Math 310 SAMPLE Test #1B Statistics & Probability NAME _____

SHOW WORK *as if making a solutions guide for students*. Use backs of paper for scratch.

Don't *forget titles, labels, scales, & legends!*

#1-3 Mr. Oversite weighed his students (in pounds). Below are the results:

42 41 58 45 53 68 52 65 47 81

(5) 1. Classify the data in an ordered stem-and-leaf diagram so there are at least five classes.

(6) 2. Draw a complete histogram using classes which correspond to the classes you created in problem (1).

(12) 3. For the data in problem 1, **find** the quartile marks, and present a **box and whiskers** plot.

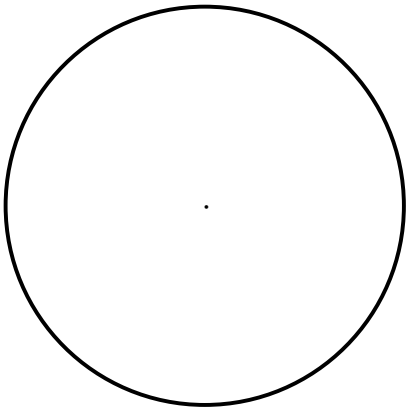
(12) 4. Showing your work, for the six quiz scores in the frequency table at right:
a. calculate the **mean**. b. calculate the **standard deviation**.

Quiz score	Number of students
6	4
8	1
10	1

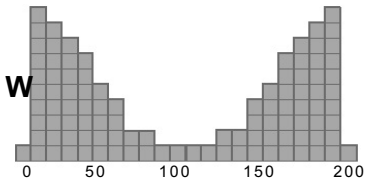
(9) 5. Construct a pie chart illustrating the distribution of school fund sources.
In the LASQ Unified School District, sources of revenue for school year 2007-8 were:

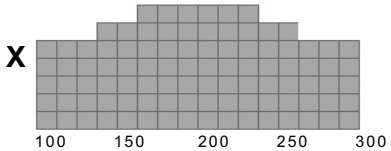
State education funds:	$ 3 billion
Federal education funds:	$ 0.5 billion
Reserves from prior years:	$ 1 billion
Local and other sources:	$ 1.5 billion

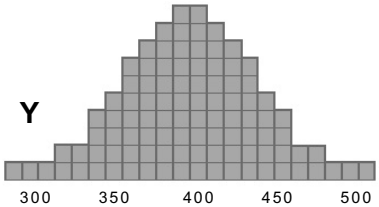
In the box below, show in detail how you find the portion to allocate for Reserves.



(5) 6. The mean value of the 142 art objects in the Sing collection in 1960 was $40,000, with a standard deviation of $12,000. No changes have been made to the collection, but art objects of this type have tripled in value over the last 48 years. What are the MEAN and STANDARD DEVIATION of those values now?







(5) 7a. Of the three distributions shown above, which one (W, X or Y)...

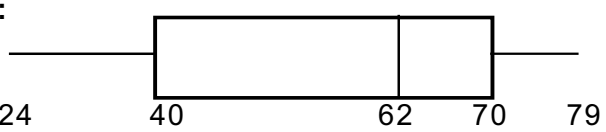
(i). has the smallest standard deviation? ____ (ii) has the greatest standard deviation? ____

7b. Data B has the box plot shown at right.

The range of B is _____.

The interquartile range for B is _____.

The middle fifty-percent of data lies between _____ and _____.



MATH 310 Test #1B: Probability SHOW appropriate work! NAME:

(6) 8. The names (in English) of the days of the week are placed on identical cards in a bag. One card is selected at random from the bag.

a. List a Sample Space (SS) for this experiment.

b. List the event that the FIRST letter of the word on the card is "S".

c. What is the probability of the event in part b?

(6) 9. A fair die is rolled. What is the probability that:

a. the number that turns up is EVEN?

b. the number that turns up is a MULTIPLE OF 3?

c. the number that turns up is EVEN or a MULTIPLE OF 3?

(13) 10. A jar contains three marbles: one RED, two BLUE. Two marbles are randomly drawn from the jar, first with the right hand, second with the left.

a. Draw a complete tree diagram for this experiment.

b. What is the probability the marbles drawn are both RED?

c. What is the probability the first marble is RED and the second one is BLUE?

d. What is the probability that two DIFFERENT-COLOR marbles are obtained?

(6) 11a. (2) A fair die has been rolled four times; the numbers that have turned up are 1,2,3,and 4. What is the probability the next roll of the die will turn up 5?

11b. Find the probability of obtaining EXACTLY ONE five in three rolls of a fair die, showing how you got your answer.

TEST #1B Probability SHOW appropriate work! NAME:

(8) 12a. In a game in which your odds are **1:3**, what is the probability of winning?

