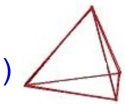
1. a. The number of distinct points necessary to determine a specific line is **2** Draw lines through this point: •
   b. The number of distinct points necessary to determine a specific plane is **3** Think tripod, or 3-legged stool.
   c. The number of distinct triangles with sides of lengths 2cm, 3cm and 7cm is **0** $2 + 3 < 7$
   d. The minimum number of faces on a convex polyhedron is **4** (tetrahedron)

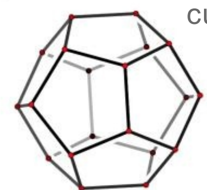


   e. The number of faces on a prism is **5** $2 + 3$
   f. The number of line segments connecting nine points, with no 3 collinear, is **36** $8 + 7 + 6 + \cdots + 1$

2. Multiple choice. For each statement, choose the BEST completion of the statement from this list:

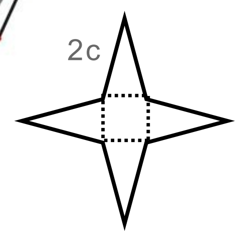
A circle	B cube	C dodecahedron	D line	E octahedron	F parallelogram
G plane	H point	I polygon	J polyhedron	K prism	L pyramid
M rectangle	N rhombus	O segment	P sphere	Q square	R simple closed curve

   a. A simple closed curve consisting of line segments is a **I** POLYGON
   b. The polyhedron illustrated at right is a **C** DODECAHEDRON 2b



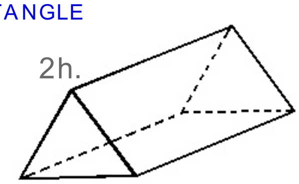
   c. The net, 2c, can be folded up into a polyhedron known as a **L**

2c

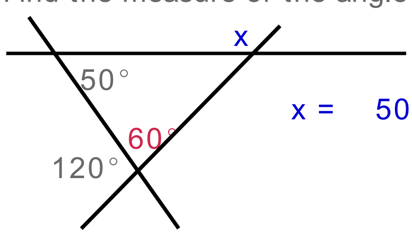


   d. The set of all points in a plane equally distant from a given point is a **A** CIRCLE
   e. A figure that is both a rhombus and a rectangle must be a **Q** SQUARE
   f. A parallelogram with an interior angle measuring 90° is a **M** RECTANGLE
   g. A quadrilateral with all sides congruent is a **N** RHOMBUS
   h. The figure illustrated at right is a **K** PRISM

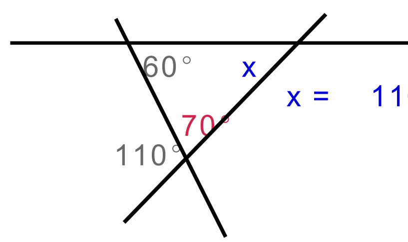
2h.



(6) 4. Find the measure of the angle marked x

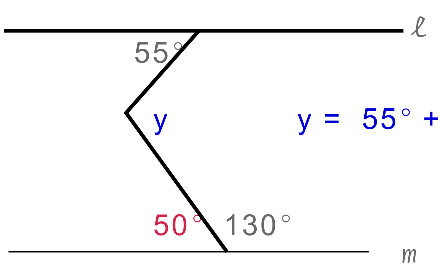


$x = 50° + 60° = 110°$

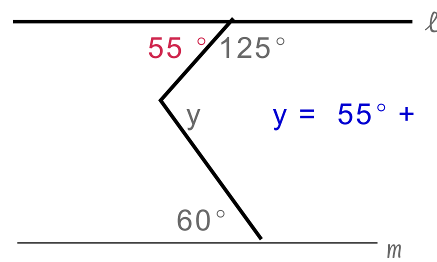


$x = 110° - 60° = 50°$

Given ℓ and m are parallel lines, find the measure of the angle marked y.



$y = 55° + 50° = 105°$



$y = 55° + 60° = 115°$

(4) 5. The measure of the complement of 75°34′21″.

