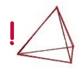
Math 310 ★ S2009★ Test #2 ★ 100 points ★ **ANSWERS TO EVEN-NUMBERED TESTS**

(14) 1. Fill in the blank to make a complete and true statement.

a. The number of triangles having sides of lengths 5cm, 6cm and 10cm is **1** That's "SSS" !!!

b. The number of triangles having sides of lengths 3cm, 6cm and 10cm is **0** 3+6=9, too short!

c. The minimum number of faces needed to form a prism is **5** See # 9 !

d. The minimum number of faces on a convex polyhedron is **4** Tetrahedron ! 

e. The number of distinct points necessary to determine a specific plane is **3** (3 noncollinear pts.)

f. The number of distinct points necessary to determine a specific line is **2**

g. The number of line segments connecting nine points, with no 3 collinear, is **36** (8+7+6+5+4+3+2+1)

(16) 2. Multiple choice. For each statement, choose the BEST completion of the statement from this list:

A circle	B line	C plane	D polyhedron	E quadrilateral	F sphere
G cube	H octahedron	I point	J prism	K rectangle	L square
M dodecahedron	N parallelogram	O polygon	P pyramid	Q rhombus	R tetrahedron

and place its LETTER in the box.

a. The polyhedron illustrated in figure 2a is called a **H**

b. Figure 2b can be folded up into a polyhedron called a **J**

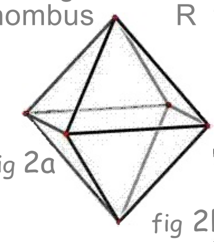


fig 2a

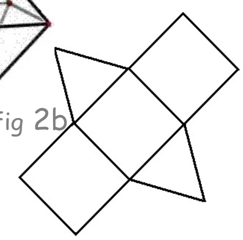


fig 2b

c. A simple closed curve consisting of line segments is a **O**

d. The set of all points in a plane equally distant from a given point P is a **A**

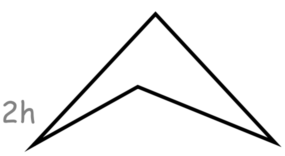
e. A single figure that is both a rhombus and a rectangle must be a **L**

f. A parallelogram with an interior angle measuring 90° is a **K**

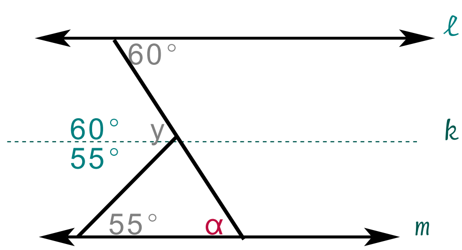
g. A quadrilateral with all of its sides congruent must be a **Q**

h. The figure illustrated at right is a **E**

fig 2h



(6) 4. Given ℓ and m are parallel lines, find the measure of the angle marked y.

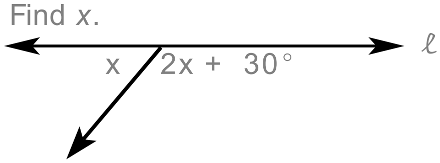


Method ONE:

Picture a line k || to both ℓ & m; use alt. int. ∠s to get y = 60°+ 55°

Method TWO:
Use alt int ∠s at ℓ and m to get α = 60°
So the 3rd angle in that triangle is 180° – 115°. Thus y = 115°

Find x.



x and 2x+ 30° form a straight angle, so

x + 2x + 30° = 180°

3x + 30° = 180°

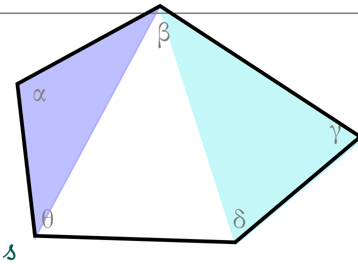
x = 50° ("degrees" *necessary* in the answer !)

(4) 5. The measure of an angle is 23°51′51″. Find the measure of its supplement.

$$\begin{array}{r} 180^\circ \\ -\ \underline{23^\circ\ 51'\ 51''} \end{array} = \begin{array}{r} 179^\circ\ 59'\ 60'' \\ -\ \underline{23^\circ\ 51'\ 51''} \\ 156^\circ\ \ 8'\ \ 9'' \end{array}$$

/40

(5) 6. *Without using a protractor*, and *showing your work*, find the **sum** of the measures of the interior angles in the polygon at right:



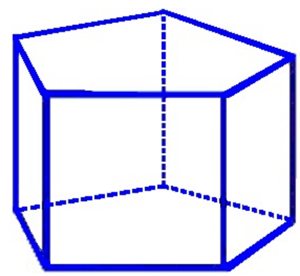
$m(\angle\alpha) + m(\angle\beta) + m(\angle\gamma) + m(\angle\delta) + m(\angle\theta) =$ **540°**

Work: As you can see, 3 triangles cover this polygon without overlap. The triangles' interior angles add up to the pentagon's interior angles. Thus the total is 3·180°. Or (5-2)·180°.