12b. If the probability of rain on tomorrow is 30%, what are the odds against rain tomorrow?

(4) 13. A probability experiment has four possible outcomes: e_1, e_2, e_3, e_4.
The outcome e_1 is three times as likely as each of the remaining outcomes.
Find the probability of e_4.

(4) 14. A pair of fair dice is tossed. What is the probability that the dice will NOT turn up two sixes?

(4) 15. Given:
1/4 of the class are athletes; 1/2 of the class are girls; 1/3 of the girls are also athletes.
What part of the athletes are girls?

Math 310 Sample Test #1 Statistics & Probability Solutions Guide

Don't forget labels, scales, & legends!

#1-3 Given weights of MR. OVERSITE's students, in pounds:

42 41 58 45 53 68 52 65 47 81

1. Classify the data in **a** stem-and-leaf diagram. so there are at least five classes.

Weights of MR. OVERSITE's students, in pounds

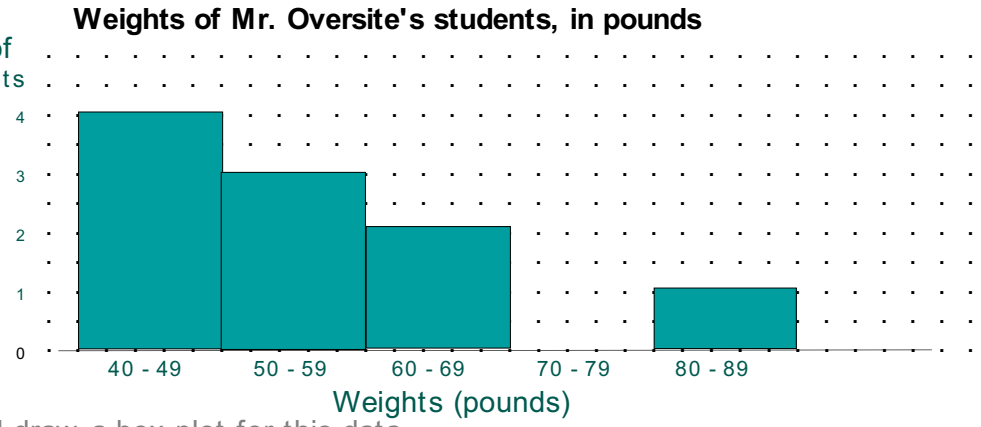
4	1 2 5 7
5	2 3 8
6	5 8
7	
8	1

Legend

6 | 5 8

represents weights 65 & 68 pounds, for 2 students

2. Draw a histogram using classes which correspond to those in problem (1).



3. Find the quartile marks and draw a box plot for this data.

Weights, in pounds, of ten students in Mr. Oversite's class

A box plot MUST show values spaced proportionally

WORK →

41 42 45 47 52 53 58 65 68 81

Min Q_1 Q_2 Q_3 Max

IQR = 65 – 45 = 20

Test value for outlier = 1.5 (20) = 30 ⇨ No outliers.

4. Showing your work, for the six quiz scores in the frequency table at right:
a. calculate the **mean**. b. calculate the **standard deviation**.

quiz score	*frequency*
6	4
8	1
10	1

4a. Mean = $\frac{\Sigma(x)}{n}$

= $\frac{4\cdot 6 + 8 + 10}{6}$ = $\frac{42}{6}$ = **7**

For this data, a mean lower than 6 or higher than 10 is clearly impossible!

Note: Data higher by 1 yields a mean higher by 1, etc.

4b. Variance = $\frac{\Sigma(x - \text{mean})^2}{n}$

= $\frac{4\cdot(6-7)^2 + (8-7)^2 + (10-7)^2}{6}$ = $\frac{4\cdot 1 + 1 + 9}{6}$ = $\frac{14}{6}$

Std Dev = $\sqrt{7/3}$

Std dev for this data should be more than 0 and less than 2. And it IS !

5. Construct a pie chart illustrating the distribution of school fund sources.
In the LASQ Unified School District, sources of revenue for school year 2007-2008 were:

State education funds:	$ 3 billion
Federal education funds:	$0.5 billion
Reserves from prior years:	$ 1 billion
Local and other sources:	$1.5 billion
Total revenue:	$6 billion