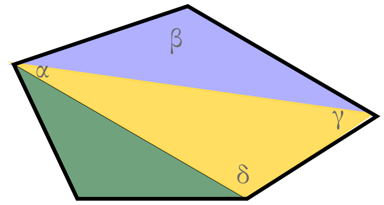
= 89°+ 60' = 89°+ 59'+ 60"

$$\begin{array}{r} 90° \\ -\ 75°\ 34'\ 21'' \end{array} \qquad \begin{array}{r} 89°\ 59'\ 60'' \\ -\ 75°\ 34'\ 21'' \\ \hline 14°\ 25'\ 39'' \end{array}$$

Find the measure of the supplement of 73°24′31″.

$$\begin{array}{r} 180° \\ -\ 73°\ 24'\ 31'' \end{array} \qquad \begin{array}{r} 179°\ 59'\ 60'' \\ -\ 73°\ 24'\ 31'' \\ \hline 106°\ 35'\ 29'' \end{array}$$

(6) 6. *Without* using a protractor, *showing your work*, find the **sum** of the measures of the interior angles in the polygon at right:



$m(\angle\alpha) + m(\angle\beta) + m(\angle\gamma) + m(\angle\delta) + m(\angle\varepsilon) = 3 \bullet 180° = 540°$

What is the measure of one interior angle of a **regular pentagon**?

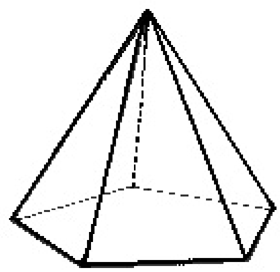
Since the interior angles of a regular polygon are all congruent (same measure), in a regular pentagon, each of the 5 interior angles must have measure $\frac{540°}{5} = 108°$

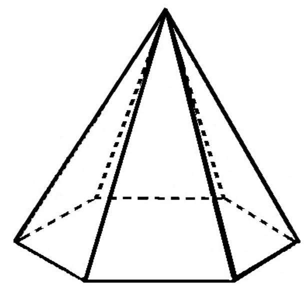
(8) 7a. Sketch (neatly & correctly) a... pyramid with pentagonal base. ... prism with hexagonal base.

b. State the number of faces: 6 | 8

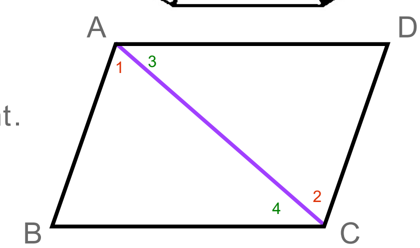
the number of edges: 10 | 18

and vertices: 6 | 12





(10) 8. Given parallelogram ABCD, and assuming we know ONLY THE DEFINITION OF THE PARALLELOGRAM, prove that opposite sides AB and CD are congruent. (You can mark the illustration to supplement your statements.)
Hint: Create two congruent triangles in this illustration.
Prove they are congruent. Then use them to draw the conclusion.



Suppose ABCD is any parallelogram.
Then AB || CD and BC || AD because, by definition, opposite sides of a parallelogram are parallel !

(1) Since AB || CD, ∠BAC ≅ ∠DCA (ie ∠1 ≅ ∠2) by alternate interior angles theorem
Similarly,
(2) since BC || AD, ∠DAC ≅ ∠BCA (ie ∠3 ≅ ∠4) by alternate interior angles theorem

(3) AC ≅ CA

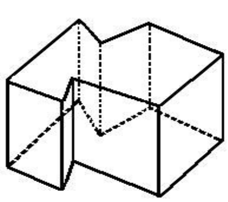
Based on (1), (2) and (3), △CAB ≅ △ACD by "ASA", or "angle-side-angle" theorem of triangle congruence.

Therefore AB ≅ CD, since "CPCTC" (corresponding parts of congruent triangles are congruent).

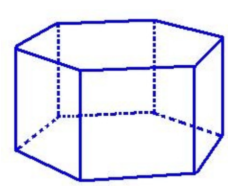
The essential points to mention are:

∠1 ≅ ∠2 by alternate interior angles theorem since AB || CD
∠3 ≅ ∠4 by alternate interior angles theorem since BC || AD
AC ≅ CA
△CAB ≅ △ACD by "ASA"
AB ≅ CD, since "CPCTC"

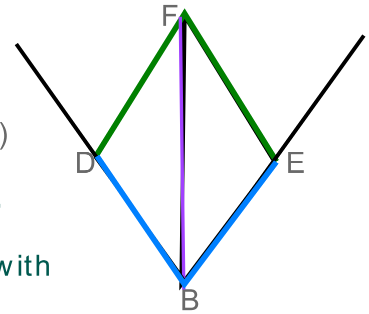
(3) 9.



