1a. A regular icosahedron has a 60 mL capacity.
If a new regular icosahedron is constructed with every edge twice as long as the original, what is the capacity of the new icosahedron?
If every edge is doubled, then all "lengths" and "heights" and "depths" are doubled, so the volume is doubled, and doubled again, and doubled again. Thus the new icosahedron has a 480 mL capacity.

1b. If the edge of a cube is increased by 2 cm, what is the effect on the surface area of the cube?
This is the "unanswerable one". Actually, it is answerable, however:
The surface area is increased by 6(2)(2cm)(old edge) + 6(2cm)(2cm).
This can be demonstrated algebraically. The surface area of the cube (with side "s") is $6s^2$.
With the edge increased by 2cm, the surface area becomes $(s + 2cm)^2$.
The difference in the surface area is $6(s + 2cm)^2 - 6s^2 = 6(s^2 + 2(2cm)s + (2cm)^2) - 6s^2$

If the edge of a cube is increased by 20%, what is the effect on the surface area of the cube?

Original SA = $6(s^2)$

New SA = $6(1.20s)^2 = 6s^2(1.2)(1.2(1.2) = 6s^2(1.728) = (\text{old SA}) \bullet 1.728$

The surface area is multiplied by the factor 1.728, and thus increases by 72.8%.

(12) 2. Convert each of the following, showing your work.

a. 5.2 hm = ____________ dm

We need to know only that there are 100 meters in 1 hectometer, and 10 decimeters in a meter.

$$5.2\text{ hm} = 5.2\text{ hm} \times \frac{100\text{m}}{1\text{ hm}} \times \frac{10\text{dm}}{1\text{ m}} = 5200\text{ dm}$$

b. 53×10^6 cm^3 = ________ m^3

We need to know only that there are 100 centimeters in 1 meter.

$$53 \times 10^6\text{ cm}^3 = 53 \times 10^6\text{ cm}^3 \times \frac{1\text{m}}{100\text{cm}} \times \frac{1\text{m}}{100\text{cm}} \times \frac{1\text{m}}{100\text{cm}} = \frac{53000000\text{ m}^3}{1000000} = 53\text{ m}^3$$

c. 500 mL water (at 4°C) = ______ kg.

We need to know that 1 mL [is 1 cc or cm^3] and, if water at most dense point, has mass 1 g. And.... we need to know there are 1000 grams in a kilogram.

$$500\text{ mL}_{\text{water@4°C}} = 500\text{ mL}_{\text{water@4°C}} \times \frac{1\text{ cc}}{\text{mL}} \times \frac{1\text{ g}}{1\text{cc}_{\text{water@4°C}}} \times \frac{1\text{kg}}{1000\text{ g}} = ,5\text{ kg}$$

3. How many liters of water are needed to fill a tank 1 meter wide, ½ meter high and ½ meter deep? Show the dimensional analysis that leads to your answer.

Volume of tank is 1m x .5m x .5m = .25m^3

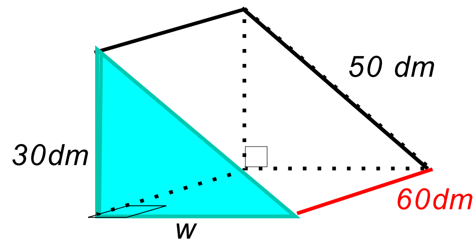
Now to convert to liters: one must know some equivalency between liters and ordinary volume. That might be that 1 cc (1 cm^3) is 1 mL (one cubic centimeter is one milliliter.)

$$.25\text{ m}^3 = .25\text{ m}^3 \times \frac{(100\text{cm})^3}{\text{m}^3} \times \frac{1\text{ mL}}{1\text{cm}^3} \times \frac{1\text{ Liter}}{1000\text{ mL}} = 250\text{ Liters}$$

Or it might be that you know that 1 cubic meter is 1000 liters.

$$.25\text{ m}^3 = .25\text{ m}^3 \times \frac{1000\text{ Liters}}{\text{m}^3} = 250\text{ Liters}$$

4. Find the volume of the prism shown at right.



The BASE of the prism is a right triangle with height 30dm and hypotenuse 50 dm. The third side, which is the base width of the triangle, has length 40 dm.

$$(30dm)^2 + w^2 = (50\ dm)^2$$
$$w^2 = 1600\ dm^2$$

Volume of prism = (Area of Base) (Height)

$= (\frac{1}{2} (40\ dm)(30\ dm))\ (60\ dm)$

$= (600\ dm^2)\ (60\ dm)$

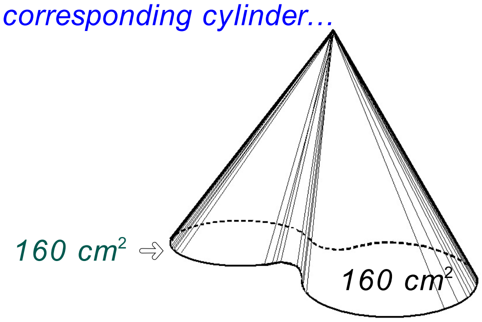
$= 36000\ dm^3$

5. The base of a cone is a 160 cm^2 region enclosed by a simple closed curve.
The base has perimeter 60cm.
The height of the cone is 40 cm.
Find the volume of the cone.

