MATH 310 ♡ Self-Test ◊ Transformation Geometry, Similarity and Symmetry ♤ s67

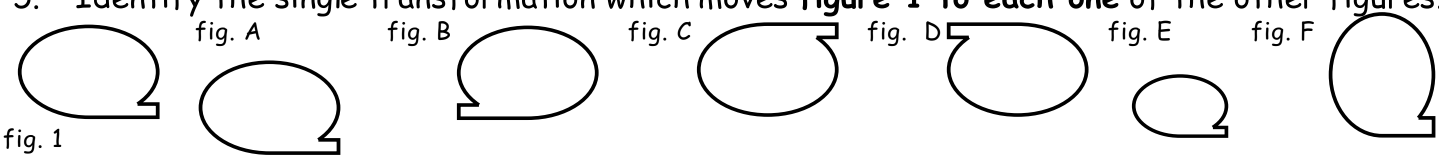
1. Explain isometry (rigid transformation or rigid motion) of the plane.
   Name and describe* the four types of isometries of the plane.
   (* what happens to the points of the plane under this transformation? How are they moved? What happens to figures in the plane? In what way are they changed? Be specific for each different type of transformation.)

2. Any transformation of the plane can be accomplished by a series of reflections. Explain.

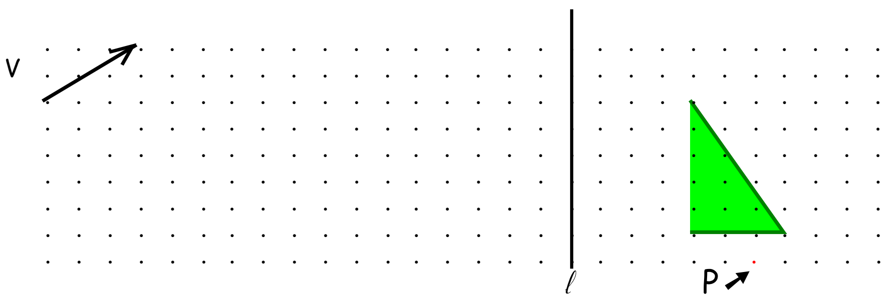
3. Which transformation(s) do not change the direction of a figure?
   (I.e. if the figure is "facing" a particular direction in the plane, the image after the transformation will face the same direction.)

4. Which transformation(s) change the orientation (clockwise sense) of a figure?

5. Identify the single transformation which moves **figure 1 to each one** of the other figures.



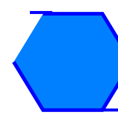
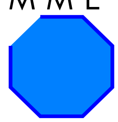
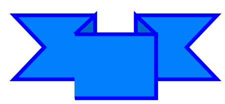
6. Sketch images of the figures under the isometries described at right. Follow directions.



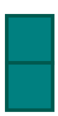
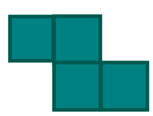
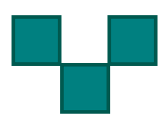
Translate the figure A by vector V, label that A'.

Then rotate A' 90° about P ; label that A''.

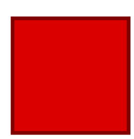
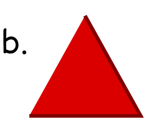
Then reflect A'' through line ℓ . Label the final result A'''

7. Briefly describe all symmetries of
   a. a square b. a regular pentagon. (Sketches are appropriate.) c. 
   d. The individual letters in "S Y M M E T R I C".
   e.  f.  g. 

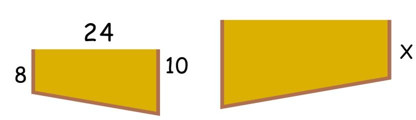
8. Add squares (congruent to existing squares) to each of the following figures to make the figure have a line symmetry, but no rotational symmetry. (What is the minimum needed?)

a.  b.  c. 

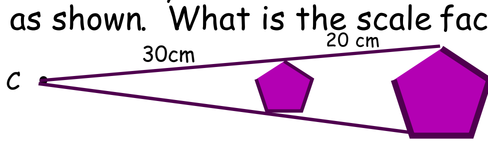
9. Add to each figure at right so the resulting figure has rotational symmetry but no line symmetry.
   (Assume these are regular polygons.)

a.  b. 

10. Given the figures at right are similar,
a. What is the scale factor from the left figure to the right?
b. Find x .

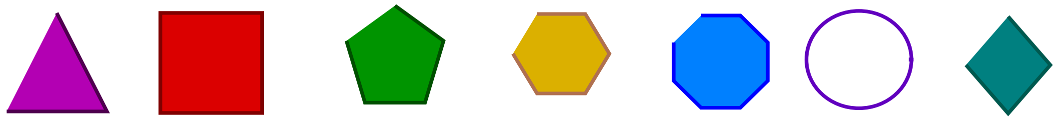


11. In a dilation of the plane with center C, the distances between C and a figure and its image are as shown. What is the scale factor of the transformation?



Some more detailed questions:

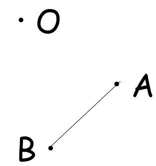
A1. How many rotational symmetries has each figure? How many line symmetries?



A2. Is it possible to generate a given translation by using a series of reflections?
Is it possible to generate a given rotation by using a series of reflections?
Is it possible to generate a given glide-reflection by using a series of reflections?
(If yes to all the above, then we'd say the set of reflections generates all possible isometries of the plane;
every type of isometry can be accomplished by a series of reflections.)

A3. Can translations be used to generate all possible isometries of the plane? Why or why not?
What about rotations? ...glide reflections?

A4. Using any of the tools we have employed:
rotate line segment AB by +45° about center O.



Now we get really esoteric:

A5. Match each description or attribute of a transformation with the most appropriate transformation (T R M GR):
- a. No point of the plane is fixed; a figure and its image have the same orientation.
- b. Every point of the plane moves exactly the same distance.
- c. Exactly one point of the plane is fixed ("fixed" means doesn't move).
- d. No point of the plane is fixed; a figure must be "flipped over" to match its image.

A6. Select the appropriate transformation(s) to fit each description below.
T = Translation, **R** = Rotation, **M** = Reflection, **GR** = Glide Reflection
- a. The composition of two translations is (always/sometimes/never) a (T R M GR).
  [How do points move if translated by vector U then V?]
- b. The composition of two reflections through parallel lines is a (T R M GR).
  [How do points move if reflected through line l, then through parallel line m? How far? In what direction?]
- c. The composition of two rotations about a common center is a (T R M GR).
- d. It is possible for the composition of two reflections to be (T R M GR).
- e. It is possible for the composition of two rotations to be (T R M GR).

A7. Is the composition of rotations commutative? (You should know what that means:
given any two rotations, is the result the same regardless of the order in which they are performed? ,,.or does order matter?)

A8. Match each description or attribute of a transformation with the most appropriate transformation (T R M GR):
- a. Moves every point of the plane the same distance and direction.
- b. Determined by a vector, or arrow, that specifies the distance and direction each point is moved.
- c. Composition of two of these is always one of these. Composition is commutative (order doesn't matter).
- d. Moves points on parallel straight paths. Figures are never reversed.
- e. Moves points on concentric circular paths. Figures are never reversed.
- f. Determined by a directed angle that specifies the center, amount and direction of __.
- g. Points further from the center move a greater distance.
- h. Determined by a line. Distances moved are proportional to distances from this line.
- i. Points move on parallel paths, but not equal distances. Figures are reversed.
- j. Points do not move on parallel paths, but figures are reversed.
- k. Determined by a line and a vector.