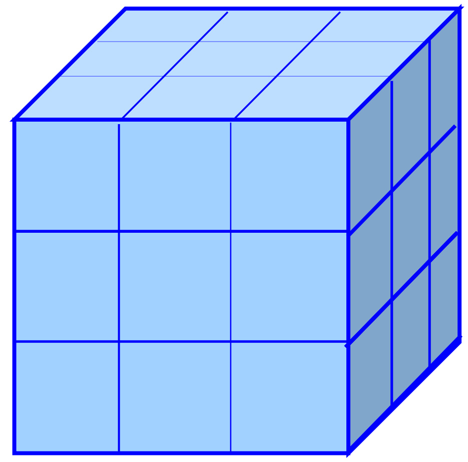
1. Use the drawing below to illustrate the relationship between cubic yards and cubic feet.
2. Show ↓ the dimensional analysis for the conversion of 2 cubic yards to cubic feet. 2 yd^3 = __54__ ft^3



27 cubic feet are required to fill the cubic yard–
"three layers,with nine 1-foot-cubes making up each layer".
(The nine one-ft-cubes are visible on the top layer.)

Here is the dimensional analysis:

$$2\ yd^3 = 2\ yd^3 \bullet \frac{3\ ft}{1\ yd} \bullet \frac{3\ ft}{1\ yd} \bullet \frac{3\ ft}{1\ yd}$$

$$= 2 \bullet 3^3\ ft^3$$

$$= 54\ ft^3$$

3. Convert each of the following, showing the dimensional analysis.

a. 5.2cm = __0.0052__ dam

$$5.2\ cm = 5.2\ cm \bullet \frac{1\ m}{100\ cm} \bullet \frac{1\ dam}{10\ m} = 5.2 \times 10^{-3}\ dam = 0.0052\ dam$$

b. 0.67 m^3 = __670000__ cm^3

$$0.67\ m^3 = 0.67\ m^3 \bullet \frac{100cm}{1\ m} \bullet \frac{100cm}{1\ m} \bullet \frac{100cm}{1\ m} = 0.67 \bullet 100^3\ cm^3 = 670000\ cm^3$$

OR $0.67\ m^3 = 0.67m^3 \bullet \left(\frac{100cm}{1\ m}\right)^3 = .67m^3 \bullet \frac{10^6\ cm^3}{1\ m^3} = .67 \bullet 10^6\ cm^3 = 670000cm^3$

c. 2500 mL water = _____ kL

$$2500\ mL = 2500\ mL \bullet \frac{1\ L}{1000\ mL} \bullet \frac{1\ kL}{1000\ L} = .0025\ kL$$

d. 36 ft^3 = ______ in^3

$$36\ ft^3 = 36\ ft^3\ \frac{12\ in}{1\ ft}\ \frac{12\ in}{1\ ft}\ \frac{12\ in}{1\ ft} = 62208\ in^3$$

4. Find the mass, in kilograms, of water (at 4°C) needed to fill a tank 1 meter wide, 1 meter high and 1 meter deep, including the dimensional analysis that leads to your answer.

So we have 1 cubic meter of water at its most dense state. We know that 1 cm^3 of such water has a mass of 1 gram, so if we convert this to cubic centimeters, we can then convert to grams.

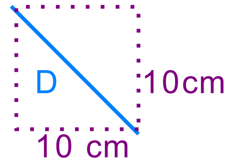
$$1\ m^3 = 1\ m^3 \bullet \frac{100cm}{1\ m} \bullet \frac{100cm}{1\ m} \bullet \frac{100cm}{1\ m} \bullet \frac{1\ g}{cm^3} \bullet \frac{1\ kg}{1000g} = \frac{1000000\ kg}{1000} = 1000\ kg$$

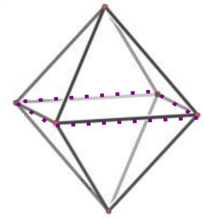
Notice at this point, we have 1,000,000 cm^3
... and at the next point, we have 1,000,000 grams

Of interest: 1000 kg is a METRIC TON. 1 cubic meter of water (at its most dense) has a mass of one metric ton. How many pounds (approximately) is a metric ton?

5. **A REGULAR OCTAHEDRON has 10cm edges. Show your work on each of the following:**

If we slice through the regular octahedron on a plane containing four edges (think equator), we see a square. Find the diagonal length of that square.





