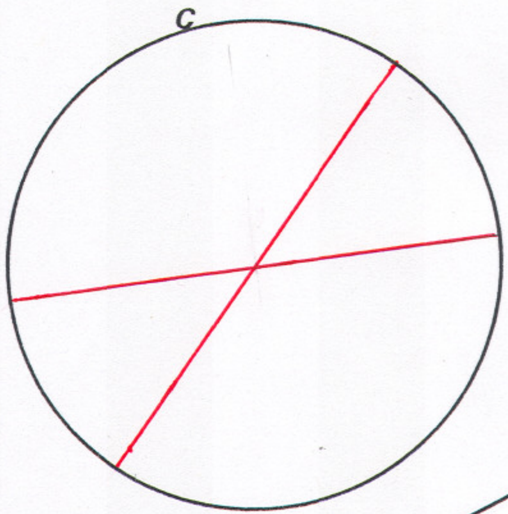
Many of the constructions we have done with compass & straightedge involved bisectors and perpendicular lines. These constructions can be very simple with the mira.

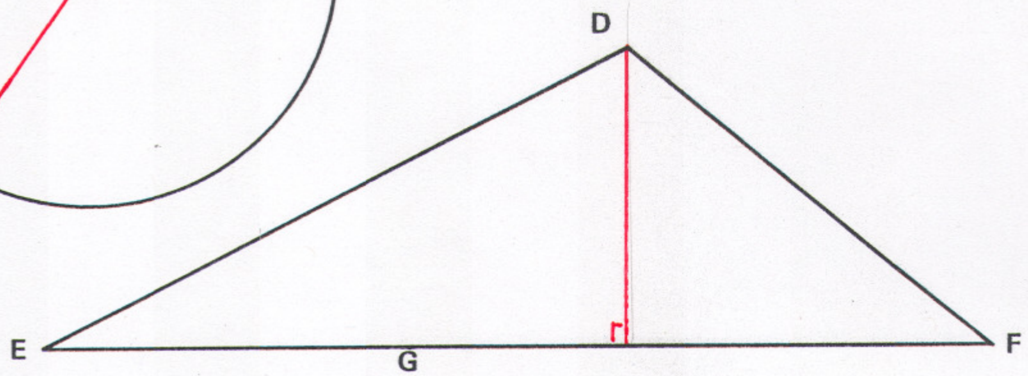


USE THE MIRA TO FIND:

MA4. the perpendicular bisector of the line segment PQ.
MA5. the bisector of the angle RST.
MA6. the center *O* of the circle *C*.
MA7. the altitude of triangle DEF passing through D.







Keep in mind:

> A **translation** of the plane is one kind of isometry– in which the **vector** (directed line segment) joining each point to its image is constant.
> In effect, all points *slide* per *one* vector (which our text calls the "slide arrow").
>
> After translation, the image of a figure in the plane is congruent to the original. Notice since every point moves the same way, *the original & image face the same direction* in the plane.

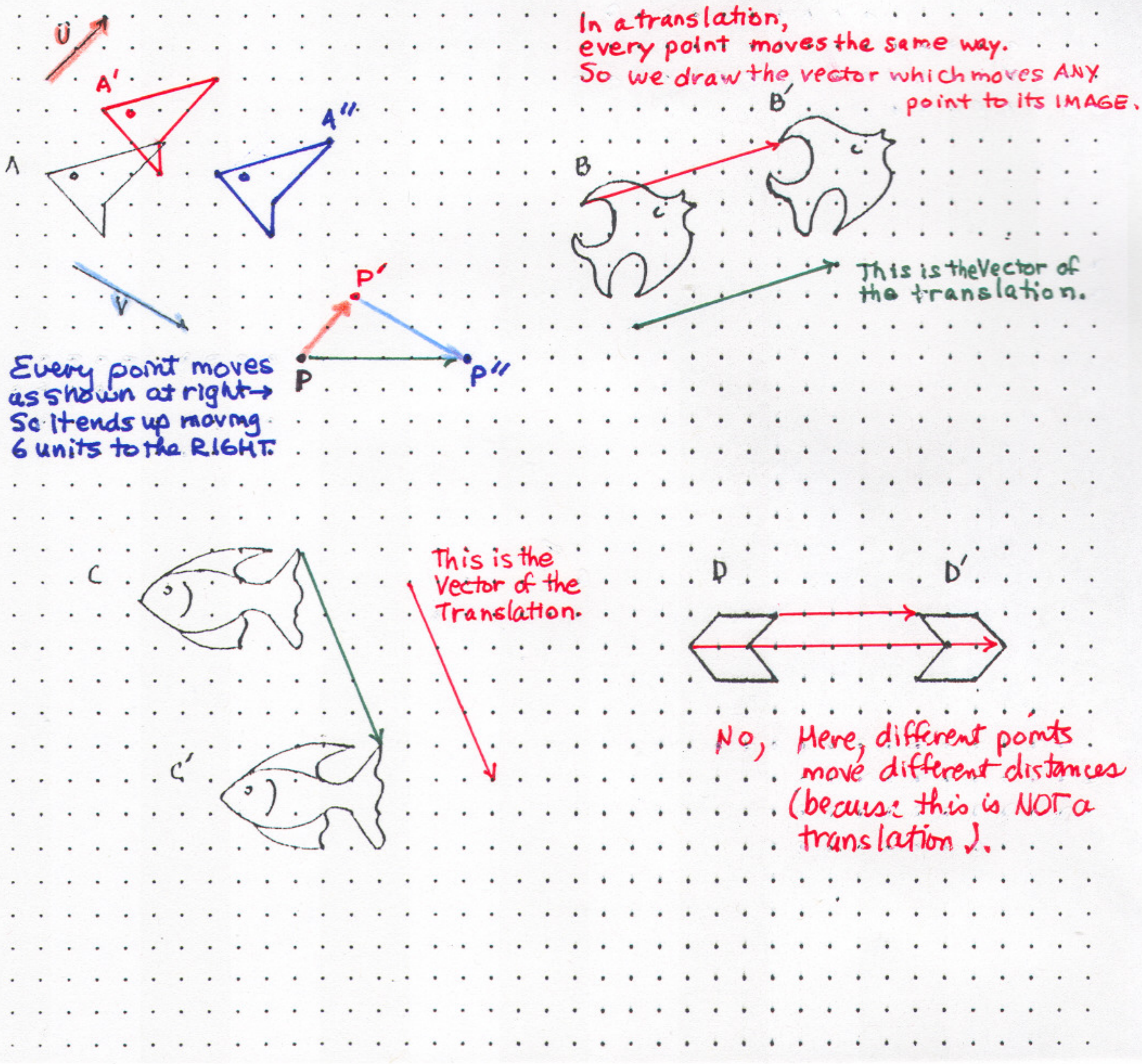
T1. Find the image of figure "**A**" under the translation indicated by vector "U". Label the image **A'**.
Hint: Locate key points, and see where the translation vector moves each one.

