**Math 310 Fall 2006 Test #1 Statistics & Probability ANSWERS**

SHOW WORK *as if making a solutions guide for students.* Use extra paper if necessary.

*Don't forget labels, scales, & legends!*

**#1-3 The scores of ten countries on the PISA* test are given below.**

5.5 5.2 5.2 5.1 5.8 4.8 4.7 5.5 5.3 3.8

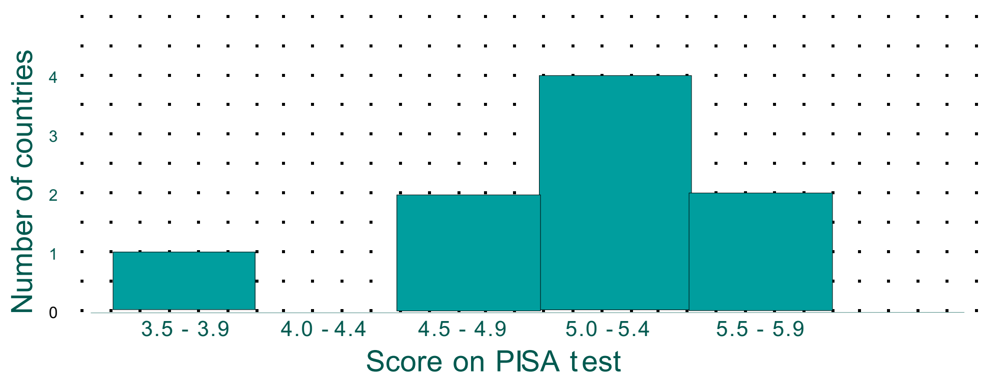
(5) 1. Classify the data in a stem-and-leaf diagram. so there are at least five classes.

Scores of ten countries on the PISA

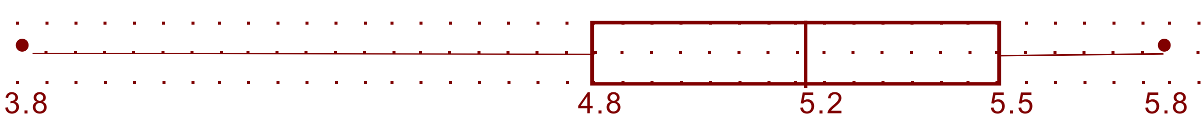
| | |
|---|---|
| 3 | 8 |
| 4 | |
| 4 | 7 8 |
| 5 | 1 2 2 3 |
| 5 | 5 8 |
| | |
| | |
| | |

Legend
4 | 7 8
represents scores of 47 & 48 for some countries

(6) 2. Draw a histogram using classes which correspond to those in problem (1).




(12) 3. Draw a box plot for this data.




*A box plot is not a line plot !*

WORK →

| 3.8 | 4.7 | 4.8 | 5.1 | 5.2 | 5.2 | 5.3 | 5.5 | 5.5 | 5.8 |
|---|---|---|---|---|---|---|---|---|---|
| Min | | $Q_1$ | | $Q_2$ | | | $Q_3$ | | Max |

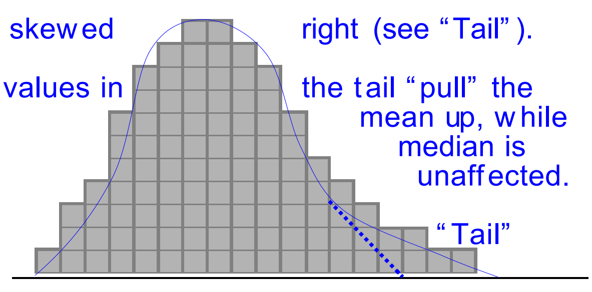
IQR = 5.5 – 4.8 = .7

Test value for outlier = 1.5 (.7) = 1.05 No outliers....

(6) 4. For the data below, choose (circle the letter of) the best completion of these two statements:

It appears:
the mean and median are the same.
the median is greater than the mean.
the mean is greater than the median.
there is no median, but there is a mean.
there is insufficient information to draw any conclusion about the relative positions of the mean and median.

The data is skewed right (see "Tail").
The higher values in the tail "pull" the mean up, while median is unaffected.