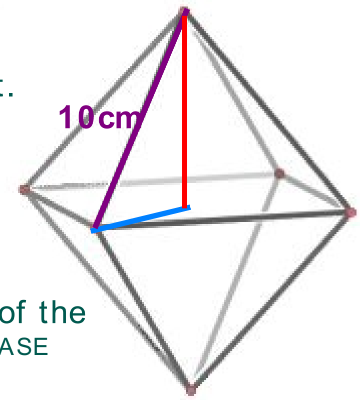
The edges of that square are 10cm each.
The diagonal is the hypotenuse of a right triangle with sides of length 10cm.

$D^2 = (10cm)^2 + (10cm)^2$

$D^2 = 2\,(10cm)^2$ or $D^2 = 200\ cm^2$

$D = 10\sqrt{2}\ cm$ or $D = \sqrt{200}\ cm$

Find the height of the octahedron.



There are a number of ways to do this. The SIMPLEST is to recall that in a regular polyhedron, all edges, angles, vertices are congruent. So the distance from vertex to diametrically opposite vertex is the same for any of the three diametrically opposed pairs of vertices. Therefore, the "HEIGHT" of the regular octahedron is the same as the "DIAMETER" of the octahedron, $10\sqrt{2}$ cm or $\sqrt{200}$ cm.

METHOD 2:
Using the diagram at right, we see that the height of the TOP HALF of the octahedron is part of a RIGHT TRIANGLE with HYPOTENUSE 10 cm, and BASE half the diameter we found above.

$h^2 + (5\sqrt{2}\ cm)^2 = (10cm)^2$

$h^2 + 50\ cm^2 = 100\ cm^2$

$h^2 = 50\ cm^2$

$h = 5\sqrt{2}\ cm$

And of course the HEIGHT OF THE OCTAHEDRON is twice this value, or $10\sqrt{2}$ cm

Find the volume of the octahedron.

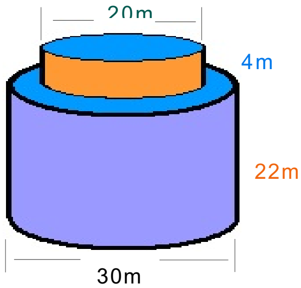
The octahedron's volume can be viewed as that of two square-based pyramids joined at their bases. Thus volume is one-third of the product of the area of base and the height.

$V = (⅓)\ (\text{Area of Base})\ (\text{Height})$

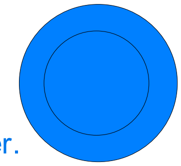
$V = (⅓)\ (10\ cm)^2\ (10\sqrt{2}\ cm)$

$V = \frac{1000\sqrt{2}}{3}\ cm^3$

6. Find the ENTIRE SURFACE AREA of *the SOLID object illustrated below*.
Assume all curves are circular, and all edges that appear vertical are.



There are four surfaces to add up:



(1) The "roof": consisting of a 20m circular disc surrounded by a ring extending the disc to 30m diameter.
Area is $\pi\,(15\text{m})^2 = 225\pi\ \text{m}^2$

(2) Upper lateral wall: a band* 20πm long, 4m high.
Area = $(20\pi\text{m})(4\text{m}) = 80\pi\ \text{m}^2$

(3) Lower lateral wall: a band* 30πm long, 22m high.
Area = $(30\pi\text{m})(22\text{m}) = 660\pi\ \text{m}^2$

(4) The base (not seen in this view): same size as the "roof".

Thus the entire surface area is

$$SA = 225\pi\ \text{m}^2 + 80\pi\ \text{m}^2 + 660\pi\ \text{m}^2 + 225\pi\ \text{m}^2 = 1190\pi\ \text{m}^2$$

* band = long rectangle, such as the label wrapped around a can.