T2. Next translate the figure **A'** (*NOT* A) by the vector "V", to figure **A''**.

T3. You have just found the image of A under the **composition** of the two translations, U *followed by* V. [The notation for this composition is V∘U.]

✱ What transformation of the plane would move figure **A** directly to **A''**?

T4. Can you find a translation of the plane (ie find the *vector* of translation), that transforms figure **B** to **B'**? **C** to **C'**? **D** to **D'**?



TG-2 Defn: A **reflection** of the plane through the line *l* is an isometry in which *l* is a perpendicular bisector of the segment connecting each point with its image. *A reflection "FLIPS" the plane about the line l.* Note the points on *l are fixed (do not move when the plane is reflected through l).*

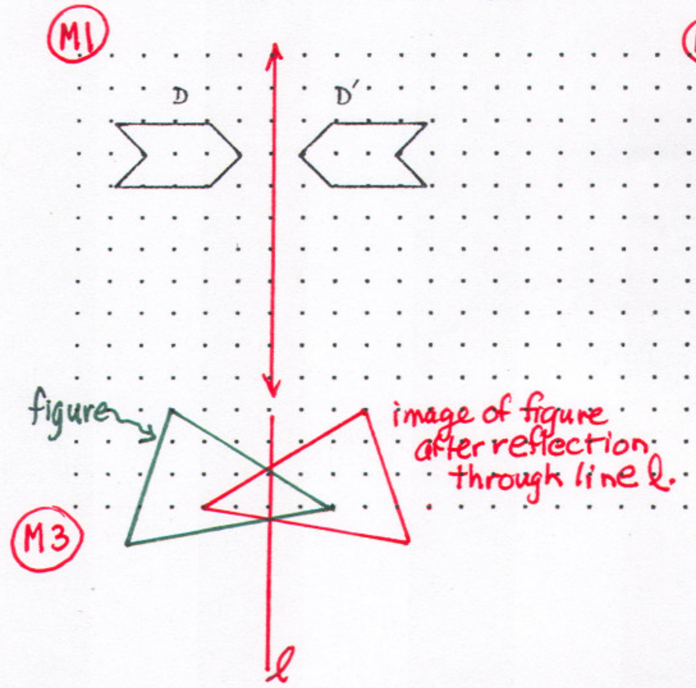
M1. Drawing the perpendicular bisector of one such segment suffices to determine the line of reflection, *l*.
Find the line of reflection for the isometry taking D to D'.

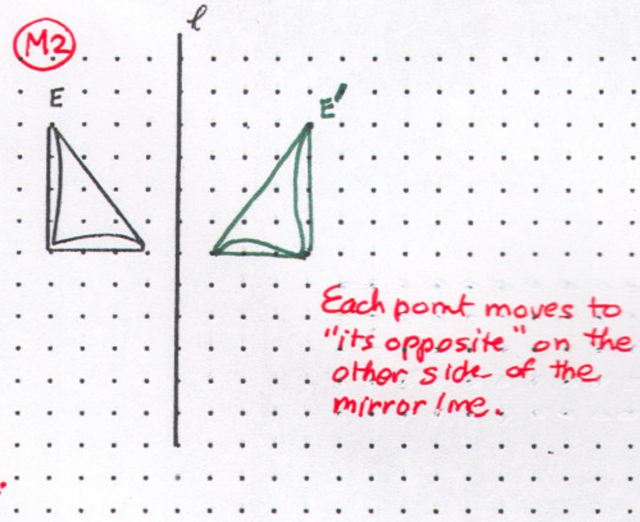
M2. Use a mira to draw the reflection of figure E through the line *l*. (*Note: dots are not aligned in squares.*)

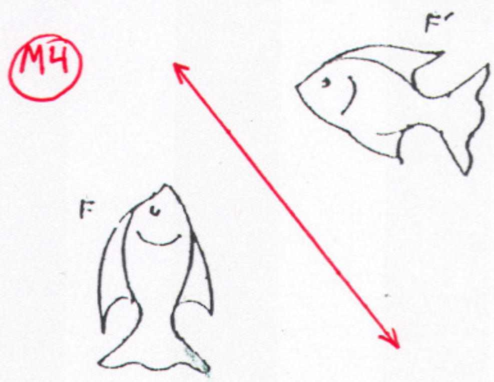
M3. Draw a figure on a piece of clean translucent paper. a. Draw a *separate* line. Use paper folding to draw the reflection of the figure through *l*. b. Draw a line *through* the figure and find the reflection.

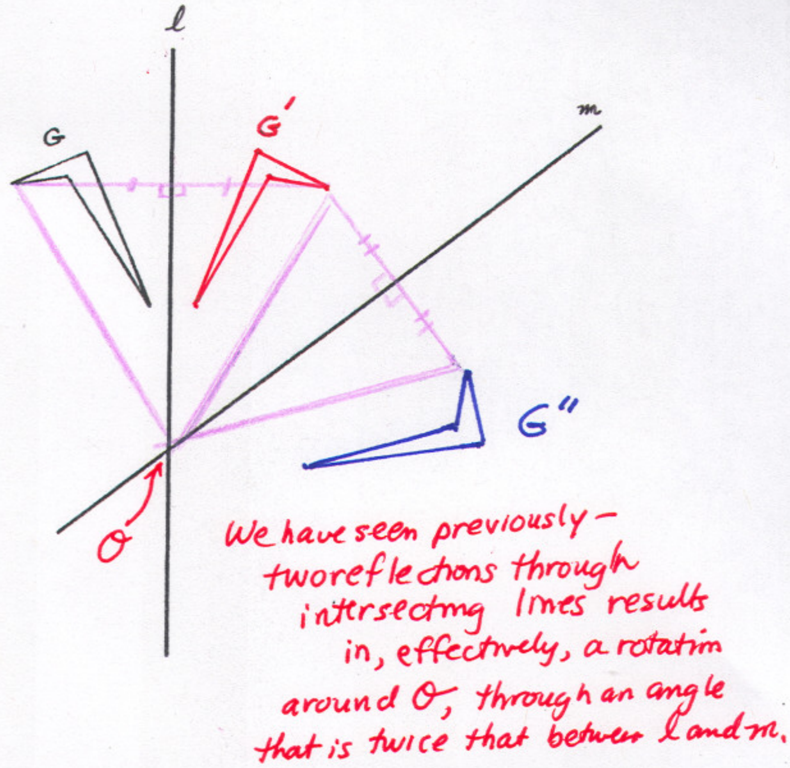
M4. How can you use paper folding to find the line of reflection between figures D and D'? Between F and F'?