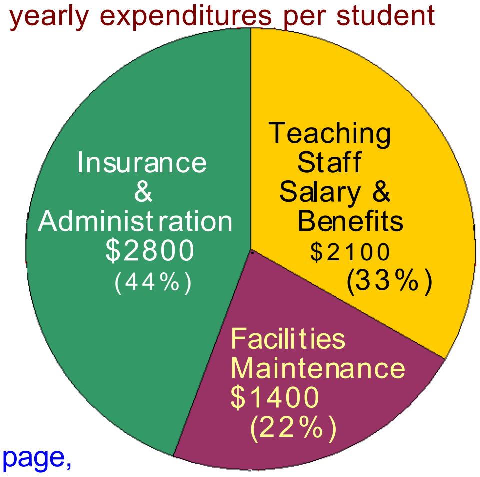



The standard deviation *could* be:
0 ← Data shown is not all one value!
*about* 10 ← Most of the data is further than 10 from the mean.
*about* 50 ← Most of the data falls within 100 units of the mean.
*about* 200 ← All the data is closer to the mean than 200.
There is a standard deviation, but it could not be near any of the values above.
There is insufficient information to draw *any* conclusion about the standard deviation.

(9) 5. Showing ALL your work, construct a pie chart illustrating the distribution of school funds / student. In the UFSD Unified School District, the following yearly expenditures are made for each student:

| | |
|---|---|
| Teaching staff salary & benefits: | $2100 |
| School plant maintenance: | $1400 |
| Insurance & Administration: | $2800 |
| Total expenditure per student: | $6300 |

UFSD Unified School District
yearly expenditures per student




Teaching salary etc. merits 2100/6300, or
1/3 of the circle.
1/3 of 360° or 120° sector

School plant and maintenance should have
1400/6300 = 2/9 ...2/9 of 360° = 80° sector

Ins. & admin. get 2800/6300 = 4/9 of the pie,
or a sector with 4/9 of 360° = 160° central angle.

Note: the above constitutes "showing your work".
Notice this does not mean scribbling computations all over the page, but making clear how the work is being done.

(12) 6. Showing your work, for the quiz scores in the frequency table at right:
a. calculate the **mean**. b. calculate the **standard deviation**.

| *quiz score* | *frequency* |
|---|---|
| *6* | *4* |
| *8* | *1* |
| *10* | *1* |

6a. Mean $= \dfrac{\Sigma(x)}{n}$

$$= \frac{4 \cdot 6 + 8 + 10}{6} = \frac{42}{6} = 7$$

NOTE: A mean lower than 6 or higher than 10 is clearly impossible! (Why?)

6b. Variance $= \dfrac{\Sigma(x - \text{mean})^2}{n}$

$$= \frac{4 \cdot (6-7)^2 + (8-7)^2 + (10-7)^2}{6} = \frac{4 \cdot 1 + 1 + 9}{6} = \frac{14}{6}$$

Std Dev $= \sqrt{7/3}$

NOTE: Std dev should be more than 0 and less than 2. It is!

(5) 7. The 25 students in Miss Horne's 4th-graders averaged 70 on the state reading test; the remaining 15 4th-graders, in Mr. King's class, averaged only 50. What is the average of all the fourth graders?

mean $= \dfrac{\text{Total (of all tests) points}}{\text{\# of tests}}$

$= \dfrac{25 \cdot 70 + 15 \cdot 50}{25 + 15}$ ←Total points from Horne's class, plus total from King's class.
← # of tests from Horne's plus # from King's class.
Notice Horne's class should have more "weight" in computing the mean, and it does....

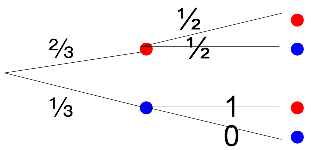
$= \dfrac{1750 + 750}{25 + 15}$

$= \dfrac{2500}{40}$

$= 62.5$

Notice the distance to 70 (average of 25 students) is 7.5 and the distance to 50 is 12.5 (average of 15 students).
‼The 25 students in Horne's class have more influence on the location of the mean than the 15 students in King's class.

(9) 1. A jar contains three marbles: two red, one blue. Two marbles are drawn from the jar.
  a. What is the probability the marbles drawn are both red?
  b. What is the probability the marbles drawn are both blue?
  c. What is the probability the two different-color marbles are obtained?