7. Find the volume of the observatory illustrated at right, consisting of a 10-foot high right circular cylinder topped by a hemispherical dome with diameter 12 feet. (Ignore the steps!)
See also http://commons.wikimedia.org/wiki/Image:Aldershot_observatory_01.JPG

The volume of the observatory may be found via
Volume of hemispherical top + Volume of cylindrical base
The diameter of both is 12 feet.
The height of the cylinder is 10 feet.

$$V_{hemisphere} = (\tfrac{1}{2})\, V_{sphere} = (\tfrac{1}{2})\,(\tfrac{4}{3})\,\pi\, r^3$$

$$= (\tfrac{1}{2})\,(\tfrac{4}{3})\,\pi\,(6\text{ft})^3$$

$$= 144\pi\ \text{ft}^3$$

$$V_{cylinder} = (\text{Area of Base})(\text{Height}) = \pi\, r^2\,(\text{Height})$$

$$= \pi\,(6\text{ ft})^2\,(10\text{ ft})$$

$$= 360\pi\ \text{ft}^3$$

Volume of the observatory = $V_{hemisphere} + V_{cylinder}$

$$= 144\pi\ \text{ft}^3 + 360\pi\ \text{ft}^3$$

$$= 504\pi\ \text{ft}^3$$

8. What is the Surface Area (of the exposed surface) of the building above? (Ignore the steps!)

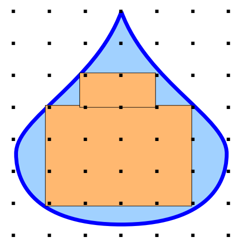
As in #7, we find two parts— SA of hemispherical dome & SA of cylinder's lateral wall

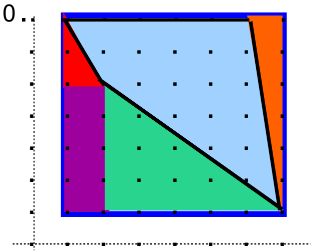
$$SA_{dome} = (\tfrac{1}{2})\text{ of } SA_{sphere} = (\tfrac{1}{2})\,4\,\pi\, r^2 = (\tfrac{1}{2})\,4\,\pi\,(6\text{ ft})^2 = (\tfrac{1}{2})\,144\,\pi\ \text{ft}^2 = 72\,\pi\ \text{ft}^2$$

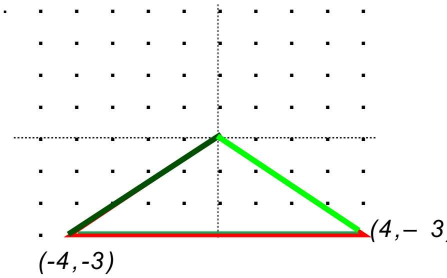
$$\text{Lateral } SA_{cylinder} = \text{Circumference} \bullet \text{Height} = 2\,\pi\, r\, h = 2\,\pi\,(6\text{ ft})\,(10\text{ ft}) = 120\,\pi\ \text{ft}^2$$

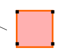
Surface Area = $SA_{dome} + LSA_{cylinder}$

$$= 72\,\pi\ \text{ft}^2 + 120\,\pi\ \text{ft}^2 = 192\,\pi\ \text{ft}^2$$

9.


10.


11.


In all the above, the area shown here is one square unit.


9. ESTIMATE THE AREA of the figure in #9.

We can see 3•4 + 2 = 14 square units lie entirely within the curve, and sixteen square units have part within the curve, and part outside it.

We estimate that the partial squares within the curve average half the area of each of those sixteen squares, so that half of sixteen square units lie within the curve.

Thus our estimate is 14 + (½) 16 square units = 22 square units.

10. FIND THE AREA enclosed by the figure in #10. (Curve turns at points: (7,1) & (6,7) & (1,7) & (2,5))

We begin with the area of the rectangle covering the entire polygon, then subtract the areas of the four peripheral figures that are outside the given curve.

$A_{curve} = A_{rectangle} - (A_{\text{Upper right triangle}} + A_{\text{Lower right rectangle}} + A_{\text{large triangle}} + A_{\text{Left triangle}})$

$= 6 \cdot 6\ u^2 - ((½)\ 2 \cdot 1 u^2 + 4 \cdot 1\ u^2 + (½)\ 5 \cdot 4\ u^2 + (½)\ 1 \cdot 6\ u^2)$

$= 36\ u^2 - (1u^2 + 4\ u^2 + 10\ u^2 + 3\ u^2)$

$= 18\ u^2$

11. FIND THE PERIMETER of the triangle in figure #11 above

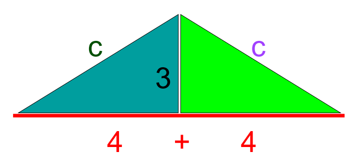
PERIMETER is distance around triangle = sum of lengths of sides of triangle

P = length from (– 4,– 3) to (4,– 3) + length from (4,– 3) to (0,0) + length from (0,0) to (– 4,– 3)