M5. Reflect figure G through line *l* (call the image G'). Reflect G' through line *m* to G''. Is G'' congruent to the original, G? What rigid transformation of the plane would move G to G'' directly? (Can this be generalized?)







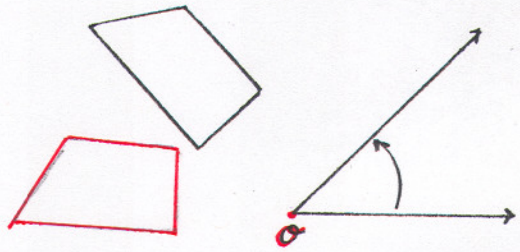


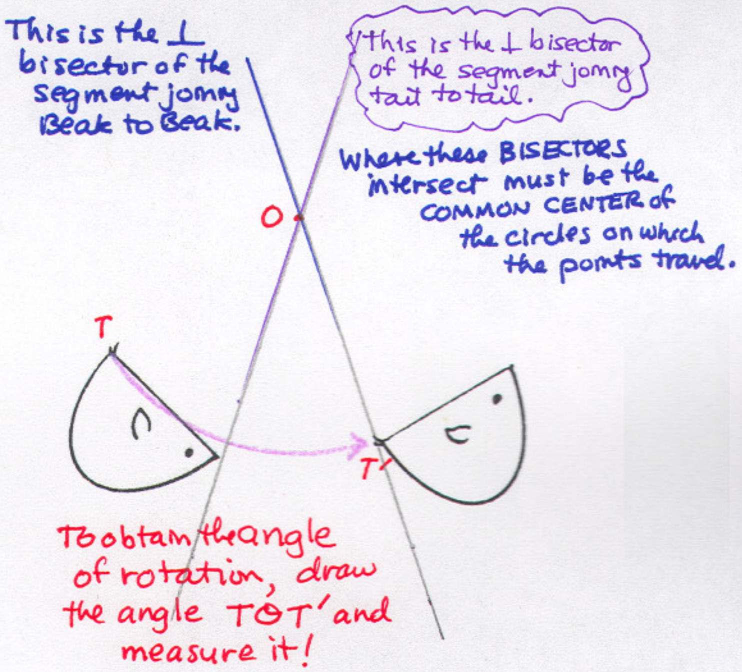
TG-3 Defn: A **rotation** of the plane is an isometry which fixes one point, O, (the center of rotation); all other points move so that the point and its image are equidistant from O, and segments from the point to O and from O to the image, form constant (congruent) angles.
(A rotation "turns" or "pivots" the plane, through some angle, around some fixed point.)

R1. Use tracing paper to rotate the given trapezoid 45° about the point O.

R2. Roughly estimate the center of rotation between the "birds". Connect two corresponding points on the birds. Notice the triangle formed with O. What kind of triangle? Where is O with respect to the line segment connecting the two points? connect another pair of points. How can we pinpoint O?

R3. Find (construct) the center and angle of rotation for the boomerang figures in M5.





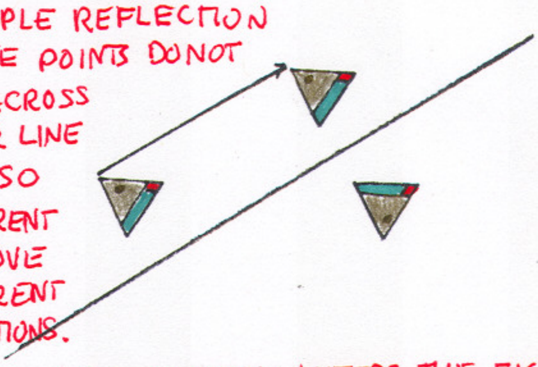
Defn: The fourth, and last, type of isometry of the plane is called a **glide reflection**– which reflects the plane through a line, then moves the plane in a direction parallel to the line of reflection.

(This isometry is a unique transformation in its own right, but may be thought of as the composition of a translation and a reflection through a line parallel to the translation vector.)

G1. Consider the example below.
How does the transformation differ from translation? from simple reflection? from rotation?

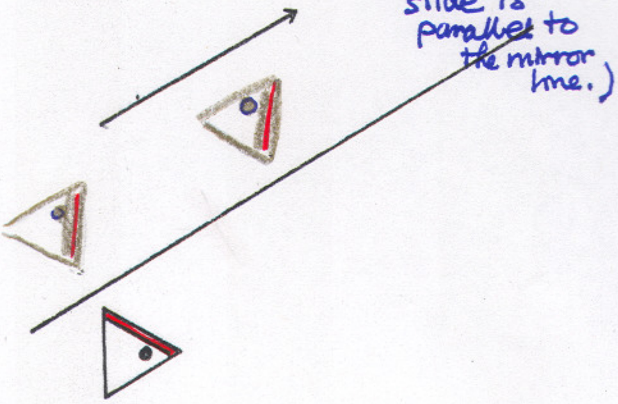
G2. Given glide vector U, & line of reflection m, find the image of the figure under the corresponding glide reflection.

NOT SIMPLE REFLECTION BECAUSE POINTS DO NOT MOVE ACROSS MIRROR LINE (⊥) SO DIFFERENT PTS. MOVE DIFFERENT DIRECTIONS.

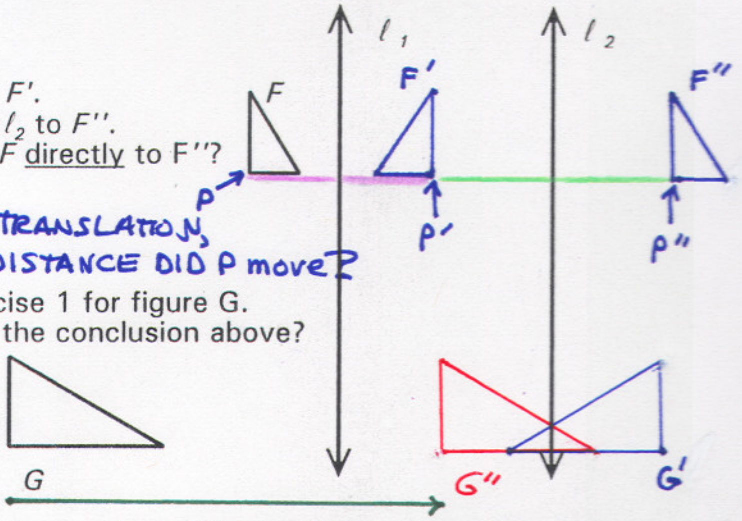


TRANSLATION KEEPS THE FIGURE "LOOKING" IN THE SAME DIRECTION, THE GLIDE REFLECTION FLIPS THE FIGURE, CHANGING ITS DIRECTION AND ORIENTATION. ROTATION KEEPS SAME ORIENTATION (clockwise).