Volume of a cone or pyramid is (1/3) of the volume of the corresponding cylinder...
That makes the volume (1/3) (Area of Base) (Height)

$= (1/3)\ (160\ cm^2)\ (40\ cm)$

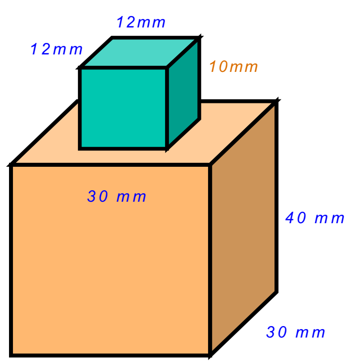
$= \frac{6400\ cm^3}{3}$



(12) 6. *Find the surface area of the solid illustrated below.*
Assume all angles between connected segments are right angles.

We have five faces of the small box on top... and five full faces of the large box, plus the exposed shoulders of the top of the large box. However, we can combine the top of the small box with the exposed shoulders of the top of the large box, to obtain a simple square region, 30 mm by 30 mm, so combined area is 900 mm^2.

Areas of lateral sides of the small box add up to 4 (12mm)(10mm) = 480 mm^2



Areas of lateral sides of the large box add up to
4 (30 mm) (40 mm) = 4800 mm^2

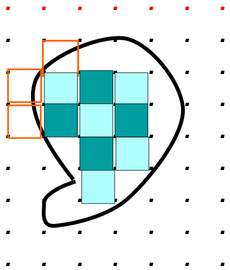
Base of large box (Bottom!) has area (30mm)(30mm) = 900 mm^2

Total Surface area =

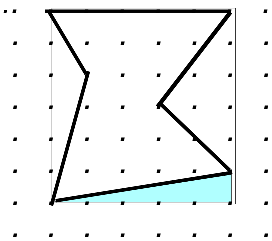
900 mm^2 + 480 mm^2 + 4800 mm^2 + 900 mm^2 =

7080 mm^2

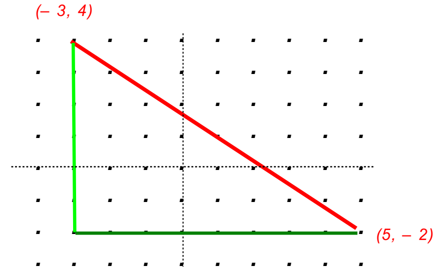
7.



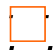
8.



9.



In all the above, the area shown here is one square unit.



7. *Estimate the area of the figure in #7.*

AS SHOWN HERE : there are 9 complete square units enclosed within the curve, and 14 to 16 additional squares needed to cover the partial squares (3 shown) around the border...*

We assume the additional squares average ½ u^2 inside the curve, so the partial squares should total (½)(14 to 16 u^2) = 7 to 8 u^2

Thus the area is approximately 16.5 square units , or 16-17 square units.

** If shown differently, answers may differ.*

8. *Find the area enclosed by the figure in #8.*

The region inside the curve is the region inside the surrounding rectangle, less the three triangular concavities at the bottom and two sides (the one at the bottom is highlighted).

Thus the region has area = 5x6 square units – ((½) 6•1 + (½) 5•1 + (½) 5•2) sq. units

= 30 u^2 — (10.5) u^2

= 19.5 u^2

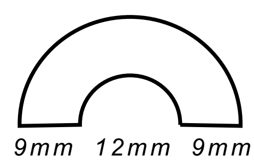
9. *Find the distance between the points (– 3,4) and (5,– 2).*

$6^2 + 8^2 = c^2$

$36 + 64 = c^2$

$100 = c^2$

$c = 10$ *The distance between (– 3,4) and (5,– 2) is 10 units.*



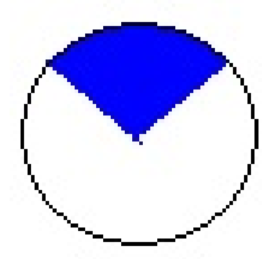
10. *Find the perimeter of the figure at right. given all arcs are semicircular.*

The figure is composed of a larger, outer semicircular arc with radius 15 mm, and a smaller, inner semicircular arc with radius 6 mm, and two segments each 9 mm long.

Total length = (½) (2 π (15 mm)) + (½) (2 π (6 mm)) + 9 mm + 9mm

= (21π + 18) mm

11. Find the area of a 30° sector of a circle with radius 40 cm.



The area enclosed by the entire circle is: $\pi(40cm)^2 = 1600\pi\ cm^2$

So the area enclosed within a 30° sector of that circle is

$$\frac{30}{360} \text{ of } \pi(40cm)^2 = \frac{1}{12} \bullet 1600\pi\ cm^2 \quad \text{or} \quad \frac{400\pi}{3}\ cm^2$$

12. Which of the following is the Volume of a sphere? ________

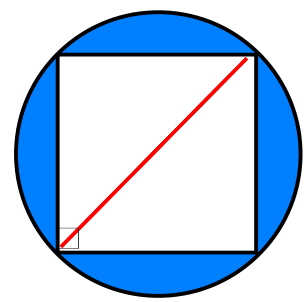
Which of the following is the Surface Area of a sphere? ________

$2\pi r$ πr^2 $2\pi r^2$ $\frac{4}{3}\pi r^2$ $4\pi r^2$ $\frac{4}{3}\pi r^3$ $2\pi r^3$ $4\pi r^3$

$4\pi r^2$: Surface Area of a sphere of radius r.

$\frac{4}{3}\pi r^3$: Volume enclosed by a sphere of radius r

13. Find the area of the shaded region within the circle:



The diameter of the circle is the hypotenuse of an isosceles right triangle with sides of length 2m.

$$(2m)^2 + (2m)^2 = c^2$$

$$8m^2 = c^2$$

$$c = 2\sqrt{2}\ m$$

So the radius of the circle is $\sqrt{2}$ m.

The area of the shaded region is the area of the circle less the area of the square...

area shaded $= \pi(\sqrt{2}\ m)^2 - (2m)^2 = (2\pi - 4)\ m^2$