Name the figure illustrated as specifically as possible.
Octagonal right prism | Hexagonal right prism



(7) 10. We bisected angle B by swinging an arc centered at B across the rays that comprise B. That arc crosses the rays at D and E. We then created arcs with identical radii centered at D & E. The intersection of these arcs is F. PROVE that BF bisects the original angle B. (Hint: what triangles are congruent, and why?)



Since we located D & E by creating an arc of a circle centered at B,
BD ≅ BE, by construction.
Since we located F by creating arcs of circles centered at D and E with identical radii (we used the same compass opening)
DF ≅ EF, by construction.
BF ≅ BF

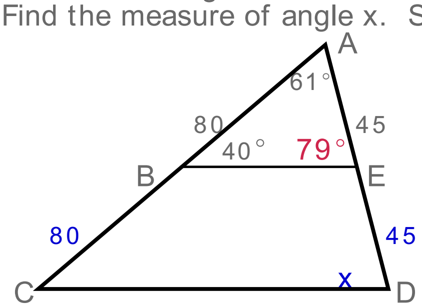
Therefore, △BDF ≅ △BEF , by "SSS" (the side-side-side theorem of triangle congruence).

It follows that ∠DBF ≅ ∠EBF , since "CPCTC" (corresponding parts of congruent △s are ≅).

A bare bones proof:

BD ≅ BE, by construction.
DF ≅ EF, by construction.
BF ≅ BF
△BDF ≅ △BEF , by "SSS"
∠DBF ≅ ∠EBF , since "CPCTC"

(4) 11. Given the triangle and measurements illustrated,
Find the measure of angle x. SHOW all the angles you find in order to determine the measure of x.

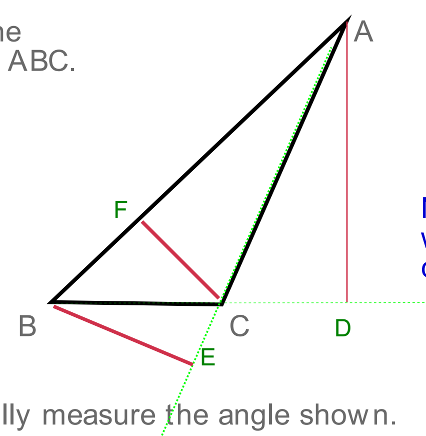


x = 79°

← We know this since 61° + 40° + 79° = 180°

B and E are the midpoints of AC and AD, respectively, so by the Midpoint Theorem (see page Congruence 103)...
CD is parallel to BE. Therefore x is the same as the 79° angle we just found, since they are corresponding angles of || lines..

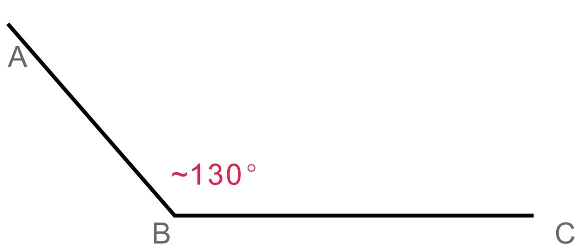
(5) 12. Carefully illustrate (sketch) the three altitudes of the triangle ABC.



NOTE: D, E & F are the FEET of the altitudes, where the altitudes meet the bases (the lines containing the opposite sides.

(3) 13. Using your protractor, carefully measure the angle shown.

m (∠ABC) ≈



(5) 14. *Showing all necessary marks*, **construct** a line through A parallel to BC.

A .

There are many methods of constructing the parallel line through A.

You were given three methods in this class.

They are illustrated on the Practice Final solutions, Part H, #2, and can be seen at http://www.csun.edu/~cas24771/m310/310PF_H1.pdf .