We can slide & reflect, or reflect and slide. (This works only because the slide is parallel to the mirror line.)



C1. l_1 and l_2 are parallel.
Reflect figure F through l_1 to F'.
Reflect the image F' through l_2 to F''.
What transformation moves F <u>directly</u> to F''?



Consider P→P'→P''.
We can see the result is a TRANSLATION,
BUT HOW FAR? WHAT DISTANCE DID P move?

C2. Repeat the activities of exercise 1 for figure G.
Do your observations match the conclusion above?

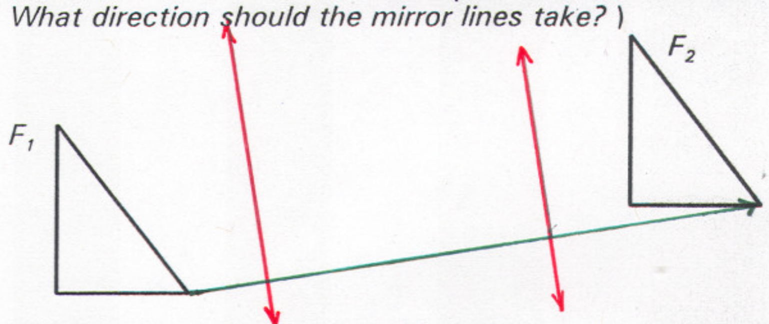
the double reflection, through parallel lines, translates all points twice the distance between....

C3. In M5, two reflections caused a rotation. What's different here?

Answer →

[*Two reflections through non-parallel lines cause a rotation, with center at the intersection, angle measure twice that of the angle between. Two reflections through parallel lines result in a translation, perpendicular to the two lines, twice the distance between.*]

C4. Find two reflections which accomplish the translation that moves F_1 to F_2:
(*What direction should the mirror lines take?*)

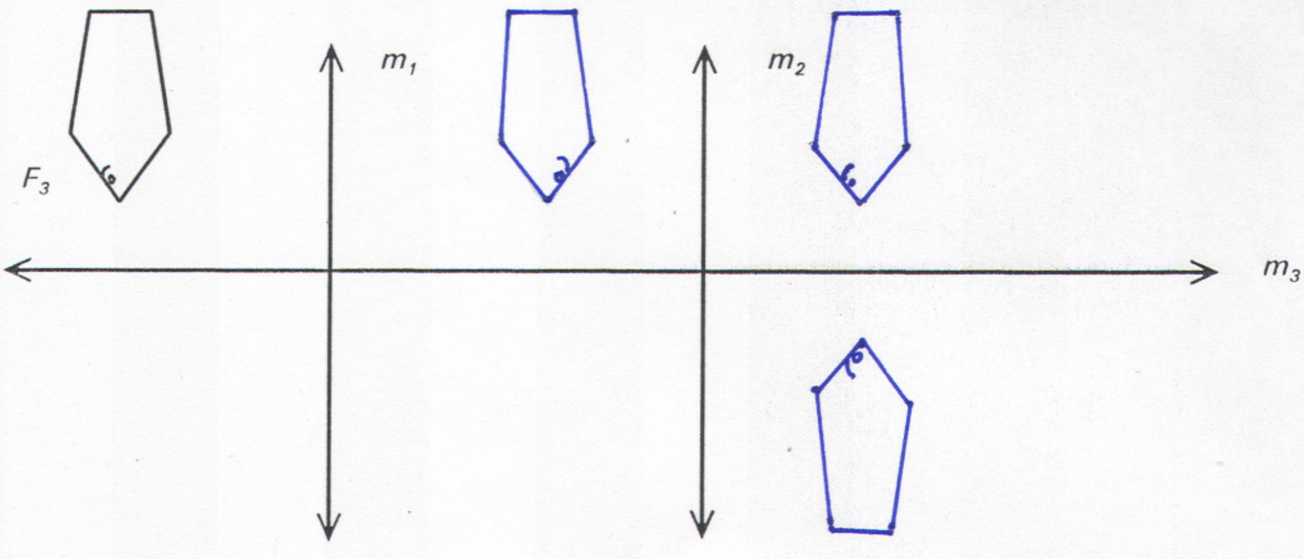


to translate F_1 to F_2 using reflections, choose a line ⊥ vector of translation... then a parallel line so the distance between the two is half the length of the vector.

C5. Note that m_1 and m_2 are parallel, and
m_3 is perpendicular to m_1 and m_2.
Reflect F_3 through m_1 to F_3', then
reflect F_3' through m_2 to F_3'', then
reflect F_3'' through m_3 to F_3'''.

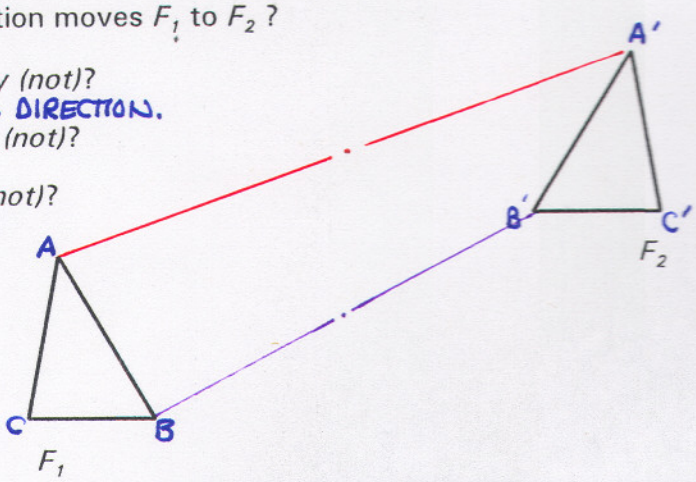
} → SLIDE } GLIDE REFLECTION
→ & FLIP

What transformation moves F_3 to F_3''' directly?
Why is this not surprising?



✤ A series of reflections can be used to duplicate a translation, a rotation, or a glide reflection.

C6. Given $F_1 \cong F_2$... what transformation moves F_1 to F_2?