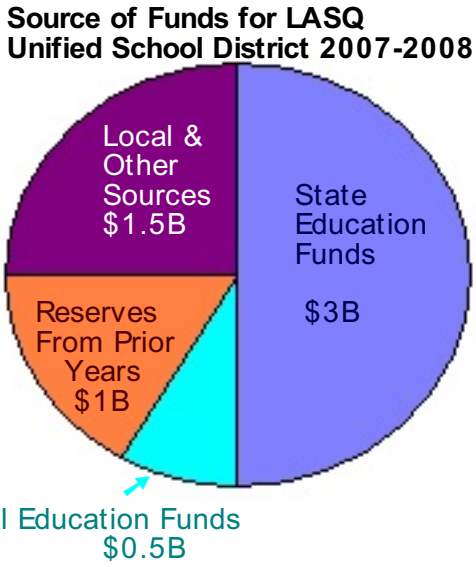
State Ed Funds. Accounts for 3/6, or 1/2 of the circle.
1/2 of 360° or 180° sector

Federal Ed Funds make up .5/6 = 1/12 of the total:
...1/12 of 360° = 30° sector

Reserves are 1B/6B= 1/6 of the pie,
or a sector with 1/6 of 360° = 60° central angle.

Local & Other cover 1.5/6 = 1/4 1/4 of 360° = 90°

Note: the above constitutes "showing work".
Show work does not mean scribble computations all over the page, but make clear how the work is done.



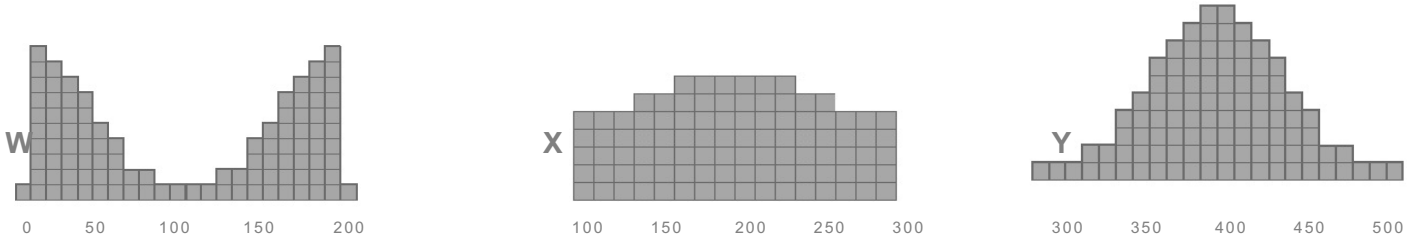
6. The mean value of the 142 art objects in the Sing collection in 1960 was $40,000, with a standard deviation of $12,000. No changes have been made to the collection, but art objects of this type have tripled in value over the last 48 years. What are the mean and standard deviation of those values now?

If each object tripled in value, then the mean value has tripled also.

The new mean is 3 · the old mean = 3· $40,000 = **$120,000** (3·$50,000 = $150,000)

The new standard deviation is also tripled, since the spread in the values tripled.

The new standard deviation = 3·(the old s.d.) = 3·$12,000 = **$36,000** (3·$15,000 = $45,000)



(5) 7a. Of the three distributions shown above, which one (W, X or Y)...

(i). has the smallest standard deviation? **Y** (ii) has the greatest standard deviation? **W**

Since these distributions are symmetric, the means are at the center.
Each distribution has a range of 200.
So comparison of the spread away from the mean is uncomplicated.
The data in Y is most clustered about the mean.
The data in W has by far the most values spread out to the maximum distance from the mean.

7b. Data B has the box plot shown at right.

The range of B is maximum – minimum = 79 – 24 = **55**

The interquartile range for B is Q_3 – Q_1 = 70 – 40 = **30**

The middle fifty-percent of data lies between **40** and **70**.

B:

(6) 8. The names (in English) of the days of the week are placed on identical cards in a bag. One card is selected at random from the bag.

a. List a Sample Space (SS) for this experiment.

{ Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday }

b. List the event that the FIRST letter of the word on the card is "S".

{ Sunday, Saturday }

c. What is the probability of the event in part b?

2/7

(6) 9. A fair die is rolled. What is the probability that:

a. the number that turns up is EVEN? P({2,4,6}) = **3/6**

b. the number that turns up is a MULTIPLE OF 3? P({3,6}) = 2/6 = **1/3**

c. the number that turns up is EVEN or a MULTIPLE OF 3? P({2,3,4,6}) = 4/6 = **2/3**

(13) 10. A jar contains three marbles: one RED, two BLUE. Two marbles are randomly drawn from the jar, first with the right hand, second with the left. (Note: you never let go of your marbles!)

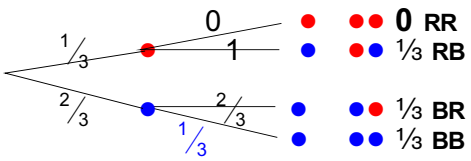
a. Draw a complete tree diagram for this experiment.

b. What is the probability the marbles drawn are both RED?

c. What is the probability the first marble is RED and the second one is BLUE?

d. What is the probability that two DIFFERENT-COLOR marbles are obtained?

a. tree diagram:



b. P(●●) = P(● 1st) P(● 2nd | ● 1st) = $(\frac{1}{3})\cdot 0$ = **0**

This work was already done in the tree diagram.

c. P(●●) = P(● 1st) P(● 2nd | ● 1st) = $\frac{1}{3}$

d. P(two different) = P(●●) + P(●●) = $\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$

The branch that contains the 0 probability generally would be omitted. It is shown here only to be as inclusive as possible.