●● ⅓
●● ⅓
●● ⅓
●● 0

a. P( ●● ) = P(● 1st) P(● 2nd | ● 1st ) = (⅔)(½) = ⅓

(See part c below)

b. P( ●● ) = P(● 1st) P(● 2nd | ● 1st ) = (⅓)(0) = 0

c. P(two different) = P(●●) + P(●●) = (⅔)(½) + (⅓)(1) = ⅔

Notice the probabilities on the branches from a single point must always add up to 1.
Anything else would not make sense! (WHY?)

Notice that the events listed in parts a and b and c make up the entire sample space with no overlap--therefore these probabilities must all add up to 1.

(21)2. You take a card at random from the partial deck shown.
Let "D" be the event the card drawn is a diamond (♦).
Let "H" be the event the card drawn is a heart (♥).
Let "K" be the event the card is a King.

| | | | |
|---|---|---|---|
| A♥ | A♦ | A♣ | A♠ |
| K♥ | K♦ | K♣ | K♠ |
| Q♥ | Q♦ | Q♣ | Q♠ |
| J♥ | J♦ | J♣ | J♠ |

a. P(H) = P(♥) = $^4/_{16}$ = $^1/_4$
P(D) = same as above, obviously
P(K) = $^4/_{16}$ = $^1/_4$
b. P(D or K) = P(♦) + P(K) – P(K ♦) = $^4/_{16}$ + $^4/_{16}$ – $^1/_{16}$
c. P(D and K) = P(K ♦) = $^1/_{16}$

d. Are H & D *independent events*? NO!!! How do you know?

"Independent" means each one has no effect on the other.
If the card drawn is a heart(H) it cannot be a diamond... (Now that's an effect!!!) [ P(D| H) = 0 ]

When events are independent, P( H & D) = P(H) P(D) But 0 ≠ (½)(½)

e. Are H & D *mutually exclusive events*? ________ How do you know?

Yes, H & D are mutually exclusive– if one happens, the other has Probability 0.
P(D| H) = 0
P(D| H) = 0

Yes, P(H & D) = 0

Notice that independence and mutual exclusivity are NEARLY OPPOSITE in their meaning.
If events are independent, the occurrence of one has NO EFFECT on the probability of the other.
If events are mutually exclusive, one's occurrence virtually PRECLUDES the other.

(6) 4a. Cars approaching a certain red light from the west were monitored. Out of 400 cars approaching the light, 32 ran the red light. What is the probability that the next that car approaches the red light from the west will run the red light?

Based on the best information we have (which is past history): 32/400 or 2/25 or 8%.

4b. A fair coin has been tossed seven times; heads have turned up four times. What is the probability the next toss of the coin will turn up tails?

It's a fair coin, so P(tails) is always ½

NOTE in part a, our past experience has a bearing on the probability we state. Not in part b! WHY?

(3) 5. Find the probability of obtaining *exactly* 1 *head* in 3 tosses of a fair coin.

You can (and SHOULD) draw a tree diagram and see that there are three ways for this to happen:

P( exactly 1 H in 3 tosses) = P( HTT or THT or TTH )

= P(HTT) + P(THT) + P(TTH)

= P(H)P(T)P(T) + P(T)P(H)P(T) + P(T) P(T)P(H)

= ( ½ )( ½ )( ½ ) + ( ½ )( ½ )( ½ ) + ( ½ )( ½ )( ½ )

= $^{1}/_{8}$ + $^{1}/_{8}$ + $^{1}/_{8}$

= $^{3}/_{8}$

(8) 7a. In a game in which the *odds against you* are **5:2**, what is the probability of winning?

☹ ☹ ☹ ☹ ☹ ☺ ☺

If the ODDS are against you, 5 to 2, it is as if there are 7 outcomes, and only 2 favor you.

So your chance of winning is 2/7.

7b. The probability of rain tomorrow is 20%. What are the odds against rain tomorrow?

P(rain) = 20% ....so P(not rain) = 80%

The odds are in fact against rain.... 8 to 2 ( or 4 to 1)

(3) 8. A probability experiment has four possible outcomes: $e_1$, $e_2$, $e_3$, $e_4$.
The outcome $e_1$ is twice as likely as each of the remaining outcomes.
Find the probability of $e_4$.

$P(e_1) + P(e_2) + P(e_3) + P(e_4) = 1$ We know the sum of all the probabilities must be 1.

$2x + x + x + x = 1$ ... and $e_1$ is twice as likely as the other outcomes.

$5x = 1$ I decided to call $P(e_4)$ “x” to make life easier....

$x = {}^{1}/_{5}$