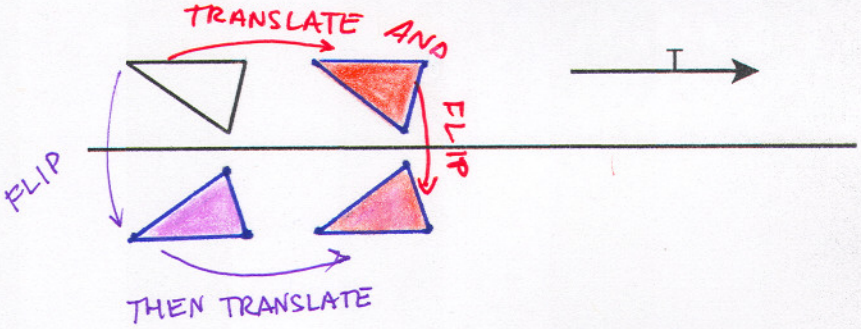


Could this be a translation? *Why (not)?*
NOT FACING THE ORIGINAL DIRECTION.
Could this be a reflection? *Why (not)?*
SEE ☆ BELOW
Could this be a rotation? *Why (not)?*
ORIENTATION REVERSED.
ABC Clockwise A'B'C' Counter-
This leaves only one possibility:
(verify it) !!

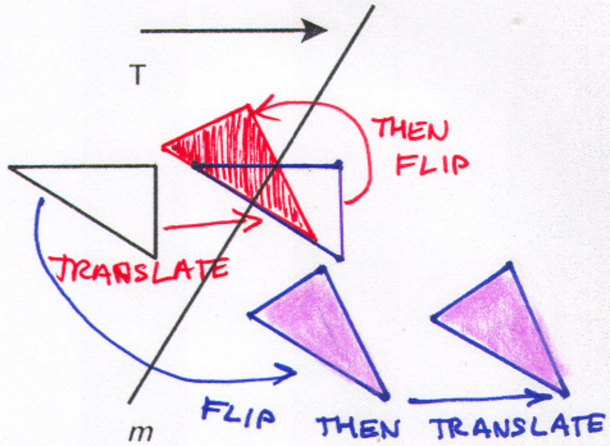
☆ $\overline{AA'}$ and $\overline{BB'}$ ARE NOT PARALLEL. IN A REFLECTION, ALL POINTS MOVE ⊥ MIRROR LINE, SO MUST MOVE PARALLEL.

!! THE LINE OF REFLECTION IS THROUGH THE MIDPOINTS OF $\overline{AA'}$ AND $\overline{BB'}$. AFTER REFLECTING, IT'S NOT HARD TO FIND THE VECTOR!

C7. If T is a translation, and R a reflection, how does T∘R compare with R∘T? (Let *V* be the translation vector, and l the line of reflection; investigate.)



m (mirror line for R)
(*Notice T is parallel to m*)
Below, T is NOT parallel to m.



NOT THE SAME RESULT!

In general, are compositions of transformations commutative? NO, NOT IN GENERAL.

C8. a. Can every type of transformation be accomplished by a series of translations? Why? (*Does a translation ever change the orientation of an object?*)

No, because no amount of translating can ever reverse the orientation.

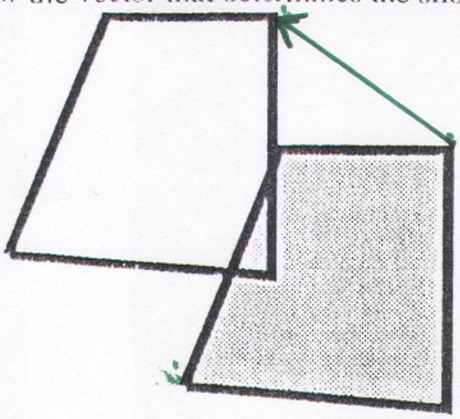
b. Same question, for rotations.

Same answer.

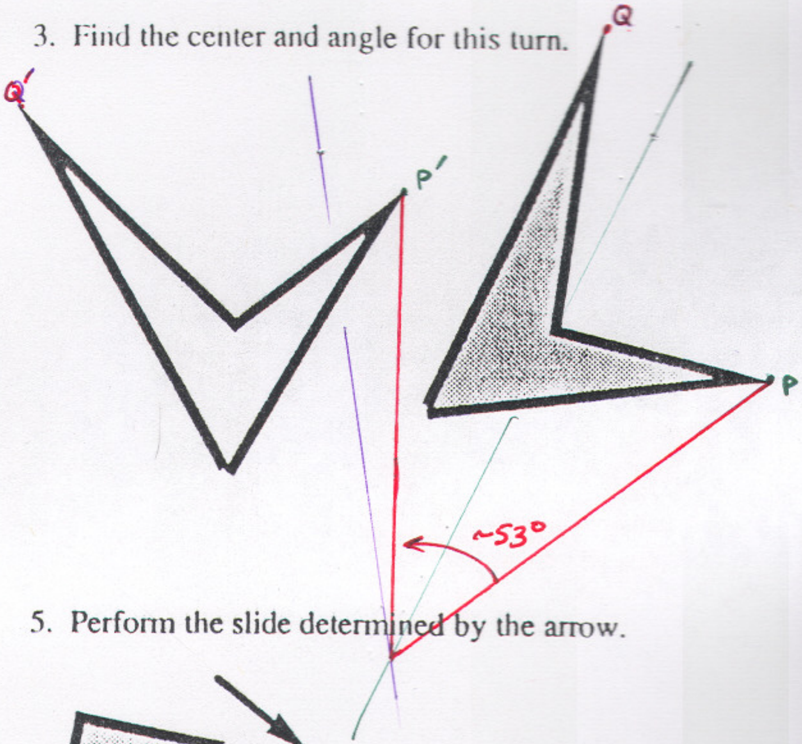
c. Same question, for reflections.

Yes, see M5, C4, C5, to see how any isometry can be done with reflections.

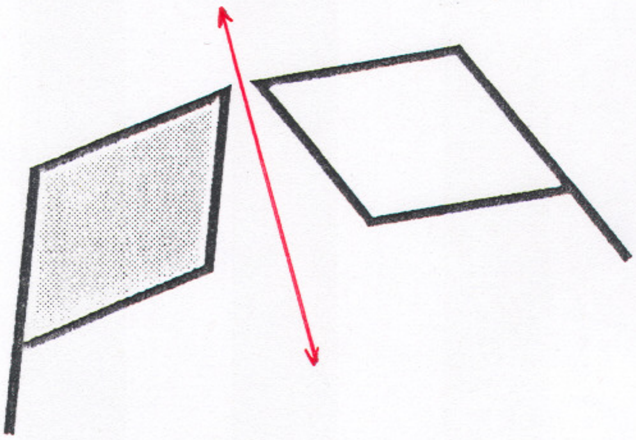
2. Draw the vector that determines the slide.