B ———————————————— C

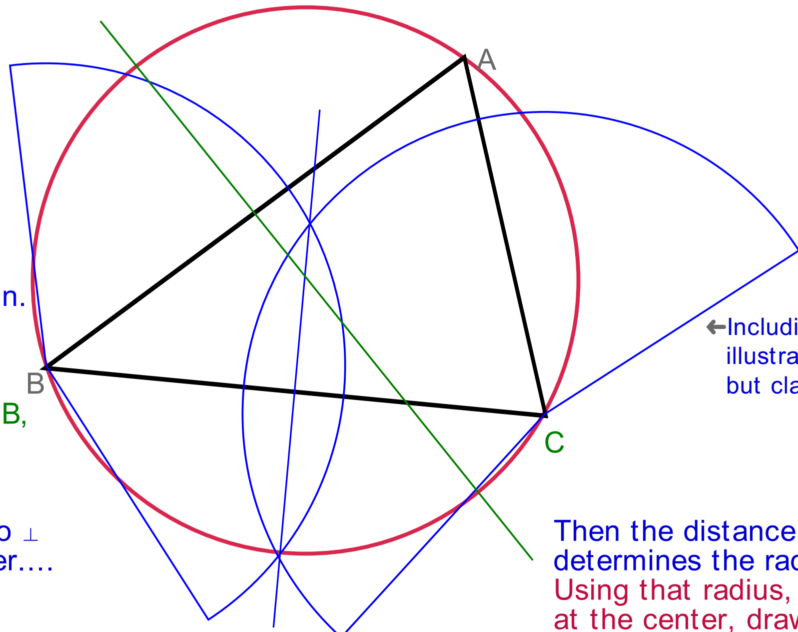
(8) 15. *Showing your work,* carefully **construct** a circle passing through the vertices of triangle ABC below.
This, likewise, was done several times in class, and is virtually the same construction as shown on the Practice Final, part H, #4, which can be seen at http://www.csun.edu/~cas24771/m310/310PF_H2.pdf .

Method:
Perpendicular bisectors of the sides meet at the point which is equidistant from the ends of the sides (ABC), locating the center.

The construction of ONE ⊥ bisector, of BC, is shown.
Radii of the arcs show the location of the centers

A second ⊥ bisector, of AB, is illustrated, but without its construction.

The intersection of the two ⊥ bisectors locates the center....

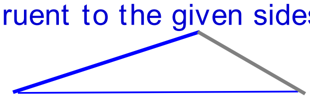


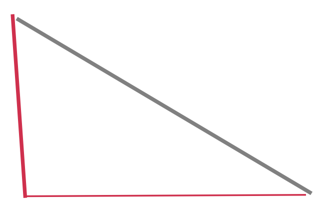
←Including the radii in this illustration clutters the picture, but clarifies location of centers.

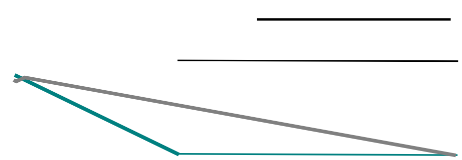
Then the distance to any vertex determines the radius of the circle.
Using that radius, with compass point at the center, draw the circle.

(4) 16. Carefully **draw** two **different** (not congruent) triangles with sides shown.
(That is, draw △ABC and △DEF where AB≅DE and BC≅EF but △ABC ≇ △DEF.)

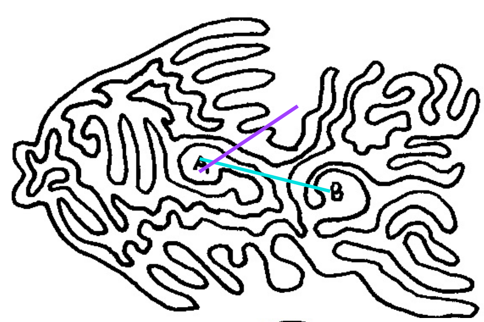
Here are three possibilities.
In each case the sides in color are congruent to the given sides.







(3) 17. Given the illustration at right* is of a simple closed plane curve, are points A & B on the SAME side, or OPPOSITE sides, of the curve? Explain (briefly) how you know.



A & B are on the SAME side of the curve, since a line segment connecting them crosses the SCPC an even number of times.
This can also be seen by the fact that A & B are both INSIDE the SCPC.... any line segment connecting them to the outside crosses the curve an odd number of times.

* curve does not include "A" & "B"

Although the hope was that you would answer as shown above, shading the entire interior of the curve, as shown at right, also clarifies that A and B are both inside the curve.