(6) 11a. (2) A fair die has been rolled four times; the numbers that have turned up are 1,2,3,and 4. What is the probability the next roll of the die will turn up 5?

$\frac{1}{6}$ (The die does not care what already turned up, nor what happens next.)

11b. Find the probability of obtaining EXACTLY ONE five in three rolls of a fair die, showing how you got your answer. (Tree not required)

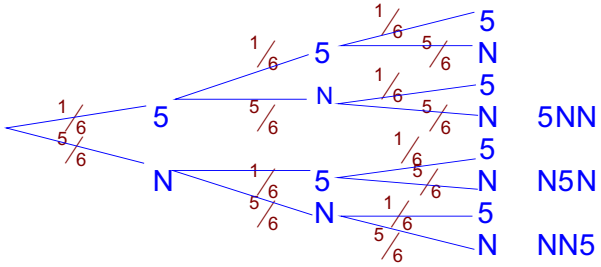
P(one "5" in 3 rolls of fair die)

= P(5NN or N5N or NN5)

= P(5)P(N)P(N) + P(N)P(5)P(N) + P(N)P(N)P(5)

= $(\frac{1}{6})(\frac{5}{6})(\frac{5}{6}) + (\frac{5}{6})(\frac{1}{6})(\frac{5}{6}) + (\frac{5}{6})(\frac{5}{6})(\frac{1}{6})$

= $\frac{75}{216}$



(8) 12a. In a game in which your odds are **1:3**, what is the probability of winning?

If the ODDS of winning are 1:3, then out of every four outcomes, 1 is favorable, 3 not....

The probability of winning is **1/4** ☺ ☹ ☹ ☹

In a game in which your odds are 2:3, the probability of winning is 2/5.

12b. If the probability of rain on tomorrow is 30%, what are the odds against rain tomorrow?

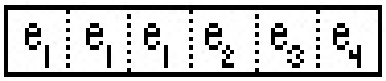
P(rain) = 3/10, P(not rain) = 7/10 ➔ 

3 for rain 7 against

The odds against rain tomorrow are 7 : 3 .

If the probability of rain tomorrow is 10%, the odds against rain tomorrow are 9:1.

(4) 13. A probability experiment has four possible outcomes: e_1, e_2, e_3, e_4.
The outcome e_1 is three times as likely as each of the remaining outcomes.
Find the probability of e_4.



All the probability (1, as in 100%) is distributed as shown.
The amount of probability allotted to each cell is the same.

"e_1 is three times as likely as..."

$6 \cdot x = 1$

$x = \frac{1}{6}$

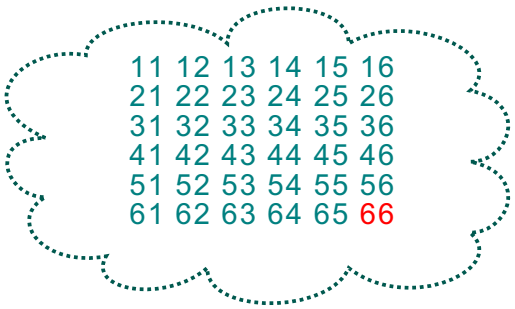
$P(e_4) = \frac{1}{6}$

(4) 14. A pair of fair dice is tossed. What is the probability that the dice will NOT turn up two sixes?

P(pair will not turn up "66") = 1 – P("66")

$= 1 - \frac{1}{36}$

$= \frac{35}{36}$



(4) 15. Given:
1/4 of the class are athletes; 1/2 of the class are girls; 1/3 of the girls are also athletes.
What part of the athletes are girls?

Using a number:

Pick a nice number to work with.
Suppose there are 60 in the class.
¼are athletes, so 15 are athletes.
½ are girls, so 30 are girls.
⅓ of the girls (⅓ of 30) are athletes,
So 10 girls are athletes.

Of 15 athletes, 10 are girls.
So **2/3** of the athletes are girls.

Using a formula:

$P(\text{girl} \mid \text{athlete}) = \frac{P(\text{girl \& athlete})}{P(\text{athlete})}$

$= \frac{(\frac{1}{2})(\frac{1}{3})}{\frac{1}{4}}$ ←P(girl)·P(athlete| girl)

$= \mathbf{\frac{2}{3}}$