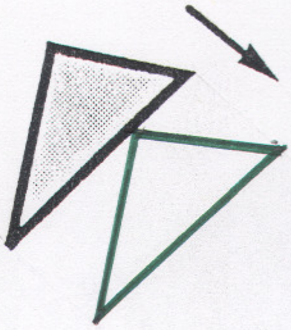
3. Find the center and angle for this turn.



4. Find the reflection line for this flip.

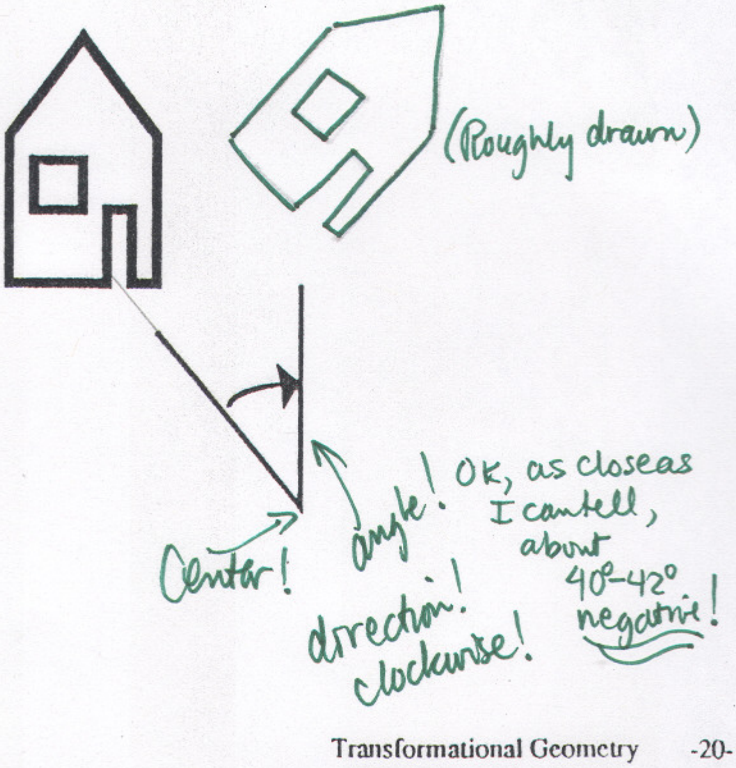


5. Perform the slide determined by the arrow.

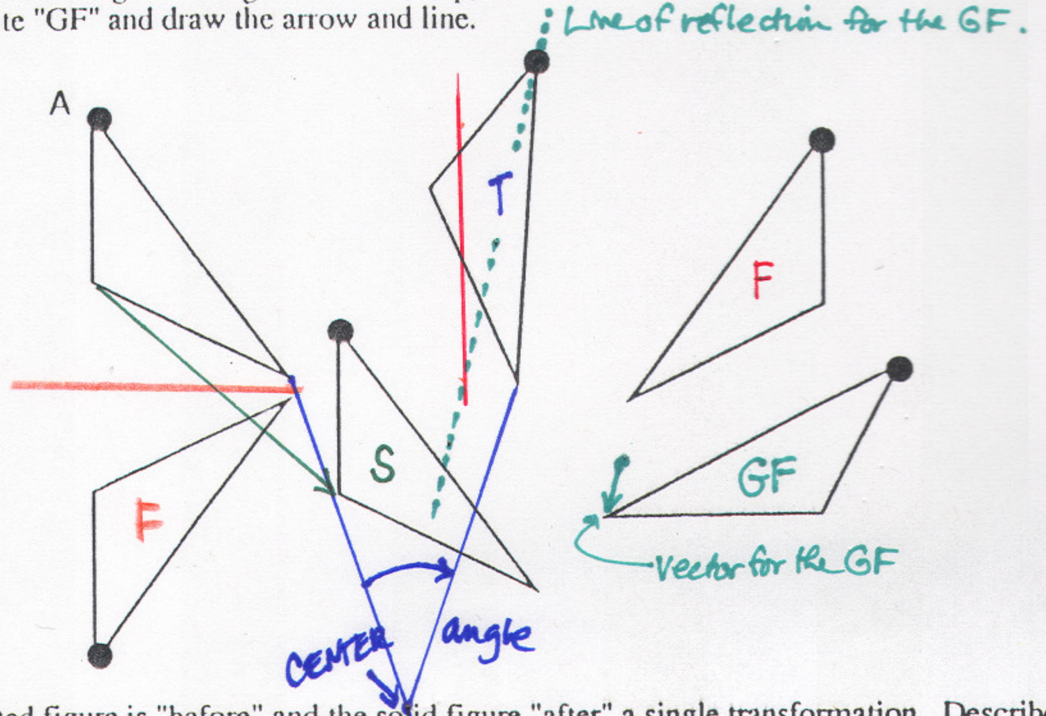


6. Find the flip image determined by the dotted line.

7. Find the turn image determined by the center, angle and direction.

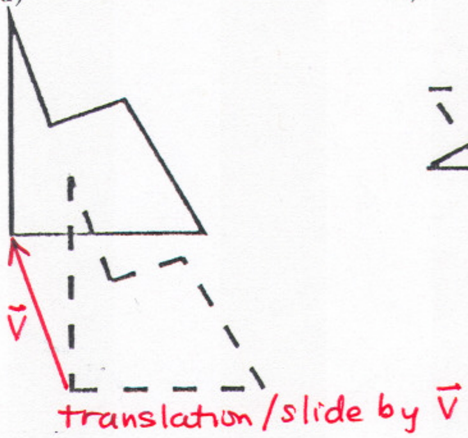


8. Each of the unmarked triangles is congruent to triangle A. If a triangle is an image of triangle A under a slide, mark it "S" and draw the vector. If a triangle is an image of triangle A under a turn, mark it "T" and draw the angle. If a triangle is an image of triangle A under a flip, mark it "F" and draw the line.. If it is the image from a glide reflection, write "GF" and draw the arrow and line.

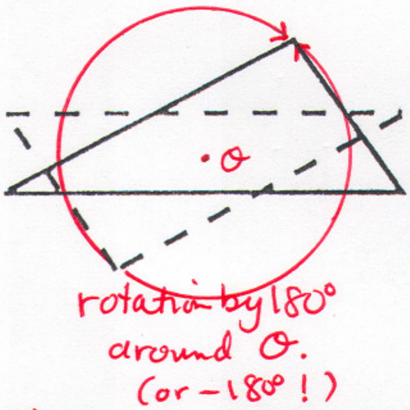


9. In each, the dotted figure is "before" and the solid figure "after" a single transformation. Describe each carefully - give the arrow for a slide, the flip line for a flip, and the center, angle, direction for a turn.