What is the measure of **one** interior angle of a **regular pentagon**?

One-fifth of the total, or $(^1/_5)\cdot 540° = 108°$ **108°**

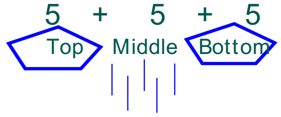
(10) 7a. **Sketch** (neatly & correctly) a prism with a pentagonal base.



b. State the number of faces: **7** 2 bases + 5 lateral faces

and vertices: **10** 5 at the top, 5 at bottom

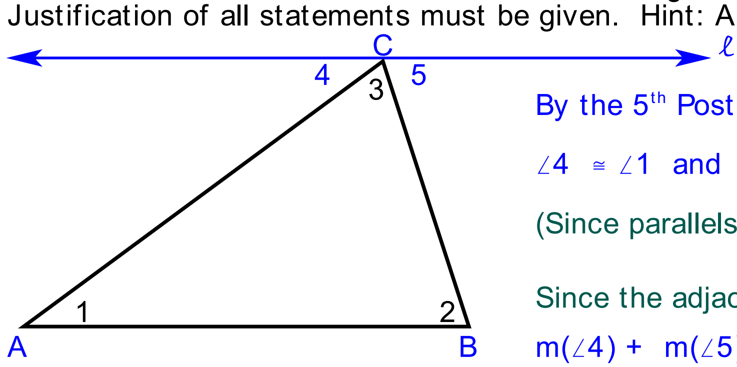
and edges: **15** 5 + 5 + 5



(2) 8. **F** All isosceles triangles are equilateral triangles. Here is an isosceles triangle that is not equilateral:

T All equilateral triangles are isosceles triangles.

(8) 8. **Prove** the sum of the measures of the interior angles of any triangle is 180°. Justification of all statements must be given. Hint: A certain additional line helps!



By the 5th Postulate, there is a line ℓ through C parallel to AB.

$\angle 4 \cong \angle 1$ and $\angle 5 \cong \angle 2$ by the alternate interior angles theorem.

(Since parallels ℓ and AB are cut by transversals AC and BC.)

Since the adjacent angles 3 & 4 & 5 form straight angle ℓ

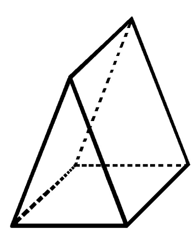
$m(\angle 4) + m(\angle 5) + m(\angle 3) = 180°$

Substituting $m(\angle 1)$ for $m(\angle 4)$, since 1 & 4 are congruent, and similarly substituting $m(\angle 2)$ for $m(\angle 5)$ we get:

$m(\angle 1) + m(\angle 2) + m(\angle 3) = 180°$

QED! ["which is what we were supposed to demonstrate"].

(4) 9. Name the figure illustrated at right as specifically as possible.



Right Triangular Prism

How many faces ? **5** (Note this helps answer #1.)

(7) 10. Construct the bisector of angle ABC. Marks that are necessary to the construction MUST be shown; unnecessary marks should NOT be shown.

First we drew an arc of a circle centered at B

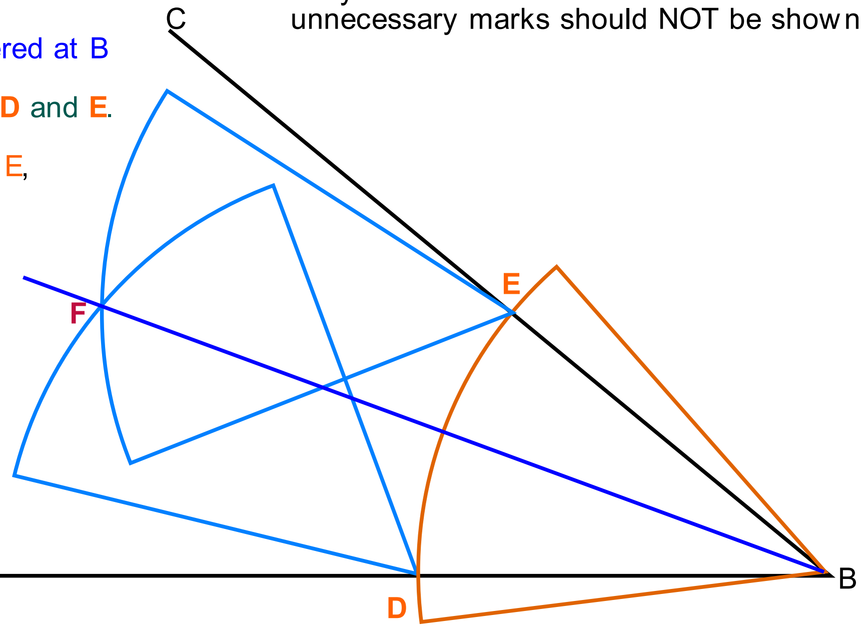
This intersected the rays of ∠ABC at **D** and **E**.