To find the latter two lengths, we use the Pythagorean theorem:

$c^2 = (3u)^2 + (4u)^2 = 25u^2$

c = 5 u (& c= 5 also)



P = 8 units + 5units + 5 units = 18 units

12. Place the following measurements in order, least to greatest:

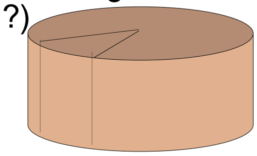
a. .003 km, 0.5 m, 2 in, 10 cm

2 in < 10 cm < 0.5 m < .003 km
.1 m   3 m

b. 0.5 qt, 1 gal, 5 dL, 100 mL, 900 cm^3

100 mL < 0.5 qt < 5 dL < 900 cm^3 < 1 gal
5 dL: 500 mL, ½ L
900 cm^3: close to 1000 cc which is 1 Liter
1 gal: 4 qts close to 4 Liters

13. Find the volume of a 40° wedge of cake cut from a 8" round cake that is $^{15}/_{\pi}$" high. (If the cake averages 25 calories per cubic inch, how many calories is that?)



A 40° wedge is $^{40}/_{360}$ or $^{1}/_{9}$ of the cake...

The volume of cake is that of a cylinder with base radius 4", height $^{15}/_{\pi}$".

$V = \ ^{1}/_{9}$ (Area of base) (Height) $= \ ^{1}/_{9} \bullet \pi\ (\text{radius})^2$ Height

$= \ ^{1}/_{9} \bullet \pi\ (4\text{ in})^2\ ^{15}/_{\pi}\text{ in}$

$= \ ^{80}/_{3}\text{ in}^3 \ = \ 26\tfrac{2}{3}\text{ in}^3$

(At 25 calories per cubic inch, that's 666.666··· calories. I see why they call it.... chocolate.)

14*. A cylinder was designed to hold 200 mL. If a new cylinder is designed with the diameter doubled, and the height tripled, what is the capacity of the new cylinder?

$V_{cylinder} = \pi\ (\text{Radius})^2$ Height $= \pi\ (R)^2\ H$

Doubling the diameter doubles the radius.

The volume of the new cylinder with double radius and triple height is

$V_{cylinder} = \pi\ (2\text{ Radius})^2\ (3\text{Height})$

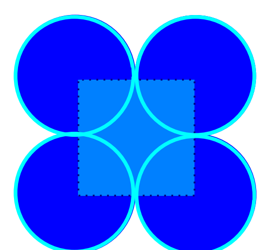
$= \pi\ (2R)^2\ (3H)$

$= \pi\ 4R^2\ 3H$

$= 4 \bullet 3\ \pi\ R^2\ H =$ *twelve times the original volume. Therefore...*

The new cylinder holds 12•200mL = 2400 mL

15. Find the AREA of the figure at right, given it was constructed by pasting four circles of diameter 4cm atop a square of side 4 cm, so that each circle's center is located at a vertex of the square.



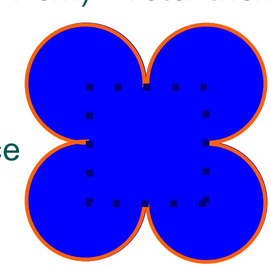
Adding the area of the square to the area of the four circles counts the area where the square overlaps the circles twice. To correct this, *we subtract the areas of the four quarter-circle overlaps*.

Area shaded = Area of square + 4• Area of circle – Area of overlap
= Area of square + 4• Area of circle – 4• ¼ • Area of circle
= Area of square + 3• Area of circle

$= (4\text{ cm})^2 + 3 \bullet \pi\ (2\text{ cm})^2 = (16 + 12\pi)\text{ cm}^2$

Note: There are several other workable methods, including noticing that the shaded region consists of four circular discs, plus the middle section that looks like ✧.The area contained in that section is $(16 - 4\pi)\text{cm}^2$ (see problem #12, replacing 2m to 4 cm). Total then would be $4 \bullet \pi\ (2\text{ cm})^2 + (16 - 4\pi)\text{cm}^2$.

Find the perimeter of the figure.



The perimeter consists of 4 arcs, each arc 3/4 of the circumference of the circle.

$P = 4 \bullet \ ^{3}/_{4} \bullet 2\pi\ (2\text{ cm}) = 12\pi\text{ cm}$