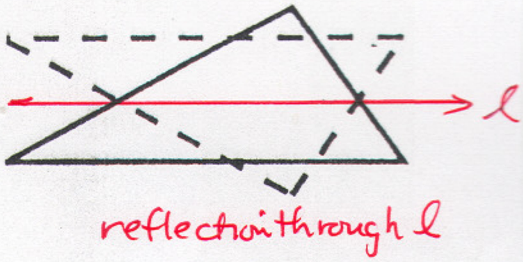
a)



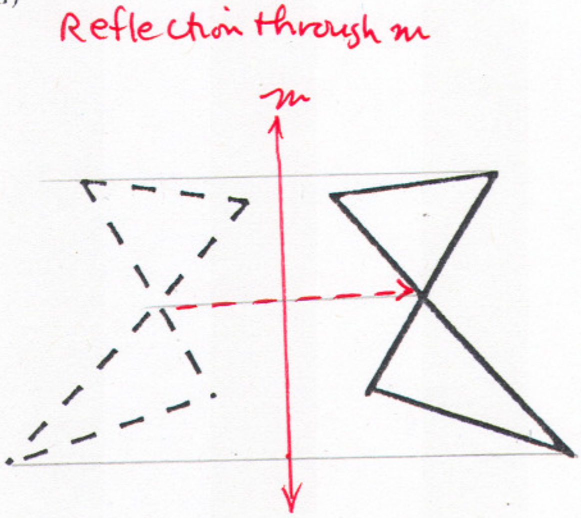
b)



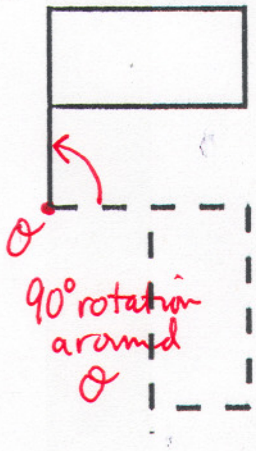
c)



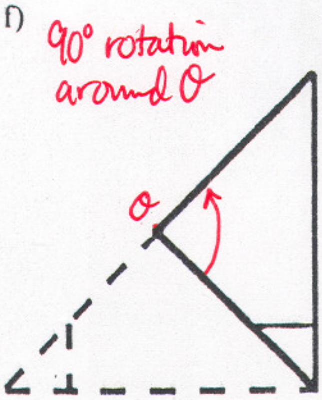
d)



e)



f)



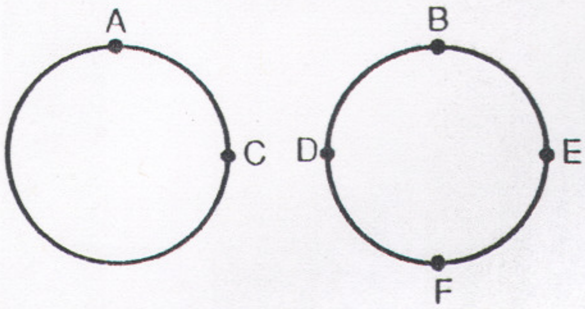
TG8

10) If a figure has a lot of symmetry, it is sometimes difficult to tell if a transformation is the result of a slide, a flip, or a turn. It is necessary to check the image of a few points to decide. The circle on the left is "before" and the circle on the right is "after".

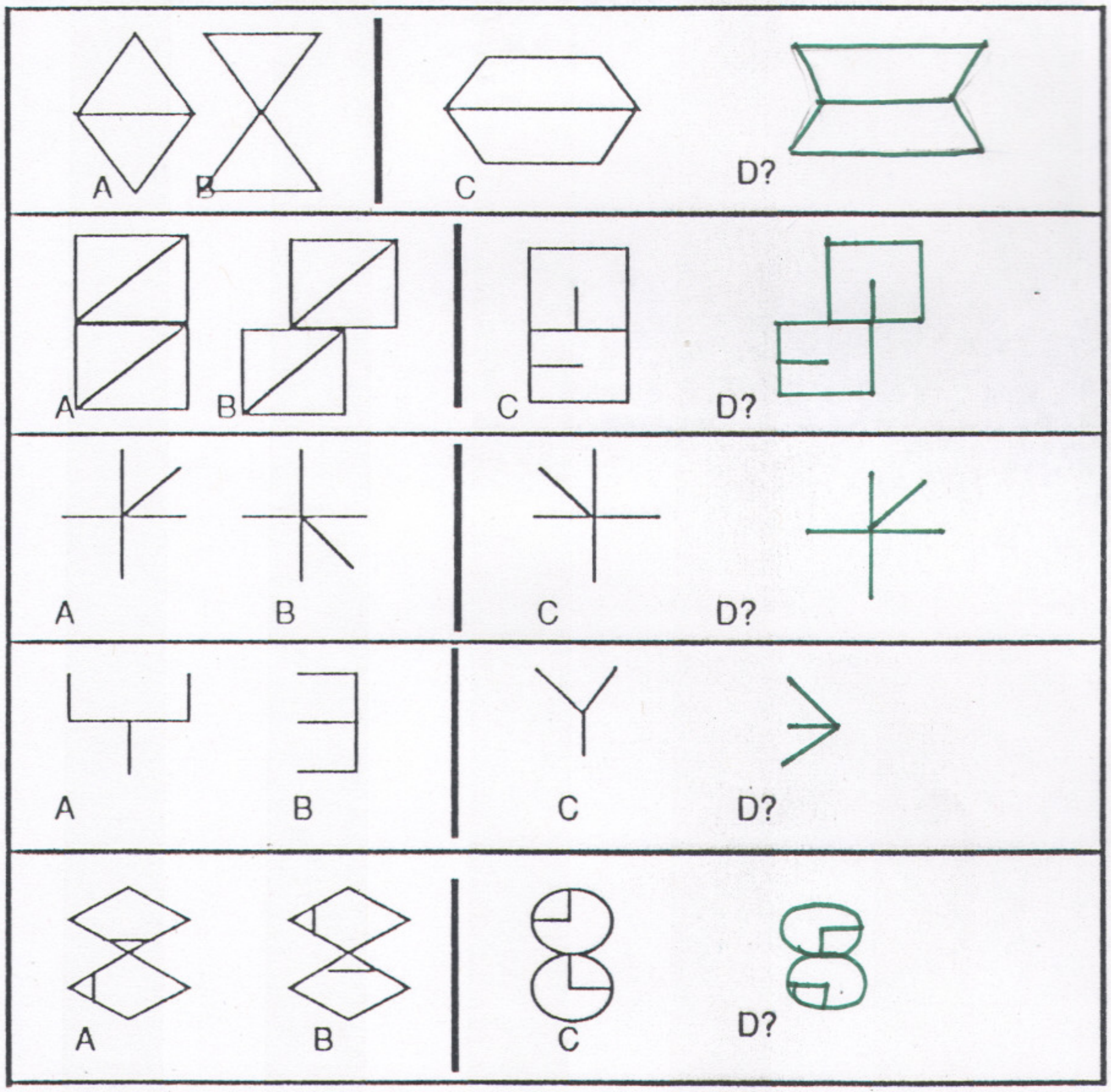
If the image of A is B and the image of C is D, what kind of transformation was used? REFLECTION