With compass centered at D, then at E, we drew arcs with equal radii to intersect at **F**.

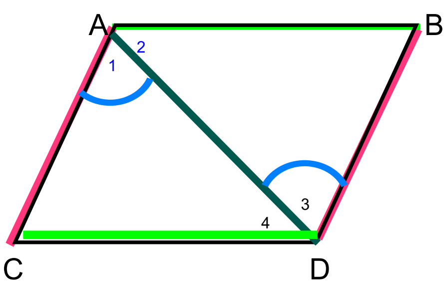
BF is the ray that bisects ∠ABC.

Note: radii on each end of the arcs drawn are included so the center is clear.

This is not the only method.



(12) 11. **Prove** the following:
In quadrilateral ABCD, if AB ≅ CD and AC ≅ BD, then ABCD is a parallelogram. *Partial Credit possible. Hints available (not free).*



GIVEN:
(1) ABCD is a quadrilateral,
(2) **AB** ≅ **CD**,
(3) **AC** ≅ **BD**.

We draw segment AD, and note that (4) **AD** ≅ **AD**.

Thus △**ACD** ≅ △**DBA** —from (2) & (3) & (4) using **SSS** theorem

Since Corresponding Parts of Congruent Triangles are Congruent,
We can now conclude that ∠**CAD** (∠1) ≅ ∠**BDA** (∠3) . **[CPCTC]**
AC || BD since AD forms a transversal and the **alternate interior angles** 1 and 3 are congruent.

Similarly, ∠CDA (∠4) ≅ ∠BAD (∠2) and, in turn, **AB || CD**

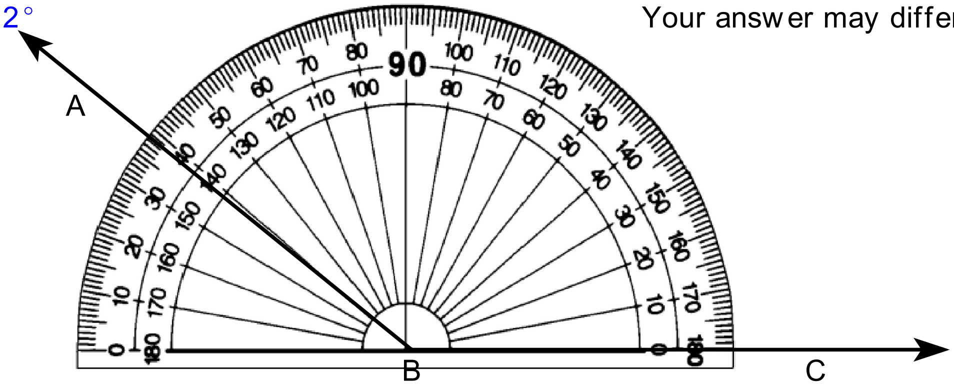
Thus the quadrilateral **ABCD is a parallelogram, because opposite sides are parallel (definition of ▱)**.

[Bolded items are the critical points to be made in the proof.]

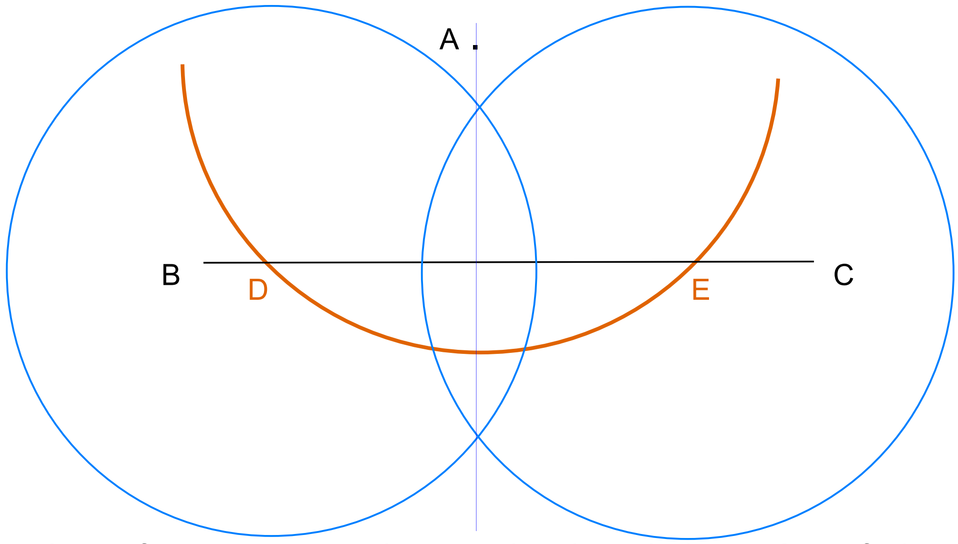
(3) 12. Using your protractor, carefully measure the angle shown.