If the image of A is B and the image of C is E, what kind of transformation was used? TRANSLATION

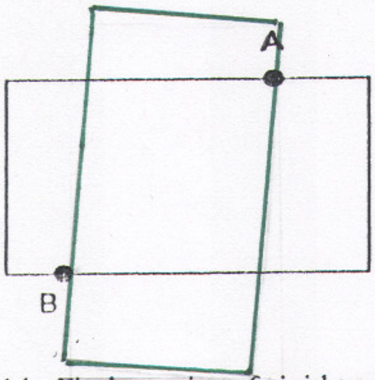
If the image of A is F and the image of C is D, what kind of transformation was used? ROTATION



11. In each, figure B is the result of performing one or more rigid transformations on part(s) of figure A. Your task is to perform those same movements on figure C to create figure D. That is, carefully sketch D so that "A is to B as C is to D."

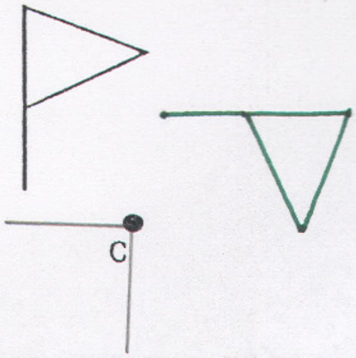


12. Reflect the rectangle through the line that would map point A onto point B.

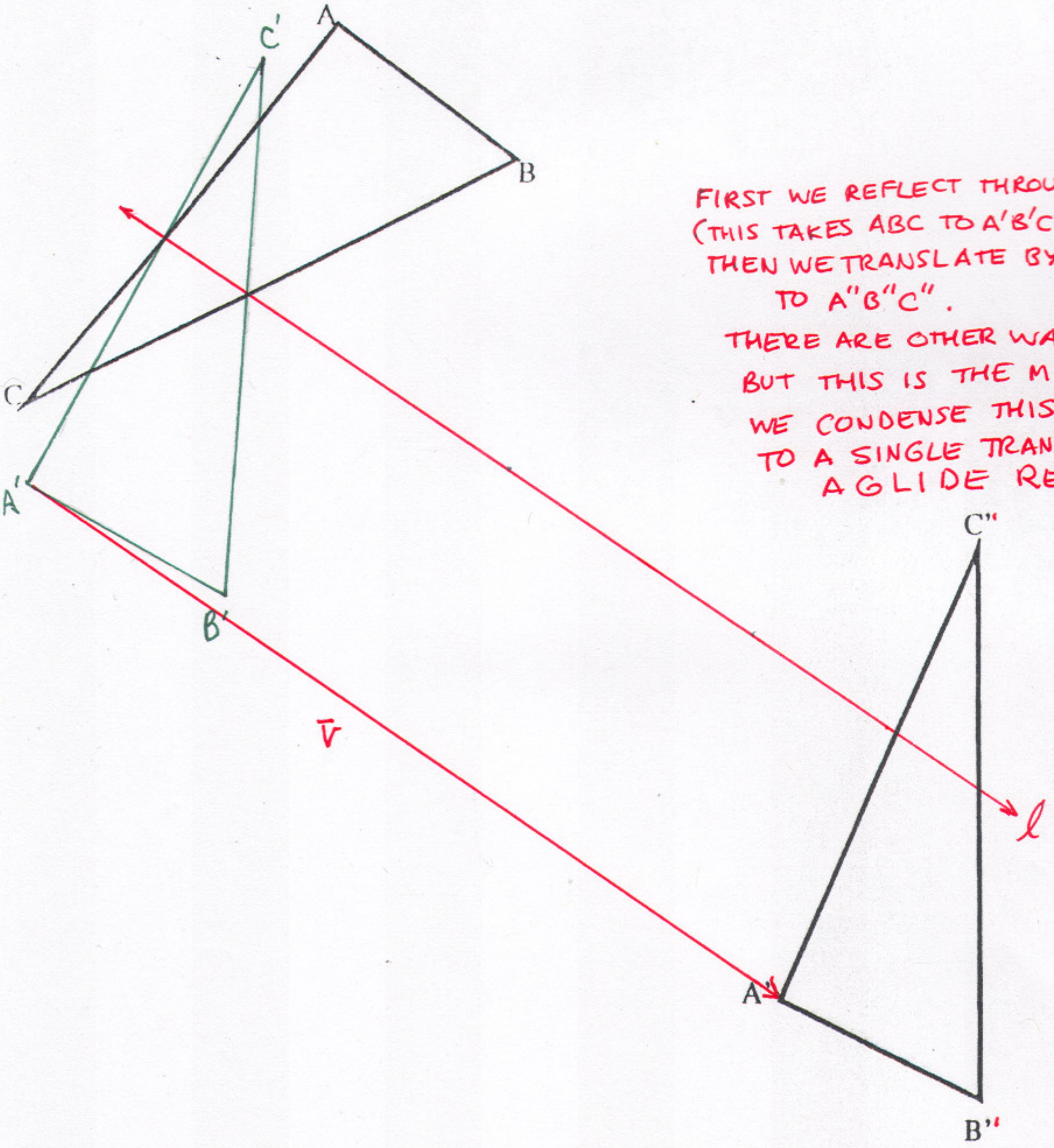


13. Rotate the figure 90° clockwise about C.

(−90°)



14. Find a series of rigid transformations that would take ΔABC onto ΔA'B'C'. Give all the necessary information for each step (lines, angles, vectors). Number and carefully sketch each intermediate position.



FIRST WE REFLECT THROUGH LINE ℓ.
(THIS TAKES ABC TO A'B'C'.)
THEN WE TRANSLATE BY VECTOR $\vec{v}$.
TO A''B''C''.
THERE ARE OTHER WAYS TO DO THIS, BUT THIS IS THE MINIMUM, IF WE CONDENSE THIS CONCEPTUALY TO A SINGLE TRANSFORMATION — A GLIDE REFLECTION.