m (∠ABC) ≈ 142°

Your answer may differ, some are 127°.

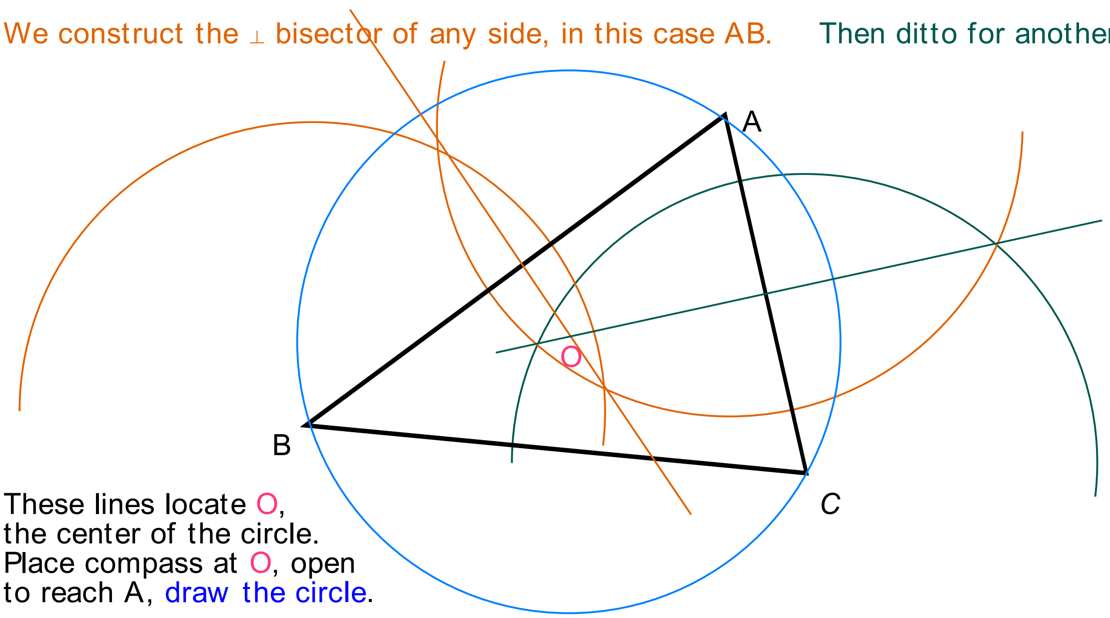


(5) 13. *Showing all necessary marks*, **construct** a line through point A perpendicular to BC.



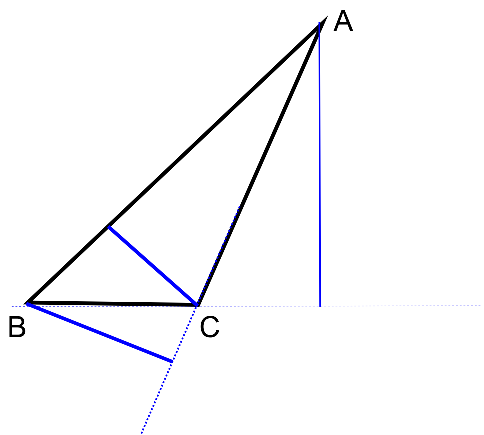
(3) 14. *Showing your work,* carefully **construct** a circle passing through the **vertices** of triangle ABC below.

We construct the ⊥ bisector of any side, in this case AB. Then ditto for another side, say AC.



These lines locate O,
the center of the circle.
Place compass at O, open
to reach A, draw the circle.

(4) 15. Carefully illustrate (sketch) the three altitudes of the triangle ABC.



(3) 16. Carefully **draw** two **different** (not congruent) triangles using lengths x and y for two sides:

That is, sketch △ABC & △DEF where both △s have sides x & y, but △ABC ≇ △DEF.

Use a ruler!